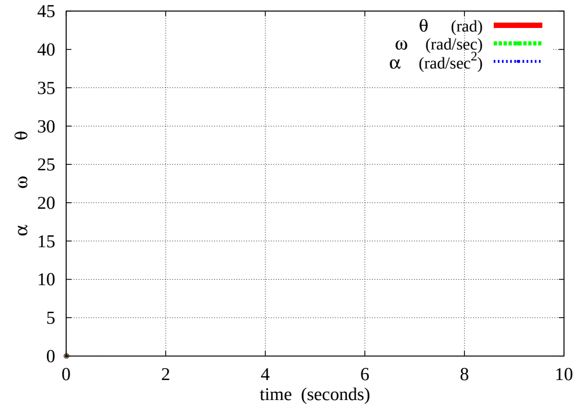
6.1 FE/EIT Review – Motion graph: $\text{T} \Rightarrow \alpha \Rightarrow \omega \Rightarrow \theta$

A wind turbine generates electricity from time-dependent aerodynamic wind forces. The wind creates a torque modeled as $\text{T} = 20 \, \frac{\text{N m}}{\text{sec}} * t$.

Measures of the wind turbine's angular acceleration α, angular velocity ω, and angle θ are related by

$$\text{T} \underset{(\mathbf{2D})}{=} \text{I}\,\alpha \qquad \alpha = \frac{d\omega}{dt} \qquad \omega \underset{(\mathbf{2D})}{=} \frac{d\theta}{dt}$$

where $\text{I} = 80 \text{ kg m}^2$ is the relevant moment of inertia. Graph α in $\frac{\text{rad}}{\text{sec}^2}$, ω in $\frac{\text{rad}}{\text{sec}}$, and θ in rad for $0 \le t \le 10$ sec. Use initial values (i.e, values at $t = 0$) of $\omega = 0$ and $\theta = 0$.



6.2 Drawing a reference frame and unit vector bases. (Section 7.2)

Draw a reference frame or rigid body B, shaped like a uniform-density doughnut (having a hole).
Draw a right-handed orthogonal bases fixed in B having unit vectors $\widehat{\mathbf{b}}_\text{x}$, $\widehat{\mathbf{b}}_\text{y}$, $\widehat{\mathbf{b}}_\text{z}$.
Draw a different right-handed orthogonal bases fixed in B with unit vectors $\widehat{\mathbf{b}}_1$, $\widehat{\mathbf{b}}_2$, $\widehat{\mathbf{b}}_3$.
Draw a properly located center of mass symbol ◐ and label this point as B_cm.
Draw a point B_o fixed on B, at a location different than B_cm.

6.3 ♣ Words and pictures for ${}^bR^a$, ${}^N\vec{\boldsymbol{\omega}}^B$, ${}^N\vec{\boldsymbol{\alpha}}^B$. (Chapters 5 and 7)

${}^bR^a$ – Description (**words**)	${}^N\vec{\boldsymbol{\omega}}^B$ – Description (**words**)	${}^N\vec{\boldsymbol{\alpha}}^B$ – Description (**words**)
Draw b and a	**Draw** B and N	

6.4 ♣ Definitions of angular velocity. (Section 7.3.3).

The definition of angular velocity of $\vec{\boldsymbol{\omega}} \triangleq \dot{\theta}\,\vec{\mathbf{k}}$ is a functional operational definition, i.e., in general, it is useful for calculating angular velocity and proving its properties (2D or 3D). **True/False**

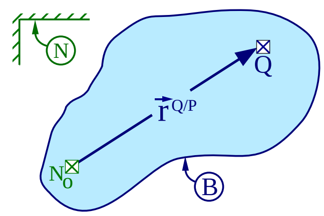
6.5 ♣ Concept: What objects have a unique angular velocity/acceleration? (Sections 7.3, 7.4).

