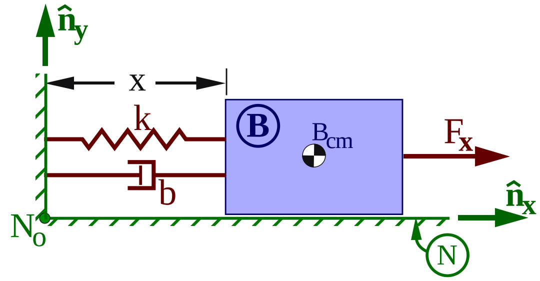
11.18 Stick-slip of a "simple" mass-spring-damper system. $\vec{\mathbf{F}} \;\Rightarrow\; \vec{\mathbf{F}} = m\,\vec{\mathbf{a}} \;\Rightarrow\; \ddot{x},\; F_n$

The following figure shows a light (massless) linear spring/damper, attached to a uniform-density rectangular block B that slides on a rough horizontal table N (a Newtonian reference frame).

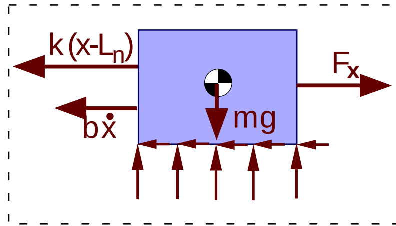
Unit vectors $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{n}}_y$ are fixed in N.
$\hat{\mathbf{n}}_x$ is horizontally-right. $\hat{\mathbf{n}}_y$ is vertically-upward.

Description	Symbol	Value
Mass of block B	m	1 kg
Earth's gravitational acceleration	g	$9.8\ \frac{\text{m}}{\text{s}^2}$
Spring constant	k	$400\ \frac{\text{N}}{\text{m}}$
Natural length of spring	L_n	4 m
Damper constant ($b = 2\,\zeta\,\sqrt{m\,k},\ \ \zeta = 0.1$)	b	$4\ \frac{\text{N s}}{\text{m}}$
Coefficient of friction between B and N	$\mu_k = \mu_s$	0.4
$\hat{\mathbf{n}}_x$ measure of force on B (**specified**)	F_x	$20\cos(t)$
$\hat{\mathbf{n}}_x$ measure of B's position from N_o	x	**Variable**



(a) Draw B's ***free-body diagram*** (showing all the **contact** and **distance** forces).

Express the resultant force on B in terms of F_n and F_f (resultant normal and friction force on B from N) and the force models for linear springs in equation (21.7), linear dampers in equation (21.9), and uniform gravity in equation (21.1).



Result:
$$\vec{\mathbf{F}}^B = \left[F_x \;-\; b\,\dot{x} \;-\; k\,(x - L_n) \;-\; F_f\right]\hat{\mathbf{n}}_x \;+\; \left(F_n - m\,g\right)\hat{\mathbf{n}}_y$$

(b) Dot-multiply $\vec{\mathbf{F}}^B = m^B\,{}^N\vec{\mathbf{a}}^{B_{cm}}$ with $\hat{\mathbf{n}}_x$ and then $\hat{\mathbf{n}}_y$ to form two scalar equations of motion.

Result: $F_x \;-\; b\,\dot{x} \;-\; k\,(x - L_n) \;-\; F_f \;=\; m\,\ddot{x}$ $\qquad F_n \;-\; m\,g \;=\; 0$

(c) Since there are **3** unknowns (F_f, F_n, $\ddot{x}$) in the previous **2** equations, **1** more equation is needed. Write an additional equation valid when B ***slides***[1] on N (i.e., ${}^N\vec{\mathbf{v}}^B \neq \vec{\mathbf{0}}$) and another valid when B is continuously ***sticking*** on N (i.e., ${}^N\vec{\mathbf{v}}^B = \vec{\mathbf{0}}$). Next, form a differential equation for x when B is ***sliding*** and an algebraic equation for F_f when B is ***sticking***.

Result:

	Sliding	***Sticking***
	$F_f \;=\; \mu_k\,F_n\ \text{sign}(\dot{x})$	$\ddot{x} \;=\; 0$
	$m\,\ddot{x} \;+\; b\,\dot{x} \;+\; k\,(x - L_n) \;+\; \mu_k\,m\,g\ \text{sign}(\dot{x}) \;=\; F_x$	$F_f \;=\; F_x \;-\; k\,(x - L_n)$

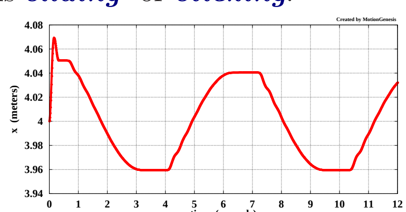
Note: One way to simulate the block's stick/slip motion is to employ if/then switching logic and a numerical integrator that has a root-finder to detect when sticking starts and ends. Usually a third equation is needed to handle the transition from sticking to slipping. The next question shows how to use the ***Continuous Friction Law*** to circumvent these complexities and ensure a smooth transition from sticking to slipping and vice-versa.

(d) A simple effective way to model ***both*** sticking and slipping is with the ***Continuous Friction Law*** in equation (21.6). Use it to form an ODE for x when B is ***sliding*** or ***sticking***.

Result:
$$m\,\ddot{x} \;+\; b\,\dot{x} \;+\; k\,(x - L_n) \;+\; \mu_k\,m\,g\,\frac{\dot{x}}{\text{abs}(\dot{x}) + \epsilon_v} \;=\; F_x$$

Note: The ***Continuous Friction Law*** uses a small positive number ϵ_v, chosen to be much smaller than a characteristic value of $\left|{}^N\vec{\mathbf{v}}^B\right| = \text{abs}(\dot{x})$.

Optional: Plot $x(t)$ for 12 sec with $x(0) = 4$ m, $\dot{x}(0) = 0$.



Simulation tip: To avoid divide by zero and numerical integrator problems, use $\epsilon_v = 1 \times 10^{-5}\ \frac{\text{m}}{\text{s}}$.

Solution at www.MotionGenesis.com ⇒ Get Started ⇒ **Stick-slip**.

[1] The function $\text{sign}(\dot{x})$ is $+1$ when $\dot{x}$ is positive, -1 when $\dot{x}$ is negative, and 0 when $\dot{x} = 0$. If $\dot{x} \neq 0$, $\text{sign}(\dot{x}) = \frac{\dot{x}}{|\dot{x}|}$.