Show work – except for ♣ fill-in-blanks.

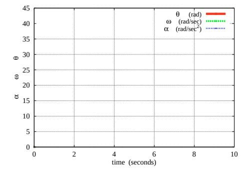
6.1 ♣ FE/EIT – Motion graph: $\mathbf{T} \underset{(2D)}{=} \mathbf{I}\,\alpha \qquad \alpha = \frac{d\omega}{dt} \qquad \omega \underset{(2D)}{=} \frac{d\theta}{dt}$

A wind turbine generates electricity from a torque (created by wind forces) modeled as $\mathrm{T} = 20 \, \frac{\mathrm{N\,m}}{\mathrm{sec}} * t$. Equations relating the wind turbine's angular acceleration α, angular velocity ω, and angle θ are shown above, where moment of inertia $\mathrm{I} = 80 \mathrm{~kg\,m^2}$. Initially, (at $t = 0$), $\omega = 0$ and $\theta = 0$.

Determine $\alpha(t)$, $\omega(t)$, $\theta(t)$.

Result:

$\alpha(t) = \frac{\quad}{\quad} \qquad \omega(t) = \frac{\quad}{\quad} \qquad \theta(t) = \frac{\quad}{\quad}$



Optional: Graph $\alpha(t)$, $\omega(t)$, and $\theta(t)$ for $0 \le t \le 10$ sec.

6.2 ♣ Draw a rigid body, center of mass, vector basis, and rigid frame. (Chapter 7)

Draw a ***rigid body*** B, shaped like a uniform-density doughnut (with a hole).

Draw a properly located ***center of mass*** symbol and label this point B_{cm}.

Draw a right-handed orthogonal ***basis*** fixed in B with unit vectors $\widehat{\mathbf{b}}_1$, $\widehat{\mathbf{b}}_2$, $\widehat{\mathbf{b}}_3$.

Draw a ***rigid frame*** with ***origin point*** B_{o} (at a location different than B_{cm}) and a right-handed orthogonal ***basis*** fixed in B with unit vectors $\widehat{\mathbf{b}}_{\mathrm{x}}$, $\widehat{\mathbf{b}}_{\mathrm{y}}$, $\widehat{\mathbf{b}}_{\mathrm{z}}$.

6.3 ♣ Words and pictures for ${}^{b}R^{a}$, ${}^{N}\vec{\boldsymbol{\omega}}^{B}$, ${}^{N}\vec{\boldsymbol{\alpha}}^{B}$. (Chapters 5 and 7)

Complete each blank below with a phrase to the right.

3D orthonormal basis	rigid body	reference frame
rotation matrix	angular velocity	angular acceleration

${}^{a}R^{b}$ R denotes ______. a is a ______. b is a ______.	${}^{N}\vec{\boldsymbol{\omega}}^{B}$ $\vec{\boldsymbol{\omega}}$ denotes ______. ${}^{N}\vec{\boldsymbol{\alpha}}^{B}$ $\vec{\boldsymbol{\alpha}}$ denotes ______. N is a ______ or ______ or ______. B is a ______ or ______ or ______.
Draw a and b.	**Draw** N and B.

6.4 ♣ Definition of angular velocity? (Section 7.3.3).

A popular definition of angular velocity is $\vec{\boldsymbol{\omega}} \triangleq \dot{\theta}\,\vec{\mathbf{k}}$. This definition is generally useful for calculating angular velocity and proving its properties (for both 2D and/or 3D analysis). **True/False**

6.5 ♣ Optional: Textbook/Internet definition of 3D angular velocity (Section 7.3).

Renowned dynamicist Thomas Kane said angular velocity is one of the "**most misunderstood concepts in kinematics**." Report a definition of angular velocity and determine if the quantities appearing in the definition are **rigorously defined** - and whether the definition is generally applicable for 3D kinematics or only applies for simple angular velocity (e.g., $\vec{\boldsymbol{\omega}} = \dot{\theta}\,\widehat{\mathbf{k}}$ described in Section 7.3.3).

Note: A definition should be able to **prove** important theorems [such as the angular velocity addition theorem of equation (7.4) and the golden rule for vector differentiation in equation (7.1)] and allow for angular velocity **calculations**.