${}^{N}\vec{\boldsymbol{\omega}}^{S}$, the angular velocity of an object S in a reference frame N is to be determined.
In general and **without ambiguity**, S could be a (circle ***all*** appropriate objects):

Real number	Point	Reference Frame	Mass center of a set of particles
Vector	Set of Points	Rigid Body	Mass center of a rigid body
Matrix	Particle	Flexible Body	Set of flexible bodies
3D Orthogonal unit basis	Set of Particles	Set of Rigid bodies	System of particles and bodies

Repeat for ${}^{N}\vec{\boldsymbol{\alpha}}^{S}$, the angular acceleration of an object S in a reference frame N [box appropriate objects].

6.6 ♣ Vector differentiation concepts "$v = \omega\, r$". (Section 7.3).



Point Q is fixed on a rigid body B. Point N_o is fixed in a reference frame N and does not move on B. Complete the following proof that shows how $\vec{\mathbf{v}}$ (Q's velocity in N) can be written in terms of ${}^{N}\vec{\boldsymbol{\omega}}^{B}$ (B's angular velocity in N) and $\vec{\mathbf{r}}$ (Q's position vector from N_o).

Mathematical statement	Reasoning (explain each step in the proof with a brief phrase)
$\vec{\mathbf{v}} \triangleq \dfrac{{}^{N}d\,\vec{\mathbf{r}}}{dt}$	Definition of Q's velocity in N
$= \square + \square$	
$= \vec{\mathbf{0}} + {}^{N}\vec{\boldsymbol{\omega}}^{B} \times \vec{\mathbf{r}}$	

6.7 ♣ Concepts: What objects have a uniquely-defined angular velocity? (Section 7.3).

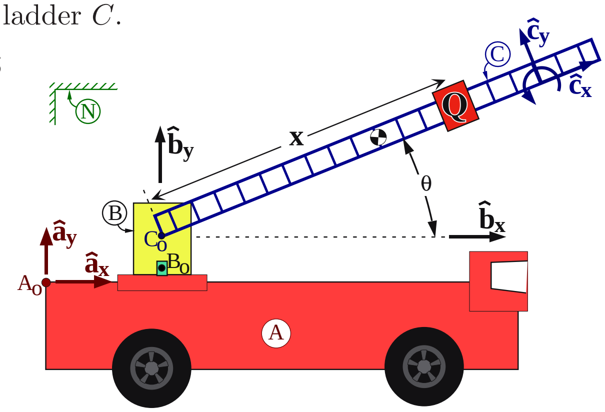
#	For: ${}^{A}\vec{\boldsymbol{\omega}}^{B}$ (B's angular velocity in A)	Object B		Object A	
a.	It is possible to find the angular velocity of a	point	in a	reference frame.	**True/False**
b.	It is possible to find the angular velocity of a	rigid body	in a	particle.	**True/False**
c.	It is possible to find the angular velocity of a	rigid body	in a	reference frame.	**True/False**
d.	It is possible to find the angular velocity of a	reference frame	in a	rigid body.	**True/False**
e.	It is possible to find the angular velocity of a	reference frame	in a	flexible body.	**True/False**
f.	It is possible to find the angular velocity of a	flexible body	in a	reference frame.	**True/False**

6.8 ♣ Rotational kinematics of a fire ladder. (Sections 7.3.3, 7.3.5, 7.3.6).

The following figure shows a fire truck chassis A traveling at constant speed in straight-line motion on Earth (A does not rotate relative to Earth). Earth is a ***Newtonian reference frame*** N.
A rigid hub B is connected to fire truck A by a revolute motor at point B_o of B.
A rigid ladder C is connected to hub B by a revolute motor at point C_o of C .
A fire-fighter Q (modeled as a particle of mass m) climbs ladder C.

