

18.14 **Dynamicist on a turntable with spinning bicycle wheel (Section 22.6).**

The pictures to the right shows a dynamicist standing on a spinning turntable and holding a spinning bicycle wheel.

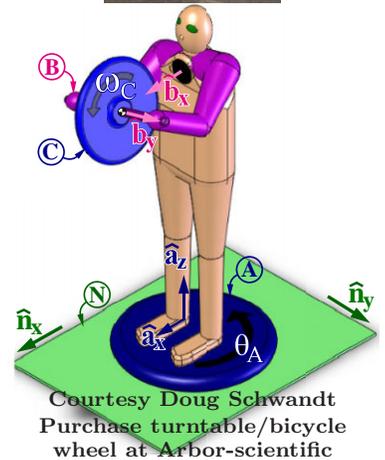
The mechanical model (below right) has a rigid body A (modeling the turntable, legs, torso, and head) that can freely rotate relative to Earth (Newtonian reference frame N) about a vertical axis that is fixed in both A and N and which passes through the center of the turntable (point N_o) and A_{cm} (A 's center of mass).

A light (massless) rigid frame B (modeling the shoulders, arms, hands, and a portion of the bicycle wheel's axle) is attached to A by a revolute motor at point B_o of B (B_o lies on the vertical axis passing through A_{cm}). The motor's revolute axis passes through points B_o and C_{cm} , is horizontal, and is parallel to $\hat{b}_x = \hat{a}_x$.

A rigid bicycle wheel C is attached to B by a frictionless revolute joint whose axis passes through C_{cm} (C 's center of mass) and is parallel to \hat{b}_y .

Right-handed orthogonal unit vectors $\hat{a}_x, \hat{a}_y, \hat{a}_z$ and $\hat{n}_x, \hat{n}_y, \hat{n}_z$ are fixed in A and N , respectively. Initially $\hat{a}_i = \hat{n}_i$ ($i = x, y, z$), and then rigid body A is subjected to a right-handed rotation characterized by $\theta_A \hat{a}_z$ where $\hat{a}_z = \hat{n}_z$ is directed vertically-upward and \hat{a}_x points from the instructors back to front (parallel to the axis of the revolute motor connecting A and B).

Unit vectors $\hat{b}_x, \hat{b}_y, \hat{b}_z$ are fixed in B . Initially $\hat{b}_i = \hat{a}_i$ ($i = x, y, z$), and then B is subjected to a right-handed rotation characterized by $\theta_B \hat{a}_x$ where $\hat{b}_x = \hat{a}_x$ and \hat{b}_y is directed along the wheel's axle from the instructor's right hand to left hand. The dynamicist changes θ_B in a **specified** (known or prescribed) sinusoid manner with amplitude 30° and period 4 seconds.

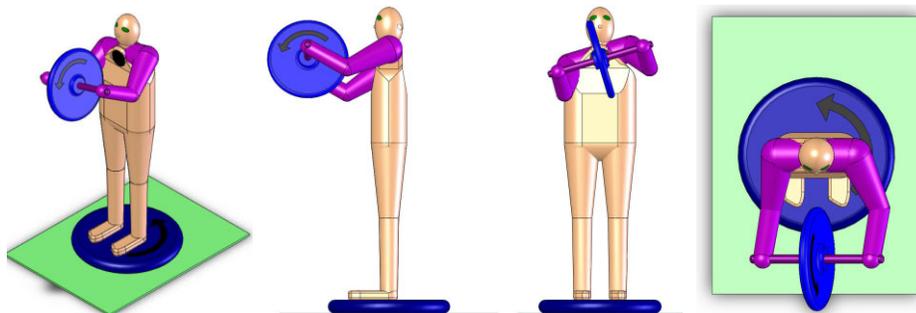


Quantity	Symbol and type		Value
Mass of C	m^C	Constant	2 kg
Distance between B_o and C_{cm}	L_x	Constant	0.5 m
A 's moment of inertia about B_o for \hat{a}_z	I_{zz}^A	Constant	0.64 kg m^2
C 's moment of inertia about C_{cm} for \hat{b}_x	I^C	Constant	0.12 kg m^2
C 's moment of inertia about C_{cm} for \hat{b}_y	J^C	Constant	0.24 kg m^2
Angle from \hat{n}_x to \hat{a}_x with $+\hat{n}_z$ sense	θ_A	Variable	
Angle from \hat{a}_y to \hat{b}_y with $+\hat{a}_x$ sense	θ_B	Specified	$\frac{\pi}{6} \sin(\frac{\pi}{2} t)$
\hat{b}_y measure of C 's angular velocity in B	ω_C	Variable	

Complete the **road map** for finding $\ddot{\theta}_A$ and $\dot{\omega}_C$ (the "about points" are not necessarily unique).

