

**18.12 Road-map, FBDs, Lagrange, and simulation: Dynamicist on a turntable**

A dynamics instructor stands on a spinning turntable and swings a heavy dumbbell inward and outward to change his spin-rate (similar to the ice-skater).

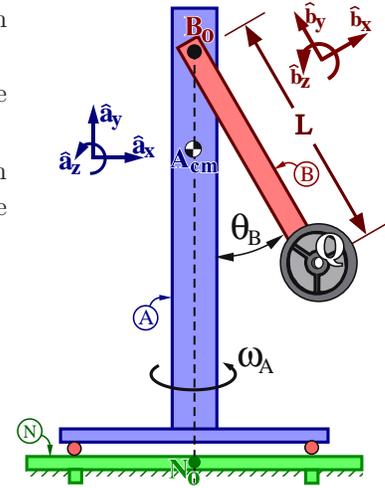
The schematic (below-right) shows a rigid body  $A$  (modeling the instructor's legs, torso, and head) that rotates (without friction) relative to Earth (a Newtonian reference frame  $N$ ) about a vertical axis that is fixed in both  $A$  and  $N$  and which passes through both point  $N_o$  of  $N$  and point  $A_{cm}$  ( $A$ 's center of mass).

A massless rigid arm  $B$  (modeling the instructor's arms and hands) attaches to  $A$  by a revolute motor (shoulder/muscles) whose revolute axis is horizontal and located at point  $B_o$  of  $B$  ( $B_o$  lies on the vertical axis connecting  $N_o$  and  $A_{cm}$ ).

The motor (muscles) **specifies**  $B$ 's angle  $\theta_B$  relative to  $A$  to change in a known (prescribed) manner from 0 to  $\pi$  rad in 4 seconds ( $\theta_B = \pi \frac{t}{4}$ ).

A heavy dumbbell  $Q$  (modeled as a particle) is rigidly attached (welded) to the end of  $B$  (the instructor's hands).

Right-handed orthogonal unit vectors  $\hat{a}_x, \hat{a}_y, \hat{a}_z$  and  $\hat{b}_x, \hat{b}_y, \hat{b}_z$  are fixed in  $A$  and  $B$ , respectively, with  $\hat{a}_y$  vertically-upward,  $\hat{b}_z = \hat{a}_z$  parallel to the revolute motor's axis, and  $\hat{b}_y$  directed from  $Q$  to  $B_o$ .



Quantity	Symbol	Type	Value
Earth's gravitational constant	$g$	Constant	$9.8 \frac{m}{s^2}$
Distance between $Q$ and $B_o$	$L$	Constant	0.7 m
Mass of $Q$	$m$	Constant	12 kg
$A$ 's moment of inertia about line $\overline{A_{cm} B_o}$	$I_{yy}$	Constant	$0.6 \text{ kg m}^2$
Angle from $\hat{a}_y$ to $\hat{b}_y$ with $+\hat{a}_z$ sense	$\theta_B$	<b>Specified</b>	$0.25 \pi t$ rad
$\hat{a}_y$ measure of $A$ 's angular velocity in $N$	$\omega_A$	Variable	

Complete the **road map** for the turntable's "spin-rate"  $\omega_A$  (Note: The "about point" is not unique)

Variable	Translate/ Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point*	Road-map equation
$\omega_A$				<b>Draw</b>		

Student/Instructor version at [www.MotionGenesis.com](http://www.MotionGenesis.com) ⇒ [Textbooks](#) ⇒ [Resources](#)

(a) Use the road map to form  $\dot{\omega}_A = \frac{-2 m L^2 \sin(\theta_B) \cos(\theta_B) \dot{\theta}_B \omega_A}{I_{yy} + m L^2 \sin^2(\theta_B)}$ .

- (b) Form the kinetic energy  $K$  of the system  $S$  formed by  $A, B, Q$  and the partial derivative of  $K$  with respect to  $\omega_A$ . Complete the following modified **Lagrange's equation** associated with  $\omega_A$ . Is  $\dot{\omega}_A$  produced by the road-map equal to  $\dot{\omega}_A$  produced by Lagrange's equation? **Yes/No**

**Result:**

$$K = \text{[Blank]} \quad \frac{\partial K}{\partial \omega_A} = [I_{yy} + m L^2 \sin^2(\theta_B)] \omega_A$$

$$\frac{d}{dt} \frac{\partial K}{\partial \omega_A} = 0 \quad \Rightarrow \quad \text{[Blank]} \omega_A + \text{[Blank]} = 0$$

- (c) Determine the gravitational potential energy of the system  $S$  formed by  $A, B, Q$ . [See equation (24.5)].

**Result:**

$$U_{\text{gravity}} = \text{[Blank]} + \text{Constant}$$

Is  $K + U_{\text{gravity}}$  constant (mechanical energy is conserved)? **Yes/No**. (See Section 23.4).

If not conserved, where is work done? [Blank]

- (d) Consider  $\vec{M}^{\bar{S}/P}$ , (moment of all forces on a generic system  $\bar{S}$  about a point  $P$  fixed in a Newtonian reference frame  $N$ ) and a unit vector  $\hat{u}$  fixed in  $N$ . The **conservation of angular momentum checklist** (Section 22.8) states: If  $\vec{M}^{\bar{S}/P} \cdot \hat{u} = 0$ , then  ${}^N \vec{H}^{\bar{S}/P} \cdot \hat{u} = \text{Constant}$ . Use this to complete the following **blanks**.

System $\bar{S}$	(not unique) Point $P$	$\hat{u}$	Symbolic expression for ${}^N \vec{H}^{\bar{S}/P} \cdot \hat{u}$	Numerical value of Constant
				$3.6 \text{ kg} \frac{\text{m}^2}{\text{s}}$

- (e)  $|{}^N \vec{H}^{\bar{S}/P}|$ , the magnitude of  $\bar{S}$ 's angular momentum about  $P$  in  $N$  is constant. **True/False**
- (f) Use a computer to numerically integrate  $\dot{\omega}_A$  [solve the ODE for  $\omega_A$  with initial value  $\omega_A(t=0) = 6 \frac{\text{rad}}{\text{sec}}$ ].  
Graph  $\omega_A$  for  $0 \leq t \leq 4$  sec. **Solution at [www.MotionGenesis.com](http://www.MotionGenesis.com)  $\Rightarrow$  [Get Started](#)  $\Rightarrow$  [Momentum](#).**

