Homework 5. Chapters 1, 7.
Calculus review and vector differentiation

Show work – except for ♣ fill-in-blanks.

5.1 ♣ Notations for derivatives (complete the blanks). (Section 1.6.1).

Symbol for 1^{st}, 2^{nd}, 3^{rd} derivative			**Idea**	Date	Name of mathematician
$\dot{y}$	$\boxed{}$	$\dddot{y}$	**Geometry/slope**	1675	$\boxed{}$
$\frac{dy}{dt}$	$\frac{d^2y}{dt^2}$	$\boxed{}$	**Differentials**	1675	$\boxed{}$ (taught Bernoullis who tutored **Euler**)
y'	$\boxed{}$	$\boxed{}$	**Functions**	1797	Euler and $\boxed{}$ (who was trained by **Euler**)
$\lim\limits_{h \to 0} \frac{y(t+h) - y(t)}{h}$	$\boxed{?}$	$\boxed{?}$	**Limits** **delta-epsilon**	1850 1872	Cauchy (trained by **Lagrange**) Weierstrass
$\frac{\partial y}{\partial x}$	$\frac{\partial^2 y}{\partial x^2}$	$\frac{\partial^3 y}{\partial x^3}$		1786 1841	Legendre (introduced partials, abandoned them) Jacobi (re-introduced partials again)

There was bitter rivalry between Newton and Leibniz about the concepts and notation for a derivative.

5.2 ♣ (1675 AD) Leibniz's shorthand notation for 3^{rd} derivatives. (Section 1.6.1).

Write the explicit expression for Leibniz's 3^{rd} derivative show right (so it contains three 1^{st} derivatives).

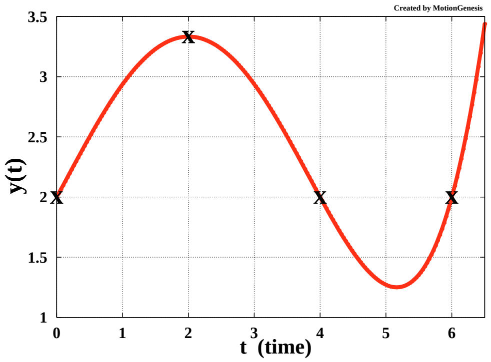
$$\underbrace{\frac{d^3y}{dt^3}}_{\text{shorthand}} \triangleq \underbrace{\frac{d}{dt}\left(\boxed{}\left(\boxed{}\right)\right)}_{\text{explicit}}$$

Write Leibniz's and Newton's shorthand expression for the 9^{th} derivative of y with respect to t.

$$\underbrace{\boxed{}}_{\text{Leibniz}} = \underbrace{\boxed{}}_{\text{Newton}}$$

5.3 ♣ (1675 AD) Newton's idea: Derivative as geometry (slope and curvature). (Section 1.6.1).

Newton related derivatives to geometry (1^{st}-derivative as slope and 2^{nd}-derivative as curvature). Estimate the slope of the function $y(t)$ shown right at $t = 0, 2, 4, 6$.



Result: Pick your answers from: $\boxed{\textbf{-1, 0, 1, 2}}$.

Slope (1^{st} **derivative**)

$$\left.\frac{dy}{dt}\right|_{t=0} \approx \boxed{} \qquad \left.\frac{dy}{dt}\right|_{t=2} \approx \boxed{}$$

$$\left.\frac{dy}{dt}\right|_{t=4} \approx \boxed{} \qquad \left.\frac{dy}{dt}\right|_{t=6} \approx \boxed{}$$

Estimate the **sign** of the curvature [2^{nd}-derivative of $y(t)$].

Result: Pick your answers from: $\boxed{<,\ \approx,\ >}$. Select $\approx$ when the curvature ≈ 0 (i.e., $\left|\frac{d^2y}{dx^2}\right| < 0.01$).