Right-handed orthogonal unit vectors $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$; $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$; $\hat{\mathbf{c}}_x$, $\hat{\mathbf{c}}_y$, $\hat{\mathbf{c}}_z$; are fixed in A, B, C, with:

- $\hat{\mathbf{a}}_x$ pointing forward on the fire truck
- $\hat{\mathbf{a}}_y$ vertically-upward and from B_o to C_o
- $\hat{\mathbf{b}}_y = \hat{\mathbf{a}}_y$ parallel to the axis of the revolute motor that connects B and A
- $\hat{\mathbf{b}}_z = \hat{\mathbf{c}}_z$ parallel to the axis of the revolute motor that connects B and C
- $\hat{\mathbf{c}}_x$ directed from C_o to Q (along C's long axis)



Note: **Visualize** C's "**Body yz**" (or "**Space zy**") rotation sequence in N (e.g., with a ruler).

Quantity	Symbol	Type
$\widehat{\mathbf{b}}_{\rm y}$ measure of B's angular velocity in A	ω_B	Constant
Angle from $\widehat{\mathbf{b}}_{\rm x}$ to $\widehat{\mathbf{c}}_{\rm x}$ with $^{+}\widehat{\mathbf{c}}_{\rm z}$ sense	θ	Variable

${}^{c}R^{b}$			

(a) Complete the previous ${}^{c}R^{b}$ rotation table (to the right).
Note: ${}^{c}R^{b}$ is unnecessary for the remainder of this problem.

(b) Clarify the process to determine ${}^{B}\vec{\boldsymbol{\omega}}^{C}$, then express it in terms of $\widehat{\mathbf{b}}_{\rm x}$, $\widehat{\mathbf{b}}_{\rm y}$, $\widehat{\mathbf{b}}_{\rm z}$. **(Section 7.3.3)**.

- C's angular velocity in B is **simple** since ____ is fixed in **both** ____ and ____.
- ____ is the time-derivative of the angle between ____ and ____.
- The sign ($\pm$) was determined using the ____-hand rule (sweep from ____ to ____).
- ${}^{B}\vec{\boldsymbol{\omega}}^{C} =$ ____

(c) B's angular velocity in A is known to be a **simple** angular velocity of ${}^{A}\vec{\boldsymbol{\omega}}^{B} = \omega_B\,\widehat{\mathbf{b}}_{\rm y}$ because $\widehat{\mathbf{b}}_{\rm y}$ is a vector fixed in **both** ____ and ____.

(d) Form C's angular velocity in N and express it in terms of $\widehat{\mathbf{b}}_{\rm x}$, $\widehat{\mathbf{b}}_{\rm y}$, $\widehat{\mathbf{b}}_{\rm z}$.
Result: $${}^{N}\vec{\boldsymbol{\omega}}^{C} \underset{(7.4)}{=} {}^{\square}\vec{\boldsymbol{\omega}}^{\square} + {}^{\square}\vec{\boldsymbol{\omega}}^{\square} + {}^{\square}\vec{\boldsymbol{\omega}}^{\square} = \vec{\mathbf{0}} + \square\,\widehat{\mathbf{b}}_{\rm y} + \square\,\widehat{\mathbf{b}}_{\rm z}$$

(e) When both ω_B and $\dot{\theta}$ are ***constant***, ${}^{N}\vec{\boldsymbol{\alpha}}^{C} = \vec{\mathbf{0}}$. **True/False**.

(f) Write the definition for C's angular acceleration in N and form ${}^{N}\vec{\boldsymbol{\alpha}}^{C}$. **(Sections 7.4, 7.3)**.
Result: $${}^{N}\vec{\boldsymbol{\alpha}}^{C} \underset{(7.8)}{\triangleq} \square \qquad {}^{N}\vec{\boldsymbol{\alpha}}^{C} \underset{(7.1)}{=} \omega_B\,\dot{\theta}\,\widehat{\mathbf{b}}_{\rm x} + \ddot{\theta}\,\widehat{\mathbf{b}}_{\rm z}$$

6.9 ♣ Theorems: Rotation matrices R, angular velocity $\vec{\boldsymbol{\omega}}$, angular acceleration $\vec{\boldsymbol{\alpha}}$? (Section 7.4).

Determine whether or not each theorem to the right is valid for general 3D motion of reference frames A, B, C, and D.

