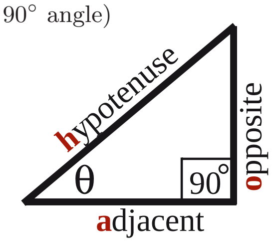
4.1 ♣ (1900 BC). Sine, cosine, tangent as ratios of sides of a right triangle. (Section 1.4)

Below is a ***right triangle*** (triangle with a 90° angle) with one angle labeled as θ. Write definitions for sine, cosine, and tangent in terms of:

- **h**ypotenuse – the triangle's longest side (opposite the 90° angle)
- **o**pposite – the side opposite to θ
- **a**djacent – the side adjacent to θ

I can draw a triangle with a negative-length side	**True/False**
Using the **limited** definition shown right, $\sin(\theta)$ (the sine of an angle) can be negative.	**True/False**



Memorize: ***Soh Cah Toa***

$\sin(\theta) \triangleq \dfrac{\square}{\square}$

$\cos(\theta) \triangleq \dfrac{\square}{\square}$

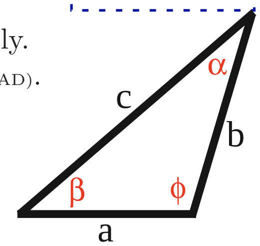
$\tan(\theta) \triangleq \dfrac{\square}{\square}$

4.2 ♣ (1900 BC - 1400 AD) Pythagorean theorem and law of cosines. (Section 1.4.2).

Draw a right-triangle with a hypotenuse of length c and other sides of length a and b. Relate c to a and b with the ***Pythagorean theorem***.

Result: Babylonians 1900 BC to Pythagoreus 525 BC. $\boxed{c^2 = \square + \square}$ ***memorize***

Shown right is a triangle with angles α, β, ϕ opposite sides a, b, c, respectively. Complete each formula below using the ***law of cosines*** (Euclid 300 BC - Al-Kashi 1400 AD).



Result: $\boxed{c^2 = \square + \square - \square}$ ***memorize***

$a^2 = \square + \square - \square$

$b^2 = \square + \square - \square$

The ***Pythagorean theorem*** is a special case of the ***law of cosines***. **True/False**. (circle one).

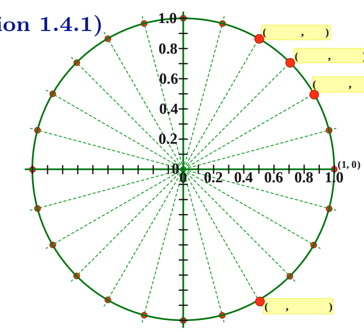
4.3 ♣ (140 BC - 1500 AD) Unit circle concept of sine and cosine. (Section 1.4.1)

Angle θ	$\sin(\theta)$	$\cos(\theta)$
0°		
30°	0.5	$\approx$ 0.866
45°	$\approx$	$\approx$
60°	$\approx$	
90°		
120°	$\approx$	
150°		$\approx$

Label the blanked coordinates on the unit circle to the right.

Note: The unit circle expands the concepts of sine and cosine to negative values and its tabulated values provide data for Euler's graphs.

Note: Negative numbers were invented $\approx$ 650 AD, developed 900 AD – 1200 AD, and widely adopted 1500 AD.



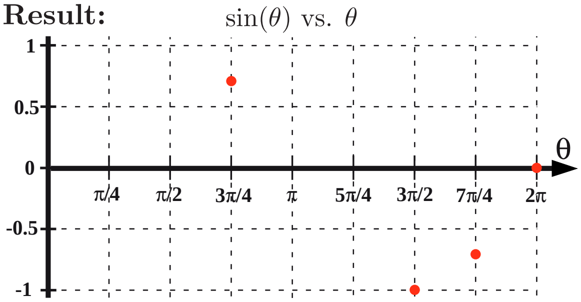
The triangle definition of sine and cosine in Hw 4.1 results in	$0° < \theta < 90°$	$0 < \sin(\theta) < 1$	$0 < \cos(\theta) < 1$
The unit circle extends the range for θ and sine and cosine to	$0° \le \theta \le 360°$	$\square \le \sin(\theta) \le \square$	$\square \le \cos(\theta) \le \square$