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point*	Road-map equation
θ_A	<input type="text"/>	<input type="text"/>	<input type="text"/>	Draw	<input type="text"/>	<input type="text"/>
ω_C	<input type="text"/>	<input type="text"/>	<input type="text"/>	Draw	<input type="text"/>	<input type="text"/>

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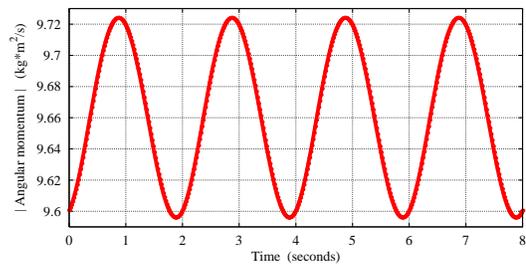
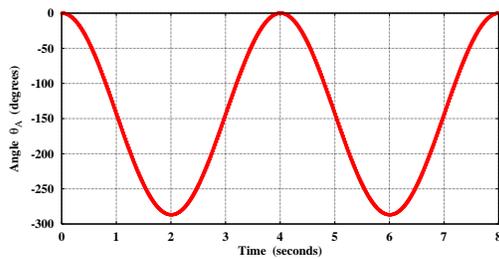
18.15 Angular momentum/conservation for dynamicist on turntable with spinning bicycle wheel.

This problem refers to Homework 18.14.

- (a) Using initial values of $\theta_A = 0^\circ$, $\dot{\theta}_A = 0 \frac{\text{rad}}{\text{sec}}$, and $\omega_C = 40 \frac{\text{rad}}{\text{sec}}$, graph θ_A for $0 \leq t \leq 8$ sec.

Optional: Graph $|\vec{H}^{S/N_o}|$ where S consists of A , B , and C and N_o is 1.2 m below B_o .

Result:



- (b) **Conservation of angular momentum** is frequently used to explain physical phenomena. Decide whether or not various measures of angular momentum are conserved (see Section 22.8). Explain your reasoning for each true/false answer. (Regard S as the system containing A , B , and C).

1	The magnitude of S 's angular momentum about N_o in N is constant.	True/False
2	The \hat{a}_z measure of S 's angular momentum about N_o in N is constant.	True/False
3	The magnitude of C 's angular momentum about C_{cm} in N is constant.	True/False
4*	The \hat{b}_y measure of C 's angular momentum about C_{cm} in N is constant.	True/False
5	The \hat{b}_y measure of C 's angular velocity in N is constant.	True/False
6	The \hat{b}_y measure of C 's angular velocity in B is constant.	True/False

Explain 1:

Explain 2:

Explain 3:

Explain 4:* (Optional) Referring to Section 17.2.3, C is an **axis-symmetric** rigid body for the line parallel to \hat{b}_y that passes through C_{cm} and $\hat{b}_y \cdot \vec{M}^{C/C_{cm}} = 0$. Alternately, q_C does not appear in \mathcal{L} in **Lagrange's equations of the second kind** so $\frac{\partial \mathcal{L}}{\partial \omega_C}$ is constant, and $\hat{b}_y \cdot {}^N \vec{H}^{C/C_{cm}}$ happens to be equal to $\frac{\partial \mathcal{L}}{\partial \omega_C}$.

Explain 5:

Explain 6:

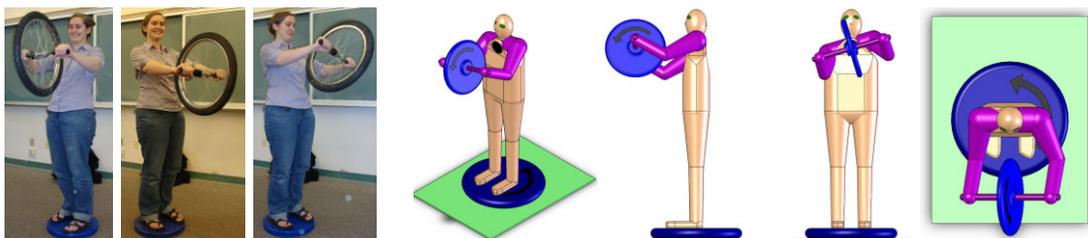
- (c) **A** gravitational potential energy for the system S formed by A , B , C is $U_{\text{gravity}} = 0$. **True/False**. (24.5)

Form the kinetic energy of S in N (in terms of symbols in the previous table and their time derivatives).

The sum $K + U_{\text{gravity}}$ is constant (i.e., mechanical energy is conserved). **True/False**. (See Section 23.4).

If not conserved, explain where work is done.

$$K = \square \dot{\theta}_B^2 + \square \omega_C^2 + \square \dot{\theta}_A \omega_C + \square \dot{\theta}_A^2$$



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