5.1 ♣ Notations for derivatives. (Section 1.6.1).

Date	Person		Symbols for 1^{st}, 2^{nd}, and 3^{rd} derivatives		
1675			$\frac{dy}{dt}$	$\frac{d^2y}{dt^2}$	$\frac{d^3y}{dt^3}$
1675			$\dot{y}$	$\ddot{y}$	$\dddot{y}$
1797		(trained by Euler)	y'	y''	y'''
1850	Cauchy/Weierstrauss		$\lim\limits_{h \to 0} \frac{y(t+h) - y(t)}{h}$	?	?

There was bitter rivalry between Newton and Leibniz, and the notations of Leibniz and Newton are not entangled. For example, $\frac{d\dot{y}}{dt}$ is written in Leibniz's notation as ___ or Newton's as ___.

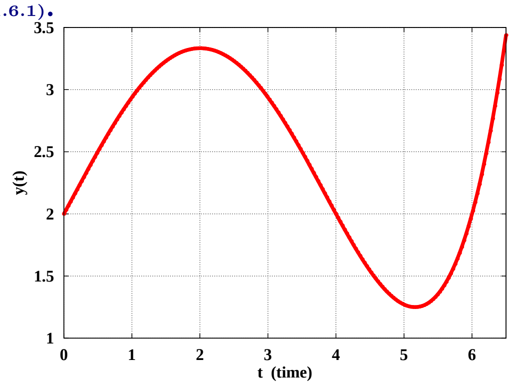
5.2 ♣ Leibniz's shorthand notation for 3^{rd} derivatives. (Section 1.6.1).

Write the explicit expression for the following 3^{rd} derivative (so it contains three 1^{st} derivatives).

Result:

$$\frac{d^3y}{dt^3} \triangleq \quad \_\_\_$$

5.3 ♣ Geometric interpretation of a derivative. (Section 1.6.1).



Estimate the 1^{st}-derivative of the function $y(t)$ shown to the right at $t = 0,\ 2,\ 4,\ 6$.

Pick your answers from: **-1**, **0**, **1**, **2**.

Result: $\left.\frac{dy}{dt}\right|_{t=0} =$ ___ $\quad \left.\frac{dy}{dt}\right|_{t=2} =$ ___

$\left.\frac{dy}{dt}\right|_{t=4} =$ ___ $\quad \left.\frac{dy}{dt}\right|_{t=6} =$ ___

Estimate the **sign** of the 2^{nd}-derivative of $y(t)$ from the answers **−**, **0**, or **+**.
Answer **0** when the absolute value of the 2^{nd}-derivative is estimated to be less than 0.5.

Result: $\left.\frac{d^2y}{dt^2}\right|_{t=0}$ is ___ $\quad \left.\frac{d^2y}{dt^2}\right|_{t=2}$ is ___ $\quad \left.\frac{d^2y}{dt^2}\right|_{t=4}$ is ___ $\quad \left.\frac{d^2y}{dt^2}\right|_{t=6}$ is ___

5.4 ♣ Derivatives of commonly-encountered functions. (Section 1.6.4).

Differentiate the following functions that depend on t (time). Ensure answers involving x are valid when x is either constant or depends on time, e.g., when $x = t^3$.

Result: $\frac{d}{dt}\, t^2 =$ ___ $\quad \frac{d}{dt}\, t^3 =$ ___ $\quad \frac{d}{dt}\, t^{47} =$ ___

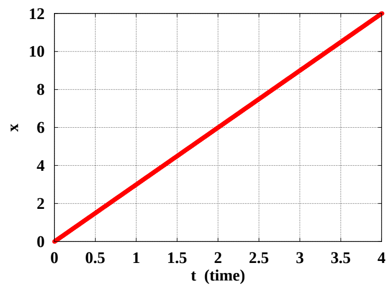
$\frac{d}{dt}\, \sin(t) =$ ___ $\quad \frac{d}{dt}\, \cos(t) =$ ___ $\quad \frac{d}{dt}\, \cos(x) =$ ___

