22.6.7 MG road-map: Instructor on turntable with spinning wheel

The pictures to the right shows a dynamicist standing on a spinning turntable and holding a spinning bicycle wheel.

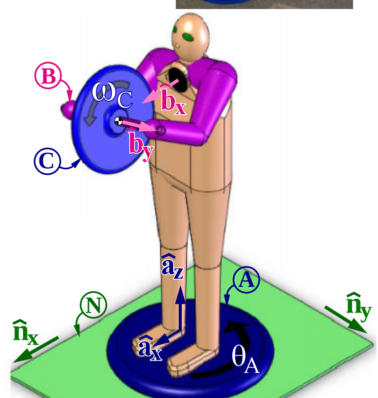
The mechanical model (below right) has a rigid body A (modeling the turntable, legs, torso, and head) that can freely rotate relative to Earth (Newtonian reference frame N) about a vertical axis that is fixed in both A and N and which passes through the center of the turntable (point N_o) and A_{cm} (A's center of mass).

A light (massless) rigid frame B (modeling the shoulders, arms, hands, and a portion of the bicycle wheel's axle) is attached to A by a revolute motor at point B_o of B (B_o lies on the vertical axis passing through A_{cm}). The motor's revolute axis passes through points B_o and C_{cm}, is horizontal, and is parallel to $\widehat{\mathbf{b}}_x = \widehat{\mathbf{a}}_x$.

A rigid bicycle wheel C is attached to B by a frictionless revolute joint whose axis passes through C_{cm} (C's center of mass) and is parallel to $\widehat{\mathbf{b}}_y$.

Right-handed orthogonal unit vectors $\widehat{\mathbf{a}}_x$, $\widehat{\mathbf{a}}_y$, $\widehat{\mathbf{a}}_z$ and $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ are fixed in A and N, respectively. Initially $\widehat{\mathbf{a}}_i = \widehat{\mathbf{n}}_i$ $(i = x, y, z)$, and then rigid body A is subjected to a right-handed rotation characterized by $\theta_A\, \widehat{\mathbf{a}}_z$ where $\widehat{\mathbf{a}}_z = \widehat{\mathbf{n}}_z$ is directed vertically-upward and $\widehat{\mathbf{a}}_x$ points from the instructors back to front (parallel to the axis of the revolute motor connecting A and B).

Unit vectors $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ are fixed in B. Initially $\widehat{\mathbf{b}}_i = \widehat{\mathbf{a}}_i$ $(i = x, y, z)$, and then B is subjected to a right-handed rotation characterized by $\theta_B\, \widehat{\mathbf{a}}_x$ where $\widehat{\mathbf{b}}_x = \widehat{\mathbf{a}}_x$ and $\widehat{\mathbf{b}}_y$ is directed along the wheel's axle from the instructor's right hand to left hand. The dynamicist changes θ_B in a **specified** (known or prescribed) sinusoid manner with amplitude 30° and period 4 seconds.



Courtesy Doug Schwandt
Purchase turntable/bicycle wheel at Arbor-scientific

Quantity	Symbol and type		Value
Mass of C	m^C	Constant	2 kg
Distance between B_o and C_{cm}	L_x	Constant	0.5 m
A's moment of inertia about B_o for $\widehat{\mathbf{a}}_z$	I^A_{zz}	Constant	$0.64\ \mathrm{kg\,m^2}$
C's moment of inertia about C_{cm} for $\widehat{\mathbf{b}}_x$	I^C	Constant	$0.12\ \mathrm{kg\,m^2}$
C's moment of inertia about C_{cm} for $\widehat{\mathbf{b}}_y$	J^C	Constant	$0.24\ \mathrm{kg\,m^2}$
Angle from $\widehat{\mathbf{n}}_x$ to $\widehat{\mathbf{a}}_x$ with $+\widehat{\mathbf{n}}_z$ sense	θ_A	Variable	
Angle from $\widehat{\mathbf{a}}_y$ to $\widehat{\mathbf{b}}_y$ with $+\widehat{\mathbf{a}}_x$ sense	θ_B	**Specified**	$\frac{\pi}{6}\sin(\frac{\pi}{2}\,t)$
$\widehat{\mathbf{b}}_y$ measure of C's angular velocity in B	ω_C	Variable	

Complete the ***MG road-map*** for θ_A and ω_C (the "about points" are not necessarily unique).

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point*	MG road-map equation
θ_A	Rotate	$\widehat{\mathbf{a}}_z$	A, B, C	**Draw**	B_o	$\widehat{\mathbf{a}}_z \cdot (\vec{\mathrm{M}}^{S/B_o} = \frac{^N d\ ^N\vec{\mathrm{H}}^{S/B_o}}{dt})$
ω_C	Rotate	$\widehat{\mathbf{b}}_y$	C	**Draw**	C_{cm}	$\widehat{\mathbf{b}}_y \cdot (\vec{\mathrm{M}}^{C/C_{cm}} = \frac{^N d\ ^N\vec{\mathrm{H}}^{C/C_{cm}}}{dt})$

θ_A	**Dot(Az> , System(A,B,C).GetDynamics(Bo))**	**M**otion**G**enesis command ©
ω_C	**Dot(By> , C .GetDynamics(Ccm))**	**M**otion**G**enesis command ©

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