6.1 ♣ FE/EIT – Motion graph:

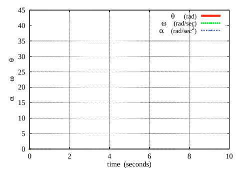
$\mathbf{T} \underset{(2D)}{=} \mathbf{I}\,\alpha \qquad \alpha = \frac{d\omega}{dt} \qquad \omega \underset{(2D)}{=} \frac{d\theta}{dt}$

A wind turbine generates electricity from a torque (created by wind forces) modeled as $\mathrm{T} = 20\ \frac{\mathrm{N\,m}}{\mathrm{sec}} * t$. Equations relating the wind turbine's angular acceleration α, angular velocity ω, and angle θ are shown above, where moment of inertia $\mathrm{I} = 80\ \mathrm{kg\,m^2}$. Initially, (at $t = 0$), $\omega = 0$ and $\theta = 0$.

Determine $\alpha(t)$, $\omega(t)$, $\theta(t)$.

Result:

$\alpha(t) = \frac{\square}{\square} \qquad \omega(t) = \frac{\square}{\square} \qquad \theta(t) = \frac{\square}{\square}$



Optional: Graph $\alpha(t)$, $\omega(t)$, and $\theta(t)$ for $0 \le t \le 10$ sec.

6.2 ♣ Draw a rigid body, center of mass, vector basis, and rigid frame. (Section 8.2.3)

Draw a ***rigid body*** B, shaped like a uniform-density doughnut (with a hole).

Draw a properly located ***center of mass*** symbol ◐ and label this point B_{cm}.

Draw a right-handed orthogonal ***basis*** fixed in B with unit vectors $\widehat{\mathbf{b}}_1$, $\widehat{\mathbf{b}}_2$, $\widehat{\mathbf{b}}_3$.

Draw a ***rigid frame*** with ***origin point*** B_{o} (at a location different than B_{cm}) and a right-handed orthogonal ***basis*** fixed in B with unit vectors $\widehat{\mathbf{b}}_{\mathrm{x}}$, $\widehat{\mathbf{b}}_{\mathrm{y}}$, $\widehat{\mathbf{b}}_{\mathrm{z}}$.

6.3 ♣ Words and pictures for ${}^bR^a$, ${}^N\vec{\boldsymbol{\omega}}^B$, ${}^N\vec{\boldsymbol{\alpha}}^B$. (Chapters 5 and 8)

Complete each blank below with a phrase to the right.

3D orthonormal basis	rigid body	reference frame
rotation matrix	angular velocity	angular acceleration

${}^aR^b$ R denotes ______. a is a ______. b is a ______.	${}^N\vec{\boldsymbol{\omega}}^B$ $\vec{\boldsymbol{\omega}}$ denotes ______. ${}^N\vec{\boldsymbol{\alpha}}^B$ $\vec{\boldsymbol{\alpha}}$ denotes ______. N is a ______ or ______ or ______. B is a ______ or ______ or ______.
Draw a and b.	**Draw** N and B.

6.4 ♣ Definition of angular velocity? (Section 8.3.3).

A popular definition of angular velocity is $\vec{\boldsymbol{\omega}} \triangleq \dot{\theta}\,\vec{\mathbf{k}}$. This definition is generally useful for calculating angular velocity and proving its properties (for both 2D and/or 3D analysis). **True/False**

6.5 ♣ Textbook/Internet definition of 3D angular velocity (Section 8.3).

Renowned dynamicist Thomas Kane said angular velocity is one of the "**most misunderstood concepts in kinematics**." Report a definition of angular velocity and determine if the quantities appearing in the definition are **rigorously defined** - and whether the definition is generally applicable for 3D kinematics or only applies for simple angular velocity (e.g., $\vec{\boldsymbol{\omega}} = \dot{\theta}\,\vec{\mathbf{k}}$ described in Section 8.3.3).

Note: A definition should be able to **prove** important theorems [such as the angular velocity addition theorem of equation (8.4) and the golden rule for vector differentiation in equation (8.1)] and allow for angular velocity **calculations**.

Source (reference). List textbook or .html link	Definition. Report the defining equation/property.	Rigorously defined?	Works for 3D kinematics?
		Yes/No	**Yes/No**

6.6 ♣ Angular velocity of a Ferris-wheel seat (courtesy of David Levinson).