$\frac{d}{dt}\, e^t =$ ___ $\quad \frac{d}{dt}\, \ln(t) =$ ___ $\quad \frac{d}{dt}\, \ln(x) =$ ___

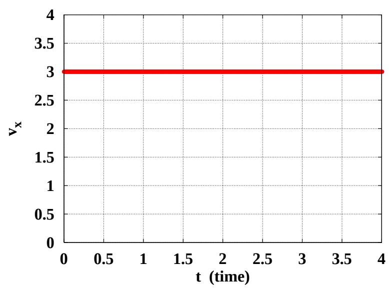
5.5 ♣ Geometric interpretations of a derivative. (Section 1.6.1).

Complete the missing analytical statements and graph the missing functions.

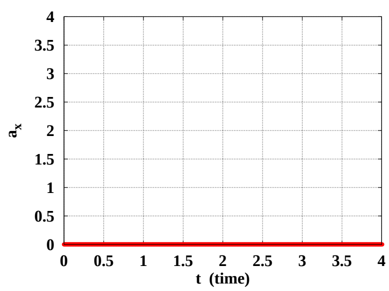
$x = 3\,t$



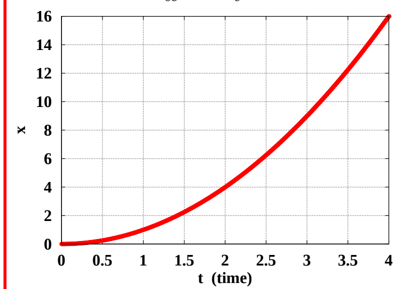
$v_x \triangleq \dfrac{dx}{dt} = $



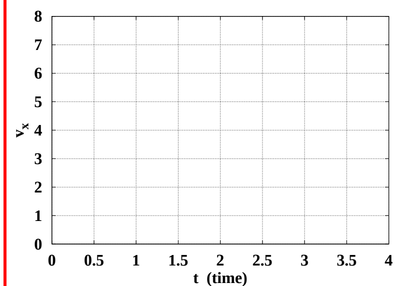
$a_x \triangleq \dfrac{d^2x}{dt^2} = $



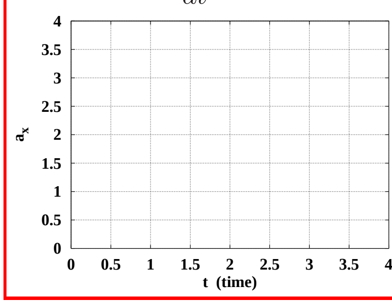
$x = t^2$



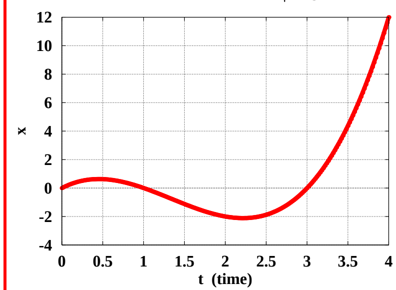
$v_x \triangleq \dfrac{dx}{dt} = $



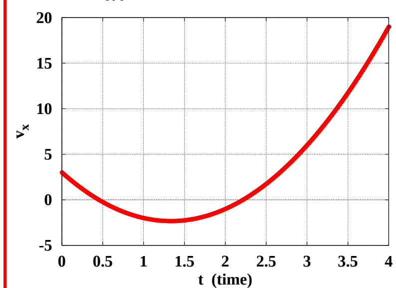
$a_x \triangleq \dfrac{d^2x}{dt^2} = $



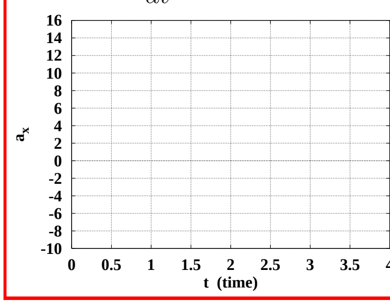
$x = t^3 - 4\,t^2 + 3\,t$

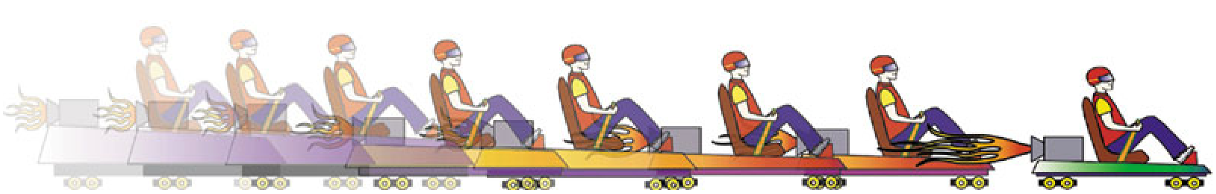


$v_x \triangleq \dfrac{dx}{dt} = $



$a_x \triangleq \dfrac{d^2x}{dt^2} = $





5.6 ♣ Good product rule for differentiation (for scalars, $\vec{\text{v}}$ectors, matrices, ...). (Section 1.6.6).

The ***good product rule for differentiation*** that works when u and v are scalars, $\vec{\text{v}}$ectors, or matrices is (circle the correct answer – and update your Calculus teacher):

$$\frac{d(u*v)}{dt} = \frac{du}{dt} * v + u * \frac{dv}{dt} \qquad \frac{d(u*v)}{dt} = u * \frac{dv}{dt} + v * \frac{du}{dt} \qquad \frac{d(u*v)}{dt} = v * \frac{du}{dt} + u * \frac{dv}{dt}$$

Knowing u, v, w are scalars or **matrices** that depend on time t, use the ***good product rule for differentiation*** to form the 1^{st} ordinary time-derivative of $y(t) = u * v * w$.

Good product rule: $\frac{dy}{dt} = \frac{d(u*v*w)}{dt} = \boxed{}$

