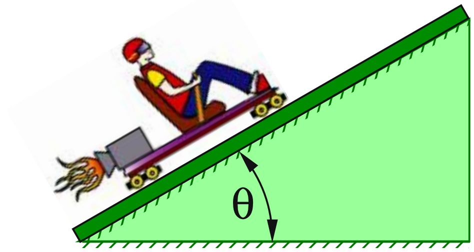
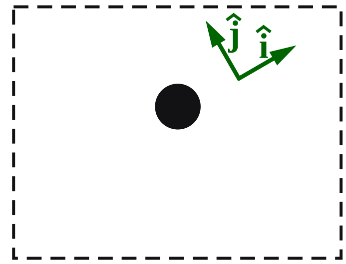
2.10 Vector components, free-body diagram (FBD), and motion graphs for a rocket-sled.

The following figure shows a rocket-sled moving along smooth (**frictionless**) inclined rails.

Description	Symbol	Type
Mass of rocket-sled and rider	m	Constant
Earth's gravitational acceleration	g	Constant
Angle between horizontal and inclined-rails	θ	Constant
$\widehat{\mathbf{i}}$ measure of thrust force on sled	F_T	Specified
$\widehat{\mathbf{j}}$ measure of normal force on sled	F_N	Variable
$\widehat{\mathbf{i}}$ measure of rocket-sled position	x	Variable



Draw a unit vector $\widehat{\mathbf{i}}$ upward-right and parallel to the rails.
Draw a unit vector $\widehat{\mathbf{j}}$ outward-normal to the rails (perpendicular to $\widehat{\mathbf{i}}$) and in the plane of the paper.



Free-body diagram

Draw a particle representing the rocket-sled.
Draw the thrust, normal, and gravity forces on the rocket-sled.
Express the net force on the rocket-sled in terms of $\widehat{\mathbf{i}}$ and $\widehat{\mathbf{j}}$.

Result: $\vec{\mathbf{F}}_{\text{Net}} = \vec{\mathbf{F}}_{\text{Thrust}} + \vec{\mathbf{F}}_{\text{Normal}} + \vec{\mathbf{F}}_{\text{gravity}}$

$= \_\_\_\_\_\_ \; \widehat{\mathbf{i}} + \_\_\_\_\_\_ \; \widehat{\mathbf{j}}$

$\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ The rocket-sled's acceleration can be calculated as $\vec{\mathbf{a}} = \frac{d^2x}{dt^2}\,\widehat{\mathbf{i}} = \ddot{x}\,\widehat{\mathbf{i}}$.

Substitute the right-hand sides of $\vec{\mathbf{F}}_{\text{Net}}$ and $\vec{\mathbf{a}}$ into $\vec{\mathbf{F}}_{\text{Net}} = m\,\vec{\mathbf{a}}$ and solve for $\ddot{x}$ and F_N.

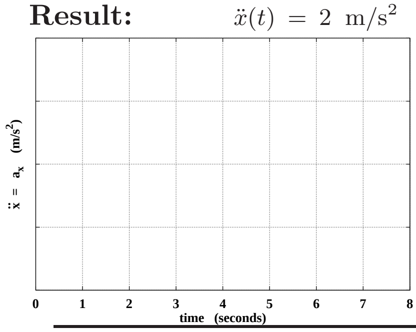
Result: $\_\_\_\_\_\_ \; \widehat{\mathbf{i}} + \_\_\_\_\_\_ \; \widehat{\mathbf{j}} = m\,\ddot{x}\,\widehat{\mathbf{i}}$

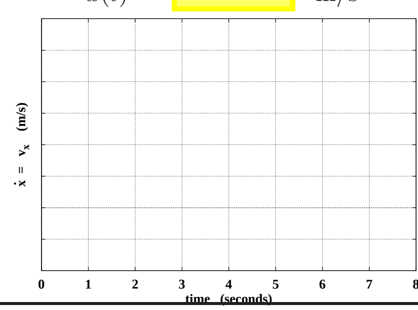
$\widehat{\mathbf{i}}: \quad \ddot{x} = \dfrac{\_\_\_ - \_\_\_\_\_\_}{m} \qquad \widehat{\mathbf{j}}: \quad F_N = \_\_\_\_\_\_$

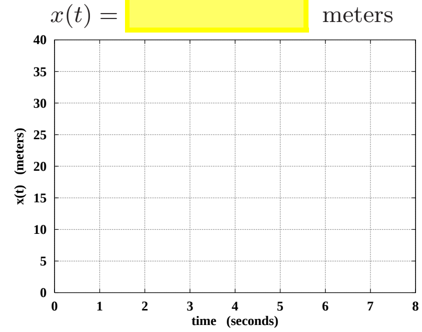
Solve for acceleration: Knowing $m = 100$ kg, $g = 10\ \frac{\text{m}}{\text{s}^2}$, $\theta = 30°$, $F_T = 700$ N, **graph** $\ddot{x}$.

Solve for velocity and position: Knowing the rocket-sled starts at $x = 8$ m, and is initially moving **downward-left** along the rail at $4\ \frac{\text{m}}{\text{s}}$, solve and **sketch** $\dot{x}(t)$ and $x(t)$ for $0 \le t \le 8$.

Result: $\ddot{x}(t) = 2\ \text{m/s}^2$ $\qquad \dot{x}(t) = \_\_\_\_\_\_$ m/s $\qquad x(t) = \_\_\_\_\_\_$ meters





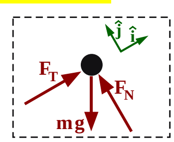


Include friction between the rails and rocket-sled, modeled via a coefficient of kinetic friction μ_k. Express $\ddot{x}$ and F_N in terms of some/all of μ_k and symbols in the table. Knowing $\mu_k \approx 0.115$ $x(0) = 8$ m, and the rocket-sled initially moves **upward-right** at $4\ \frac{\text{m}}{\text{s}}$, find $\dot{x}(t)$ and $x(t)$.

Result:

$\ddot{x}(t) = \dfrac{\_\_\_\_\_\_}{m} \approx 1\ \frac{\text{m}}{\text{s}^2} \qquad F_N = \_\_\_\_\_\_ \qquad \dot{x}(t) \approx \_\_\_ \qquad x(t) \approx \_\_\_\_\_\_$

	Calculate the minimum thrust (redraw FBDs) to:	Result
a.	Keep the rocket-sled moving uphill at constant speed	$F_T \approx$ ___ N
b.	Keep the rocket-sled moving downhill at constant speed	$F_T \approx$ ___ N



Note: Solution at www.MotionGenesis.com ⇒ Get Started ⇒ Rocket sled.