Curvature (2^{nd} **derivative**)

$$\left.\frac{d^2y}{dt^2}\right|_{t=0} \boxed{}\ 0 \qquad \left.\frac{d^2y}{dt^2}\right|_{t=2} \boxed{}\ 0 \qquad \left.\frac{d^2y}{dt^2}\right|_{t=4} \boxed{}\ 0 \qquad \left.\frac{d^2y}{dt^2}\right|_{t=6} \boxed{}\ 0$$

5.4 ♣ (1755 AD) Euler's idea: Derivative of a function is a function. (Section 1.6.5).

Differentiate the following functions that depend on t (time). Express results in terms of x, $\dot{x}$, t so the results are valid when x is constant or depends on time (e.g., when $x = 9$ or $x = t^3$ or $x = t^5$ or …).

Result:

$$\frac{d}{dt}\,t^2 = \boxed{} \qquad \frac{d}{dt}\,t^3 = \boxed{} \qquad \frac{d}{dt}\,t^{-7} = \boxed{}$$

$$\frac{d}{dt}\sin(t) = \boxed{} \qquad \frac{d}{dt}\cos(t) = \boxed{} \qquad \frac{d}{dt}\cos(x) = \boxed{} * \boxed{}$$

$$\frac{d}{dt}\,e^t = \boxed{} \qquad \frac{d}{dt}\ln(t) = \frac{\boxed{}}{\boxed{}} \qquad \frac{d}{dt}\ln(x) = \frac{1}{\boxed{}} * \boxed{}$$

5.5 ♣ Good product rule for differentiation – for scalars, $\vec{v}$ectors, [matrices], ... (Section 1.6.7).

Circle the ***good product rule*** that works when u and v are scalars or $\vec{v}$ectors, or u is a **2 × 3** matrix and v is a **3 × 5** matrix (if you did not learn the **good product rule**, update your calculus teacher).

$$\frac{d(u * v)}{dt} = \frac{du}{dt} * v + u * \frac{dv}{dt} \qquad \frac{d(u * v)}{dt} = u * \frac{dv}{dt} + v * \frac{du}{dt} \qquad \frac{d(u * v)}{dt} = v * \frac{du}{dt} + u * \frac{dv}{dt}$$

Knowing u, v, w are scalars or matrices that depend on time t, use the ***good product rule for differentiation*** to form the derivative of $y(t) = u * v * w$.

Good product rule: $\frac{dy}{dt} = \frac{d(u * v * w)}{dt} = \square \, \square \, w \; + \; \square \, \square \, w \; + \; u \, v \, \frac{dw}{dt}$

5.6 ♣ Example of the "good product rule" for differentiation (if done right, takes ≈ 2 minutes).

Differentiate the function $f(t)$ with the easy-to-use ***good product rule for differentiation***.

$$\begin{array}{llcccccccccc}
\textbf{Function:} & f(t) & = & \sin(t) & * & \cos(t) & * & t^2 & * & e^t & * & \ln(t) \\
\hline
\textbf{Derivative:} & \frac{df}{dt} & = & \cos(t) & * & \cos(t) & * & t^2 & * & e^t & * & \ln(t) \\
 & & + & \sin(t) & * & -\sin(t) & * & t^2 & * & e^t & * & \ln(t) \\
 & & + & \sin(t) & * & \square & * & \square & * & \square & * & \square \\
 & & + & \sin(t) & * & \square & * & \square & * & \square & * & \square \\
 & & + & \sin(t) & * & \square & * & \square & * & \square & * & \frac{\square}{\square}
\end{array}$$

Hint: The "**good product rule**" is an ***efficient*** way to differentiate expressions with many factors.

5.7 ♣ Alternative to quotient rule: combine product/exponent rules. (Section 1.6.8).

Although the ***quotient rule*** can be used to differentiate the ratio of functions $f(t)$ and $g(t)$, it can be easier to remember $\frac{f(t)}{g(t)} = f(t) * g(t)^{-1}$ and then use the ***product rule*** as shown below.