5.7 Derivative quotient rule? No, just use product rule and exponent. (Section 1.6.7).

Although the "***quotient rule***" can be used to calculate the derivative with respect to t of the ratio of two functions $\frac{f(t)}{g(t)}$, it can be easier to rewrite the ratio as $f(t) * g(t)^{-1}$ then use the ***product rule***. Use this idea to first rewrite the following ratio of two functions as a product and then use the ***product rule*** to calculate its derivative.

Result: $\frac{\ln(t)}{t^2} = \boxed{}$ $\qquad \frac{d}{dt}\left[\,\ln(t)\,/\,t^2\,\right] = \boxed{}$

5.8 ♣ Example of the "good product rule" for differentiation. (Takes less than 2 minutes).

The "good" product rule is easy-to-use for ***very quickly*** differentiating complex expressions. Knowing x and y are variables that depend on the independent variable t (time), determine the ordinary time-derivative of the function f when[1]

$$f(t) = \sin(t) * \cos(x+y) * (\dot{x})^2 * e^t * \ln(y) \,/\, x$$

Result:

$$\begin{aligned}\frac{df}{dt} &= \cos(t) * \cos(x+y) * (\dot{x})^2 * e^t * \ln(y) \,/\, x \\ &- \sin(t) * \sin(x+y) * (\dot{x}+\dot{y}) * (\dot{x})^2 * e^t * \ln(y) \,/\, x \\ &+ \boxed{} \\ &+ \boxed{} \\ &+ \boxed{} \\ &- \boxed{}\end{aligned}$$

5.9 Differentiation concepts. (Section 1.6.9).

Shown right is an equation relating the dependent variable y to the independent variable t.

$$y^4 - 8\,y = 3\,t^2 + \sin(t)$$

Find a general expression for the ordinary derivative $\frac{dy}{dt}$ in terms of t and y.

Find a **numerical** value for $\frac{dy}{dt}$ at $t = 0$ when y is **positive**.

Hint: The value of y is not arbitrary. If you encounter difficulty, consider ***implicit differentiation***..