4.4 ♣ (Euler 1730 AD) Sine and cosine as functions. (Section 1.4.3)

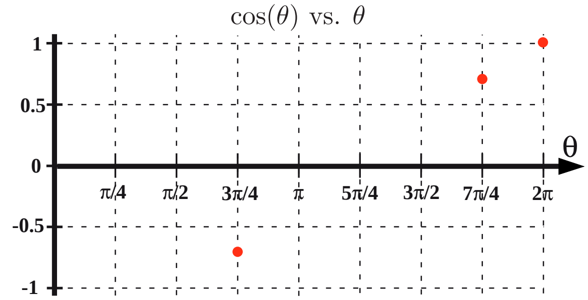
Graph sine and cosine as functions of the angle θ over the range $0 \le \theta \le 2\pi$ radians.
Note: In 1730 A.D., Euler invented the sine and cosine **functions** (more than just ratios of sides of a triangle).

Result:

$\sin(\theta)$ vs. θ



$\cos(\theta)$ vs. θ



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4.5 ♣ Ranges for arguments and return values for inverse trigonometric functions.

Determine all real return values and argument values for the following **real** trigonometric and inverse-trigonometric functions in computer languages such as Java, C^{++}, MATLAB®, **M**otion**G**enesis, …

Range of return values for z	Function	Range of argument values for x	Note
$-1 \le z \le$	$z = \cos(x)$	$< x <$	
$\le z \le$	$z = \sin(x)$	$< x <$	
$-\infty < z < \infty$	$z = \tan(x)$	$-\infty < x < \infty$	$x \ne \frac{\pm\pi}{2}, \frac{\pm 3\pi}{2}, \dots$
$\le z \le$	$z = \text{acos}(x)$	$\le x \le$	
$\le z \le$	$z = \text{asin}(x)$	$\le x \le$	
$-\pi/2 < z < \pi/2$	$z = \text{atan}(x)$	$-\infty < x < \infty$	
$\le z \le$	$z = \texttt{atan2}(y, x)$	$< y <$ $< x <$	$\texttt{atan2}(0, 0)$ is undefined

4.6 ♣ What is an angle? (Section 5.7).

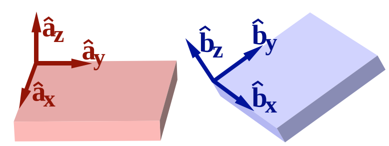
Draw the "geometry equipment" listed in the 1st column of the following table.
Complete the 2nd column with appropriate ranges for the angle θ (in degrees).

"Geometry equipment"	Draw	Appropriate range for θ
2 lines		$0^\circ \le \theta \le$
Vector and line		$\le \theta \le$
2 vectors		$\le \theta \le$
2 vectors and a sense of positive rotation		$< \theta \le$
2 vectors, a sense of $\pm$ rotation, and time-history/continuity	Not applicable	$< \theta <$

4.7 ${}^aR^b$ for dot-products, cross-products, and angles between vectors. (Section 5.4).

The ${}^aR^b$ rotation table relates two sets of right-handed, orthogonal, unit vectors, namely $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ and $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$.

${}^aR^b$	$\hat{\mathbf{b}}_x$	$\hat{\mathbf{b}}_y$	$\hat{\mathbf{b}}_z$
$\hat{\mathbf{a}}_x$	0.962	-0.084	0.259
$\hat{\mathbf{a}}_y$	0.170	0.928	-0.330
$\hat{\mathbf{a}}_z$	-0.212	0.362	0.908



Efficiently determine the following dot-products and angles between vectors (2$^+$ significant digits).
Then perform the calculations involving $\vec{\mathbf{v}}_1 = 2\,\hat{\mathbf{a}}_x$ and $\vec{\mathbf{v}}_2 = \hat{\mathbf{a}}_x + \hat{\mathbf{b}}_x$. **Show work.**