Given example:	$\frac{\sin(t)}{t} = \sin(t) * t^{-1}$	$\frac{d}{dt}\left[\sin(t) * t^{-1}\right] = \cos(t)\, t^{-1} - \sin(t)\, t^{-2}$
Complete this:	$\frac{\sin(t)}{t^2} = \sin(t) * t^{\square}$	$\frac{d}{dt}\left[\sin(t) * t^{\square}\right] = \square - \square$

5.8 ♣ Chain rule for differentiation. $\frac{d\,f[x(t)]}{dt} = \frac{df}{dx}\frac{dx}{dt}$ $\quad \frac{d\,f[x,y]}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$ (Sections 1.6.9, 1.6.4).

Differentiate the function $f(t)$ with the **chain rule** [$x(t)$ and $y(t)$ depend on the independent variable t (time)].

Function: $f(t) = \sin(x) \; + y^2 \; + (\dot{x})^2 \; + e^x \; + \ln(y) \; + \frac{1}{x} \; + \cos(x+y)$

Derivative: $\frac{df}{dt} = \cos(x)\,\dot{x} \; + 2 \, \square \, \square \; + 2 \, \square \, \square \; + \square \; + \square \; - \square \; - \square \, (\square)$

5.9 ♣ Ordinary derivative of the function $f(t) = \sin(t) * \cos(x\,y\,z)$. (Sections 1.6.7 and 1.6.9).

Differentiate the function $f(t)$ with respect to t [$x(t)$, $y(t)$, $z(t)$ depend on the independent variable t (time)].

Result: $\frac{d\,[\sin(t)\,\cos(x\,y\,z)]}{dt} = \square - \square$

5.10 ♣ Differentiation concepts. (Section 1.6.10 – implicit differentiation).

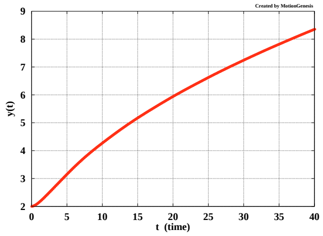
$$y^4 \;-\; 8\,y \;=\; 3\,t^2 \;+\; \sin(t)$$

Roots: $y = \boxed{}$, $y = \boxed{}$, $y \approx$ -1 ± 1.732 i

The equation to the right relates the dependent variable $y(t)$ to the independent variable t. Find two real roots to this equation when $t = 0$.

Form a general expression for $\frac{dy}{dt}$ in terms of y and t and calculate $\frac{dy}{dt}$ when $t = 0$ and $y = 2$.

Result:
In terms of t and y: $\frac{dy}{dt} = \frac{\boxed{}}{\boxed{}}$ Numerical value: $\left.\frac{dy}{dt}\right|_{\substack{t=0\\y=2}} = \frac{1}{\boxed{}}$



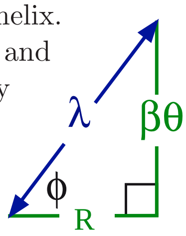
†Optional: Continuous solution of nonlinear algebraic equation.
Starting with $y = 2$, continuously solve for $y(t)$ for $0 \le t \le 40$ and plot your results as shown right. Stumped: See hint in Homework 5.32.

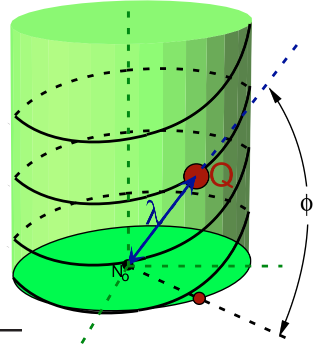
5.11 ♣ Review of explicit and implicit differentiation. (Section 1.6.10).

The figure to the right shows a point Q on a cylindrical helix. Two geometrically significant quantities are a distance λ and an angle ϕ that are related to two **constants** R and β by

$$\lambda^2 \;=\; R^2 \;+\; (\beta\,\theta)^2 \qquad\qquad \tan(\phi) \;=\; \frac{\beta\,\theta}{R}$$

Form $\dot\lambda$ and $\dot\phi$ using the two methods described below.





Explicit differentiation
1. Solve explicitly for λ and ϕ.
2. Then differentiate the resulting expressions.