Result: $\frac{dy}{dt} = \boxed{}$ $\qquad \left.\frac{dy}{dt}\right|_{t=0} = \boxed{}$

[1]Symbols for the 1^{st} and 2^{nd} ordinary time-derivatives of x include $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$ (introduced by ***Leibniz***), $\dot{x}$ and $\ddot{x}$ (introduced by ***Newton***), and `x'` and `x''` (introduced by ***Lagrange*** and used by **M**otion**G**enesis).

5.10 ♣ Review of partial and ordinary differentiation. (Section 1.6.2).

The mathematical process below is used in *Lagrange's equations of motion.*

The kinetic energy K of a bridge-crane (shown right) can be written in terms of constants M, m, L and variables x, $\dot{x}$, θ, $\dot{\theta}$, as

$$K \;=\; \tfrac{1}{2}\, M\, \dot{x}^2 \;\;+\;\; \tfrac{1}{2}\, m\, [L^2\, \dot{\theta}^2 + 2\, L\, \cos(\theta)\, \dot{x}\, \dot{\theta}]$$

- First, regard x, $\dot{x}$, θ, $\dot{\theta}$ as independent variables [so K depends separately on each, i.e., $K(x, \dot{x}, \theta, \dot{\theta})$], form the **partial derivatives** below (left).
- Next, regard x, $\dot{x}$, θ, $\dot{\theta}$ as time-dependent variables and form the **ordinary derivatives** below (right).

$\dfrac{\partial K}{\partial \theta} =$ ______ $\quad \dfrac{\partial K}{\partial \dot{\theta}} =$ ______ $\quad \dfrac{d}{dt}\left(\dfrac{\partial K}{\partial \dot{\theta}}\right) =$ ______

$\dfrac{\partial K}{\partial x} =$ ______ $\quad \dfrac{\partial K}{\partial \dot{x}} =$ ______ $\quad \dfrac{d}{dt}\left(\dfrac{\partial K}{\partial \dot{x}}\right) =$ ______

5.11 Differentiation concepts: What is dt? (Section 1.6.1).

A continuous function $z(t)$ depends on $x(t)$, $y(t)$, and time t as $\quad z \;=\; x \;+\; y^2\, \sin(t)$

At a certain instant of time, $y = 1$ and z simplifies to $\quad z \;=\; x \;+\; \sin(t)$

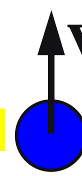
Find the time-derivative of z at the instant when $y = 1$.

Result: $\quad \left.\dfrac{dz}{dt}\right|_{y\,=\,1} \;=$ ______

5.12 ♣ Differentiation concepts – what is wrong? (Section 1.6.1 and previous problem).

The scalar v measures a baseball's upward-velocity. Knowing $v = 0$ only when the ball reaches maximum height, explain what is wrong with the following statement about v's time derivative.

$\dfrac{dv}{dt} \;=\; \dfrac{d\,(0)}{dt} \;=\; 0$ is **wrong**. You know the correct answer is: $\dfrac{dv}{dt} \;=\; g \approx 9.8\, \dfrac{\text{m}}{\text{s}^2}$.



Explain what is wrong: It is incorrect to time-differentiate as shown above because:

______

5.13 ♣ Integrals of commonly-encountered functions. (Section 1.7).

Calculate the following indefinite integrals in terms of an indefinite constant C (regard t as positive).

Result:

$\int t^2\, dt \;=$ ______ $\quad \int t^3\, dt \;=$ ______ $\quad \int t^8\, dt \;=$ ______

$\int t^{-3}\, dt \;=$ ______ $\quad \int t^{-2}\, dt \;=$ ______ $\quad \int t^{-1}\, dt \;=$ ______

$\int \sin(t)\, dt \;=$ ______ $\quad \int \cos(t)\, dt \;=$ ______ $\quad \int e^t\, dt \;=$ ______

$\int 5\, dt \;=$ ______ $\quad \int 5/t\, dt \;=$ ______ $\quad \int (5 + \frac{1}{t})\, dt \;=$ ______