Known: The rigid seat on a Ferris wheel does not change its orientation relative to ground as the Ferris wheel rotates.
Decide: The seat's angular velocity $\vec{\boldsymbol{\omega}}$ relative to ground (circle one):

Is zero $\vec{\boldsymbol{\omega}} = \vec{\mathbf{0}}$	Is constant $\vec{\boldsymbol{\omega}} = \vec{\textbf{Constant}} \neq \vec{\mathbf{0}}$	Varies $\vec{\boldsymbol{\omega}} = \vec{\boldsymbol{\omega}}(t)$	Does not exist

6.7 ♣ Concept: What objects have a unique angular velocity/acceleration? (Sections 8.3, 8.4).

The angular velocity $\vec{\boldsymbol{\omega}}$ of some object S relative to Earth is to be determined.
This object S could be a (circle ***all*** objects that have an ***unambiguously*** defined angular velocity $\vec{\boldsymbol{\omega}}$):

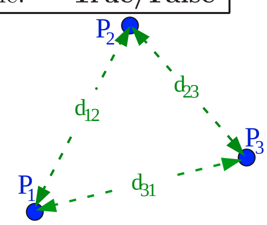
Real number	Matrix	Set of points	Mass center of a rigid body
Vector	Point	Reference frame	Flexible body
3D orthogonal unit basis	Particle	3D rigid body	System of particles and bodies

Repeat for the angular acceleration $\vec{\boldsymbol{\alpha}}$ of some object S relative to Earth box appropriate objects.

6.8 ♣ What is a reference frame, rigid body, and orthogonal basis? (Sections 4.1 and 8.2)

#	Statement (regard "***rigid body***" as a massive 2D or 3D rigid object)	True or False?
a	A reference frame has all the attributes of a rigid body.	**True/False**
b	A rigid body has all the attributes of a reference frame.	**True/False**
c	A reference frame with time-invariant distributed mass is a rigid body.	**True/False**
d	The definition of a reference frame implies a sense of time.	**True/False**
e	A rigid body B has a uniquely-defined angular velocity in a reference frame N.	**True/False**
f	A point Q has a uniquely-defined angular velocity in a reference frame N.	**True/False**
g	The reference frame B implies unique orthogonal unit vectors $\widehat{\mathbf{b}}_\text{x}$, $\widehat{\mathbf{b}}_\text{y}$, $\widehat{\mathbf{b}}_\text{z}$.	**True/False**
h	The right-handed orthogonal unit vectors $\widehat{\mathbf{b}}_\text{x}$, $\widehat{\mathbf{b}}_\text{y}$, $\widehat{\mathbf{b}}_\text{z}$ imply a unique reference frame.	**True/False**
i	The reference frame B implies a unique rigid frame.	**True/False**
j	A rigid frame with origin B_o and basis $\widehat{\mathbf{b}}_\text{x}$, $\widehat{\mathbf{b}}_\text{y}$, $\widehat{\mathbf{b}}_\text{z}$ implies a unique reference frame.	**True/False**

6.9 ♣ Concept: Reference frames and vector bases. (Sections 4.1 and 8.2)



Consider 3 distinct non-collinear points P_1, P_2, P_3 and the non-zero distances d_{12}, d_{23}, d_{31} between them. In general, determine if each object below can **always** be constructed from P_1, P_2, P_3 under the listed condition.
For each "**Yes**" answer, draw the object.

Condition	Object to be constructed	Object can be constructed?	If Yes, **Draw**
d_{12}, d_{23}, d_{31} are **constant**	Vector basis that spans 3D space	**Yes/No**	
d_{12}, d_{23}, d_{31} are **variable**	Vector basis that spans 3D space	**Yes/No**	
d_{12}, d_{23}, d_{31} are **constant**	Right-handed, orthogonal, unitary basis	**Yes/No**	
d_{12}, d_{23}, d_{31} are **variable**	Right-handed, orthogonal, unitary basis	**Yes/No**	
d_{12}, d_{23}, d_{31} are **constant**	Unique reference frame	**Yes/No**	
d_{12}, d_{23}, d_{31} are **variable**	Unique reference frame	**Yes/No**	

6.10 ♣ Concepts: What objects have a uniquely-defined angular velocity? (Section 8.3).

a.	The angular velocity of a	point	in a	reference frame	is well defined.	**True/False**
b.	The angular velocity of a	3D rigid body	in a	particle	is well defined.	**True/False**
c.	The angular velocity of a	3D rigid body	in a	reference frame	is well defined.	**True/False**
d.	The angular velocity of a	reference frame	in a	3D rigid body	is well defined.	**True/False**
d.	The angular velocity of a	3D rigid basis	in a	reference frame	is well defined.	**True/False**
e.	The angular velocity of a	reference frame	in a	flexible body	is well defined.	**True/False**
f.	The angular velocity of a	flexible body	in a	reference frame	is well defined.	**True/False**