Source (reference). List textbook or .html link	Definition. Report the defining equation/property.	Rigorously defined?	Works for 3D kinematics?
		Yes/No	**Yes/No**

6.6 ♣ **Angular velocity of a Ferris-wheel seat** **(courtesy of David Levinson).**

Known: The rigid seat on a Ferris wheel does not change its orientation relative to ground as the Ferris wheel rotates.
Decide: The seat's angular velocity $\vec{\boldsymbol{\omega}}$ relative to ground (circle one):

Is zero	Is constant	Varies	Does not exist
$\vec{\boldsymbol{\omega}} = \vec{\mathbf{0}}$	$\vec{\boldsymbol{\omega}} = \overrightarrow{\textbf{Constant}} \neq \vec{\mathbf{0}}$	$\vec{\boldsymbol{\omega}} = \vec{\boldsymbol{\omega}}(t)$	

6.7 ♣ **Concept: What objects have a unique angular velocity/acceleration?** **(Sections 7.3, 7.4).**
The angular velocity $\vec{\boldsymbol{\omega}}$ of some object S relative to Earth is to be determined.
This object S could be a (circle ***all*** objects that have an ***unambiguously*** defined angular velocity $\vec{\boldsymbol{\omega}}$):

Real number	Matrix	Set of points	Mass center of a rigid body
Vector	Point	Reference frame	Flexible body
3D orthogonal unit basis	Particle	3D rigid body	System of particles and bodies

Repeat for the angular acceleration $\vec{\boldsymbol{\alpha}}$ of some object S relative to Earth $\boxed{\text{box appropriate objects}}$.

6.8 ♣ **Concepts: What objects have a uniquely-defined angular velocity?** **(Section 7.3).**

a.	The angular velocity of a	point	in a	reference frame	is well defined.	**True/False**
b.	The angular velocity of a	3D rigid body	in a	particle	is well defined.	**True/False**
c.	The angular velocity of a	3D rigid body	in a	reference frame	is well defined.	**True/False**
d.	The angular velocity of a	reference frame	in a	3D rigid body	is well defined.	**True/False**
d.	The angular velocity of a	3D rigid basis	in a	reference frame	is well defined.	**True/False**
e.	The angular velocity of a	reference frame	in a	flexible body	is well defined.	**True/False**
f.	The angular velocity of a	flexible body	in a	reference frame	is well defined.	**True/False**

6.9 ♣ **Memorize the "golden rule for vector differentiation".** **(Section 7.3).**
The angular velocity ${}^A\vec{\boldsymbol{\omega}}^B$ characterizes the time-rate of change of orientation of a reference frame (or rigid vector basis) B in a reference frame (or rigid vector basis) A.

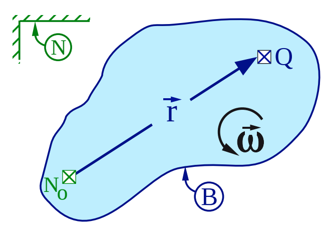
Complete the defining property of ${}^A\vec{\boldsymbol{\omega}}^B$ shown right which relates time-derivatives of **any vector** $\vec{\mathbf{v}}$ in A and B.

★ **Very important formula** ★

$$\frac{{}^A d\vec{\mathbf{v}}}{dt} \underset{(7.1)}{=} \frac{{}^B d\vec{\mathbf{v}}}{dt} + \square \times \square$$

6.10 ♣ **Vector differentiation concepts "$v = \omega\, r$".** **(Section 7.3).**
Point Q is fixed on a rigid body B. Point N_o is fixed in a reference frame N and does not move on B. Complete the following proof that shows how $\vec{\mathbf{v}}$ (Q's velocity in N) can be written in terms of ${}^N\vec{\boldsymbol{\omega}}^B$ (B's angular velocity in N) and $\vec{\mathbf{r}}$ (Q's position from N_o).

Mathematical statement	Reasoning (explain each step below with a brief phrase)
$\vec{\mathbf{v}} \triangleq \frac{{}^N d\,\vec{\mathbf{r}}}{dt}$	Definition of Q's velocity in N
$= \square + \square$	
$= \vec{\mathbf{0}} + {}^N\vec{\boldsymbol{\omega}}^B \times \vec{\mathbf{r}}$	



6.11 ♣ **Rotational kinematics of a fire ladder.** **(Sections 7.3.3, 7.3.5, 7.3.7).**
The following figure shows a fire truck chassis A traveling at constant speed in straight-line motion on Earth (A does not rotate relative to Earth). Earth is a ***Newtonian reference frame*** N.
A rigid hub B is connected to fire truck A by a revolute motor at point B_o of B.
A rigid ladder C is connected to hub B by a revolute motor at point C_o of C .
A fire-fighter Q (modeled as a particle of mass m) climbs ladder C.