$\hat{\mathbf{a}}_x \cdot \hat{\mathbf{a}}_x =$	$\angle(\hat{\mathbf{a}}_x, \hat{\mathbf{a}}_x) = \quad ^\circ$	$\hat{\mathbf{a}}_y \cdot \hat{\mathbf{a}}_z =$	$\angle(\hat{\mathbf{a}}_y, \hat{\mathbf{a}}_x) = \quad ^\circ$
$\hat{\mathbf{b}}_z \cdot \hat{\mathbf{b}}_y =$	$\angle(\hat{\mathbf{b}}_z, \hat{\mathbf{b}}_y) = \quad ^\circ$	$\hat{\mathbf{a}}_x \cdot \hat{\mathbf{b}}_x =$	$\angle(\hat{\mathbf{a}}_x, \hat{\mathbf{b}}_x) = \quad ^\circ$
$\hat{\mathbf{a}}_x \cdot \hat{\mathbf{b}}_y =$	$\angle(\hat{\mathbf{a}}_x, \hat{\mathbf{b}}_y) = \quad ^\circ$	$\vec{\mathbf{v}}_1 \cdot \vec{\mathbf{v}}_2 \approx$	$\angle(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2) \approx \quad ^\circ$

Result: $\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2 = \quad \hat{\mathbf{b}}_y + \quad \hat{\mathbf{b}}_z = \quad \hat{\mathbf{a}}_y + \quad \hat{\mathbf{a}}_z$

Express the unit vector $\hat{\mathbf{u}}$ in the direction of $3\,\hat{\mathbf{a}}_z + 4\,\hat{\mathbf{b}}_z$ in terms of $\hat{\mathbf{a}}_z$ and $\hat{\mathbf{b}}_z$.
Express $\vec{\mathbf{v}} = \hat{\mathbf{a}}_y + \hat{\mathbf{b}}_y$ in terms of $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$.

Result: $\hat{\mathbf{u}} = \quad \hat{\mathbf{a}}_z + \quad \hat{\mathbf{b}}_z \qquad \vec{\mathbf{v}} = \quad \hat{\mathbf{a}}_x + \quad \hat{\mathbf{a}}_y + \quad \hat{\mathbf{a}}_z$

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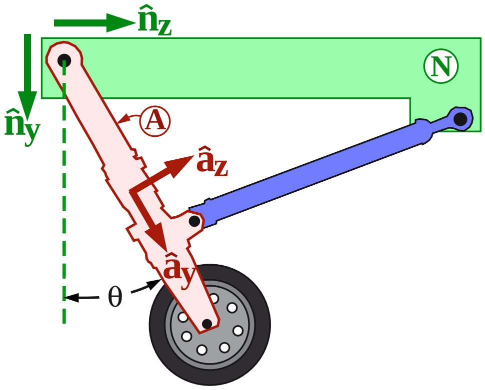
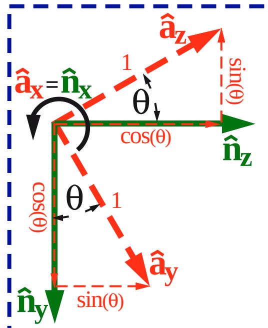
4.8 ♣ SohCahToa: Rotation tables for a landing gear system. (Section 5.5).

The figures below show three versions of the same landing gear system with a strut A that has a simple rotation relative to a fuselage N. Each figure has a different orientation for right-handed orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ (fixed in N) and $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ (fixed in A). **Redraw** $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ and $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ so it is **easy to see a right-triangle** with sines and cosines. Express each of $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ in terms of $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$, then form the ${}^aR^n$ rotation table for each figure.[1] Next, form ${}^N\vec{\boldsymbol{\omega}}^A$ (A's angular velocity in N) in terms of $\dot{\theta}$ and one of $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$.