Theorem	True or false
${}^{a}R^{d} = {}^{a}R^{b} * {}^{b}R^{c} * {}^{c}R^{d}$	**True/False**
${}^{A}\vec{\boldsymbol{\omega}}^{D} = {}^{A}\vec{\boldsymbol{\omega}}^{B} + {}^{B}\vec{\boldsymbol{\omega}}^{C} + {}^{C}\vec{\boldsymbol{\omega}}^{D}$	**True/False**
${}^{A}\vec{\boldsymbol{\alpha}}^{D} = {}^{A}\vec{\boldsymbol{\alpha}}^{B} + {}^{B}\vec{\boldsymbol{\alpha}}^{C} + {}^{C}\vec{\boldsymbol{\alpha}}^{D}$	**True/False**

6.10 Alternate formula for angular acceleration. (Section 7.3).

Prove ${}^{N}\vec{\boldsymbol{\alpha}}^{B} \triangleq \dfrac{{}^{N}d\ {}^{N}\vec{\boldsymbol{\omega}}^{B}}{dt}$ can also be calculated as ${}^{N}\vec{\boldsymbol{\alpha}}^{B} = \dfrac{{}^{B}d\ {}^{N}\vec{\boldsymbol{\omega}}^{B}}{dt}$.

6.11 ♣ Concepts: Angular acceleration for general 3D motion. (Sections 7.3, 7.4).

Determine whether or not each of the following equations generally apply to the angular acceleration $\vec{\boldsymbol{\alpha}}$ of reference frames A, B, and C in general 3D motion.

${}^{A}\vec{\boldsymbol{\alpha}}^{B} = \dfrac{{}^{A}d\ {}^{A}\vec{\boldsymbol{\omega}}^{B}}{dt}$	**True/False**	${}^{A}\vec{\boldsymbol{\alpha}}^{C} = {}^{A}\vec{\boldsymbol{\alpha}}^{B} + {}^{B}\vec{\boldsymbol{\alpha}}^{C} + {}^{A}\vec{\boldsymbol{\omega}}^{B} \times {}^{B}\vec{\boldsymbol{\omega}}^{C}$	**True/False**
${}^{A}\vec{\boldsymbol{\alpha}}^{B} = \dfrac{{}^{A}d\ {}^{B}\vec{\boldsymbol{\omega}}^{A}}{dt}$	**True/False**	${}^{A}\vec{\boldsymbol{\alpha}}^{B} = -\dfrac{{}^{A}d\ {}^{B}\vec{\boldsymbol{\omega}}^{A}}{dt}$	**True/False**
${}^{A}\vec{\boldsymbol{\alpha}}^{B} = \dfrac{{}^{C}d\ {}^{A}\vec{\boldsymbol{\omega}}^{B}}{dt}$	**True/False**	${}^{A}\vec{\boldsymbol{\alpha}}^{B} = \dfrac{{}^{C}d\ {}^{A}\vec{\boldsymbol{\omega}}^{B}}{dt} + {}^{A}\vec{\boldsymbol{\omega}}^{C} \times {}^{A}\vec{\boldsymbol{\omega}}^{B}$	**True/False**
${}^{A}\vec{\boldsymbol{\alpha}}^{B} = \dfrac{{}^{B}d\ {}^{A}\vec{\boldsymbol{\omega}}^{B}}{dt}$	**True/False**	${}^{A}\vec{\boldsymbol{\alpha}}^{B} = \dfrac{{}^{C}d\ {}^{A}\vec{\boldsymbol{\omega}}^{B}}{dt} + {}^{B}\vec{\boldsymbol{\omega}}^{C} \times {}^{A}\vec{\boldsymbol{\omega}}^{B}$	**True/False**
${}^{A}\vec{\boldsymbol{\alpha}}^{B} = {}^{B}\vec{\boldsymbol{\alpha}}^{A}$	**True/False**	${}^{A}\vec{\boldsymbol{\alpha}}^{B} = -{}^{B}\vec{\boldsymbol{\alpha}}^{A}$	**True/False**