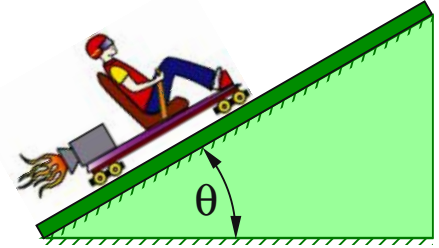


2.10 Vector components, free-body diagram (FBD), and motion graphs for a rocket-sled.

The following figure shows a rocket-sled moving along smooth (**frictionless**) inclined rails.

Description	Symbol	Type
Mass of rocket-sled and rider	m	Constant
Earth's gravitational acceleration	g	Constant
Angle between horizontal and inclined-rails	θ	Constant
\hat{i} measure of thrust force on sled	F_T	Specified
\hat{j} measure of normal force on sled	F_N	Variable
\hat{i} measure of rocket-sled position	x	Variable



Draw a unit vector \hat{i} upward-right and parallel to the rails.

Draw a unit vector \hat{j} outward-normal to the rails (perpendicular to \hat{i}) and in the plane of the paper.

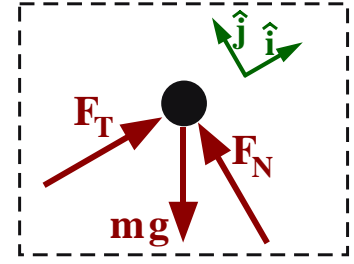
Free-body diagram

Draw a particle representing the rocket-sled.

Draw the thrust, normal, and gravity forces on the rocket-sled.

Express the net force on the rocket-sled in terms of \hat{i} and \hat{j} .

Result: $\vec{F}_{\text{Net}} = \vec{F}_{\text{Thrust}} + \vec{F}_{\text{Normal}} + \vec{F}_{\text{gravity}}$
 $= [F_T - m g \sin(\theta)] \hat{i} + [F_N - m g \cos(\theta)] \hat{j}$



$\vec{F} = m \vec{a}$ The rocket-sled's acceleration can be calculated as $\vec{a} = \frac{d^2 x}{dt^2} \hat{i} = \ddot{x} \hat{i}$.

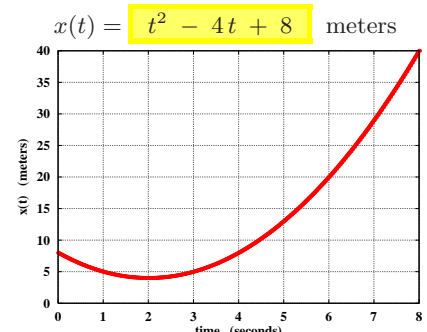
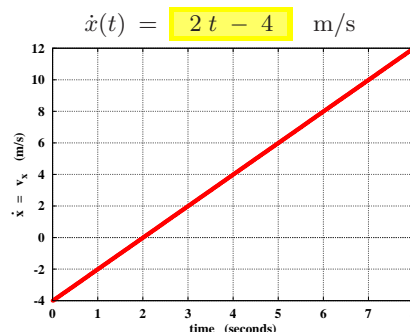
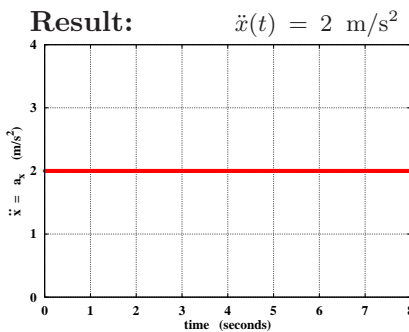
Substitute the right-hand sides of \vec{F}_{Net} and \vec{a} into $\vec{F}_{\text{Net}} = m \vec{a}$ and solve for \ddot{x} and F_N .

Result: $[F_T - m g \sin(\theta)] \hat{i} + [n - m g \cos(\theta)] \hat{j} = m \ddot{x} \hat{i}$

$$\hat{i}: \quad \ddot{x} = \frac{F_T - m g \sin(\theta)}{m} \quad \hat{j}: \quad F_N = m g \cos(\theta)$$

Solve for acceleration: Knowing $m = 100 \text{ kg}$, $g = 10 \frac{\text{m}}{\text{s}^2}$, $\theta = 30^\circ$, $F_T = 700 \text{ N}$, **graph** \ddot{x} .

Solve for velocity and position: Knowing the rocket-sled starts at $x = 8 \text{ m}$, and is initially moving **downward-left** along the rail at $4 \frac{\text{m}}{\text{s}}$, solve and **sketch** $\dot{x}(t)$ and $x(t)$ for $0 \leq t \leq 8$.



Include friction between the rails and rocket-sled, modeled via a coefficient of kinetic friction μ_k . Express \ddot{x} and F_N in terms of some/all of μ_k and symbols in the table. Knowing $\mu_k \approx 0.115$, $x(0) = 8 \text{ m}$, and the rocket-sled initially moves **upward-right** at $4 \frac{\text{m}}{\text{s}}$, find $\dot{x}(t)$ and $x(t)$.

Result: $\ddot{x}(t) = \frac{F_T - m g \sin(\theta) - \mu_k F_N}{m} \approx 1 \frac{\text{m}}{\text{s}^2}$ $F_N = m g \cos(\theta)$ $\dot{x}(t) \approx t + 4$
 $x(t) \approx \frac{1}{2} t^2 + 4t + 8$

Calculate the minimum thrust (redraw FBDs) to:	Result
a. Keep the rocket-sled moving uphill at constant speed	$F_T \approx 600 \text{ N}$
b. Keep the rocket-sled moving downhill at constant speed	$F_T \approx 400 \text{ N}$

Note: Solution at www.MotionGenesis.com \Rightarrow [Get Started](#) \Rightarrow Rocket sled.