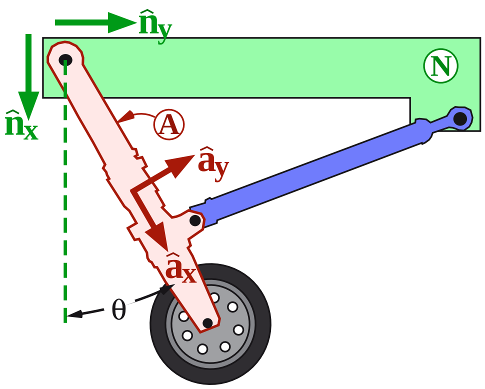


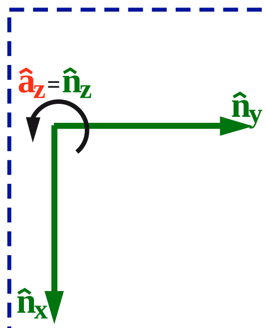


$$\hat{\mathbf{a}}_x = \hat{\mathbf{n}}_x$$
$$\hat{\mathbf{a}}_y = \cos(\theta)\;\hat{\mathbf{n}}_y + \boxed{\quad}\;\hat{\mathbf{n}}_z$$
$$\hat{\mathbf{a}}_z = \boxed{\quad}\;\hat{\mathbf{n}}_y + \cos(\theta)\;\hat{\mathbf{n}}_z$$

${}^aR^n$	$\hat{\mathbf{n}}_x$	$\hat{\mathbf{n}}_y$	$\hat{\mathbf{n}}_z$
$\hat{\mathbf{a}}_x$	1	$\boxed{\quad}$	$\boxed{\quad}$
$\hat{\mathbf{a}}_y$	0	$\cos(\theta)$	$\sin(\theta)$
$\hat{\mathbf{a}}_z$	0	$-\sin(\theta)$	$\cos(\theta)$

$${}^N\vec{\boldsymbol{\omega}}^A = +\dot{\theta}\;\hat{\mathbf{a}}_x$$

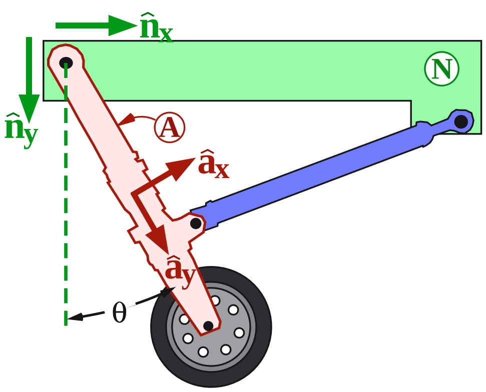




$$\hat{\mathbf{a}}_x = \boxed{\quad}\;\hat{\mathbf{n}}_x + \sin(\theta)\,\hat{\mathbf{n}}_y$$
$$\hat{\mathbf{a}}_y = \boxed{\quad}\;\hat{\mathbf{n}}_x + \cos(\theta)\,\hat{\mathbf{n}}_y$$
$$\hat{\mathbf{a}}_z = \hat{\mathbf{n}}_z$$

${}^aR^n$	$\hat{\mathbf{n}}_x$	$\hat{\mathbf{n}}_y$	$\hat{\mathbf{n}}_z$
$\hat{\mathbf{a}}_x$	$\boxed{\quad}$	$\boxed{\quad}$	$\boxed{\quad}$
$\hat{\mathbf{a}}_y$	$\boxed{\quad}$	$\boxed{\quad}$	$\boxed{\quad}$
$\hat{\mathbf{a}}_z$	$\boxed{\quad}$	$\boxed{\quad}$	$\boxed{\quad}$

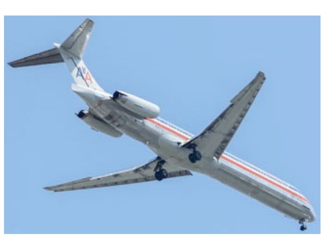
$${}^N\vec{\boldsymbol{\omega}}^A = \boxed{\quad}$$



Show work to express each of $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ in terms of $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$.

${}^aR^n$	$\hat{\mathbf{n}}_x$	$\hat{\mathbf{n}}_y$	$\hat{\mathbf{n}}_z$
$\hat{\mathbf{a}}_x$	$\boxed{\quad}$	$\boxed{\quad}$	$\boxed{\quad}$
$\hat{\mathbf{a}}_y$	$\boxed{\quad}$	$\boxed{\quad}$	$\boxed{\quad}$
$\hat{\mathbf{a}}_z$	$\boxed{\quad}$	$\boxed{\quad}$	$\boxed{\quad}$

$${}^N\vec{\boldsymbol{\omega}}^A = \boxed{\quad}$$



[1]Each figure has two missing vectors (e.g., $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{a}}_x$ are missing from the first figure). Use the fact that each set of vectors is **right-handed** to add the missing vectors to each figure.

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