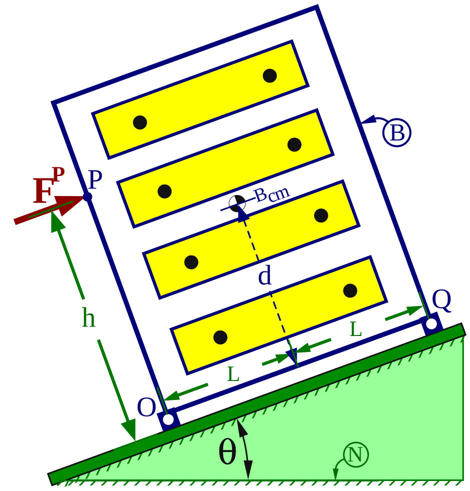
16.8 FE/EIT Review – Static bureau on rough inclined plane (2D analysis, static friction).

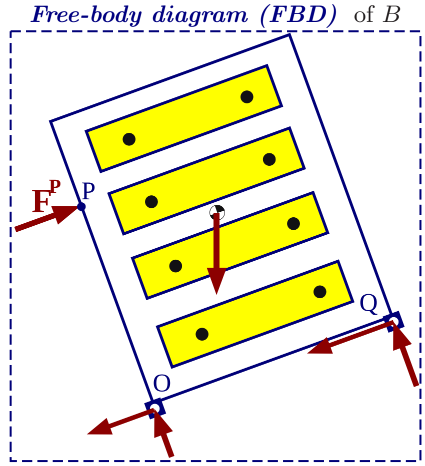
A rigid uniform-density bureau B is in contact with an inclined plane at points O and Q of B.
A person applies a force of magnitude F^P at point P of B (the force is directed up the inclined plane).

Description	Symbol	Value
Angle between inclined plane and local horizontal	θ	30°
Mass of bureau	m	10 kg
Earth's gravitational acceleration	g	10 m/s^2
Half-width of bureau	L	0.2 m
Distance between points O and P	h	0.5 m
Distance between B_{cm} and line $\overline{OQ}$	d	0.3 m
Coefficient of static friction between B and plane	μ_s	
Coefficient of kinetic friction between B and plane	μ_k	
Measure of force on B from person	F^P	**TBD**
Measure of normal force on O from inclined plane	F^O_y	**TBD**
Downhill measure of friction force on O	F^O_x	**TBD**
Measure of normal force on Q from inclined plane	F^Q_y	**TBD**
Downhill measure of friction force on Q	F^Q_x	**TBD**



- **Draw** a ***free-body diagram (FBD)*** of B.
(assume friction prevents B from sliding **up** the inclined plane).

Free-body diagram (FBD) of B



- Form equations governing F^P, F^O_y, F^Q_y, F^O_x, F^Q_x when **static friction** holds B stationary. Put the equations into the matrix form below (introduce unit vectors to facilitate your work).

Result: (in terms of θ, m, g, L, h, d) – **use static equilibrium**.

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ -h & 0 & 2L & 0 & 0 \end{bmatrix} \begin{bmatrix} F^P \\ F^O_y \\ F^Q_y \\ F^O_x \\ F^Q_x \end{bmatrix} = \begin{bmatrix} m\,g\,\sin(\theta) \\ m\,g\,\cos(\theta) \\ m\,g\,[L\cos(\theta) - d\sin(\theta)] \end{bmatrix}$$

- Solve for F^P_{min}, the minimum value of F^P to **start** clockwise tipping of B (point O loses contact).
- Determine h_{tip}, the range of values of h where F^P_{min} makes the bureau start to tip (not slide).

Result:

$$F^P_{\text{min}} = \frac{m\,g\,[d\,\sin(\theta) + L\,\cos(\theta)]}{h} \qquad h_{\text{tip}} > \frac{d\,\sin(\theta) + L\,\cos(\theta)}{\sin(\theta) + \mu_s\,\cos(\theta)}$$

Solution at **www.MotionGenesis.com** ⇒ **Get Started** ⇒ **Simple statics.**