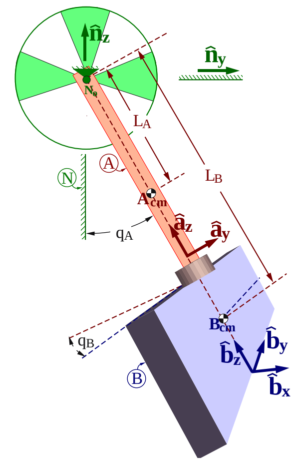
22.6.6 MG road-map: Chaotic motion of a double pendulum



The figure to the right is a schematic representation of a swinging babyboot attached by a shoelace to a rigid support. The mechanical model of the babyboot consists of a thin uniform rod A attached to a fixed support N by a revolute joint at point N_o and a uniform plate B connected to A with a second revolute joint at point B_o so B can rotate freely about A's axis.
Note: The revolute joints' axes are ***perpendicular***, not parallel.

Modeling considerations

- The plate, rod, and support are rigid.
- The revolute joints are ideal (massless, frictionless, no slop/flexibility).
- Earth is a Newtonian reference frame.
- Air resistance is negligible.
- Forces due to Earth's gravitation are uniform and constant.
- Other distance forces (electromagnetic and gravitational) are negligible.

Right-handed sets of unit vectors $\hat{\mathbf{n}}_x, \hat{\mathbf{n}}_y, \hat{\mathbf{n}}_z$; $\hat{\mathbf{a}}_x, \hat{\mathbf{a}}_y, \hat{\mathbf{a}}_z$; $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$ are fixed in N, A, B, respectively, with $\hat{\mathbf{n}}_x = \hat{\mathbf{a}}_x$ parallel to the revolute axis joining A to N, $\hat{\mathbf{n}}_z$ vertically-upward, $\hat{\mathbf{a}}_z = \hat{\mathbf{b}}_z$ parallel to the rod's long axis (and the revolute axis joining B to A), and $\hat{\mathbf{b}}_z$ perpendicular to plate B.

Complete the following ***MG road-map*** for the angle q_A (angle from $\hat{\mathbf{n}}_z$ to $\hat{\mathbf{a}}_z$ with $+\hat{\mathbf{n}}_x$ sense) and the angle q_B (angle from $\hat{\mathbf{a}}_y$ to $\hat{\mathbf{b}}_y$ with $+\hat{\mathbf{a}}_z$ sense). Note: the "about points" are not unique.[3]

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point*	MG road-map equation
q_A	Rotate	$\hat{\mathbf{a}}_x$	A, B	**Draw**	A_o	$\hat{\mathbf{a}}_x \cdot \left(\vec{\mathbf{M}}^{S/N_o} = \frac{{}^N d\, {}^N\vec{\mathbf{H}}^{S/N_o}}{dt} \right)$
q_B	Rotate	$\hat{\mathbf{b}}_z$	B	**Draw**	B_{cm}	$\hat{\mathbf{b}}_z \cdot \left(\vec{\mathbf{M}}^{B/B_{cm}} = \frac{{}^N d\, {}^N\vec{\mathbf{H}}^{B/B_{cm}}}{dt} \right)$
q_A	**Dot(Ax> , System(A,B).GetDynamics(Ao))**					**M**otion**G**enesis command ©
q_B	**Dot(Bz> , B .GetDynamics(Bcm))**					**M**otion**G**enesis command ©

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[3]The shift theorem relates moment about B_{cm} and B_o as $\vec{\mathbf{M}}^{B/B_{cm}} = \vec{\mathbf{M}}^{B/B_o} + \vec{\mathbf{r}}^{\,B_o/B_{cm}} \times \vec{\mathbf{F}}^B$.
Since $\vec{\mathbf{r}}^{\,B_{cm}/B_o}$ is parallel to $\hat{\mathbf{b}}_z$, $\hat{\mathbf{b}}_z \cdot \vec{\mathbf{r}}^{\,B_o/B_{cm}} \times \vec{\mathbf{F}}^B = 0$, so $\hat{\mathbf{b}}_z \cdot \vec{\mathbf{M}}^{B/B_{cm}} = \hat{\mathbf{b}}_z \cdot \vec{\mathbf{M}}^{B/B_o}$.