Result:
In terms of R, β, θ, $\dot\theta$.

$$\lambda \;=\; \sqrt{R^2 \;+\; (\beta\,\theta)^2} \qquad \phi \;=\; \text{atan}(\,\tfrac{\beta\,\theta}{R}\,) \qquad \text{Hint:}\;\; \frac{\partial\,\text{atan}(x)}{\partial x} \;=\; \frac{1}{1+x^2}$$

$$\dot\lambda \;=\; \frac{\boxed{}}{\boxed{}}\;\dot\theta \qquad \dot\phi \;=\; \frac{\boxed{}}{\boxed{}}\;\dot\theta$$

Implicit differentiation
1. Differentiate the equations for λ^2 and $\tan(\phi)$.
2. Then solve for $\dot\lambda$ and $\dot\phi$.

Result:
In terms of R, β, θ, $\dot\theta$, λ.

$$\dot\lambda \;=\; \frac{\boxed{}}{\boxed{}}\;\dot\theta \qquad \dot\phi \;=\; \boxed{}\;\dot\theta \;=\; \frac{\beta\,R}{\lambda^2}\;\dot\theta$$

Forming $\dot\lambda$ is easier and computationally more efficient with **explicit/implicit** differentiation.

5.12 ♣ Review of partial and ordinary differentiation. (Section 1.6.2).

The kinetic energy K of a bridge-crane (shown right) can be written in terms of constants M, m, L and variables x, $\dot x$, θ, $\dot\theta$, as

$$K \;=\; \tfrac{1}{2}\,M\,\dot x^2 \;\;+\;\; \tfrac{1}{2}\,m\,[L^2\,\dot\theta^2 + 2\,L\,\cos(\theta)\,\dot x\,\dot\theta]$$

- First, regard x, $\dot x$, θ, $\dot\theta$ as independent variables [so K depends on each separately, i.e., $K(x,\,\dot x,\,\theta,\,\dot\theta)$], form the **partial derivatives** below (left).
- Next, regard x, $\dot x$, θ, $\dot\theta$ as time-dependent variables and form the **ordinary derivatives** below (right).

The mathematical process below is used in *Lagrange's equations of motion*.

$\frac{\partial K}{\partial \theta} = \boxed{}$	$\frac{\partial K}{\partial \dot\theta} = \boxed{}$	$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot\theta}\right) = \boxed{}$
$\frac{\partial K}{\partial x} = \boxed{}$	$\frac{\partial K}{\partial \dot x} = \boxed{}$	$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot x}\right) = \boxed{}$

5.13 ♣ Differentiation concepts – what is wrong? (Section 1.6.3 and previous problem).

The scalar v measures a baseball's upward-velocity. Knowing $v = 0$ only when the ball reaches maximum height, explain what is wrong with the following statement about v's time derivative.

$\frac{dv}{dt} = \frac{d\,(0)}{dt} = 0$ is **wrong**. We know the correct answer is: $\frac{dv}{dt} = \text{-}g \approx \text{-}9.8\ \frac{\text{m}}{\text{s}^2}$.

Explain what is wrong: It is incorrect to time-differentiate as shown above because:

5.14 ♣ Leibniz's idea and differentiation concepts: What is dt? (Section 1.6.1).

A continuous function $z(t)$ depends on $x(t)$, $y(t)$, and time t as:	$z = x + y^2 \sin(t)$
At a certain instant of time, $y = 1$ and z simplifies to:	$z = x + \sin(t)$

Determine the time-derivative of z at the instant when $y = 1$.

Result: $\left.\frac{dz}{dt}\right|_{y\,=\,1} = \boxed{}$

5.15 ♣ Euler's idea: Integral of a function is a function. (Section 1.7).

Calculate the following indefinite integrals in terms of an indefinite constant C (regard t as positive).

Result:

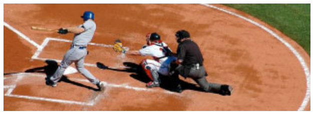
$\int t^2\,dt = \frac{\boxed{}}{\boxed{}} + C$ $\quad\int t^3\,dt = \frac{\boxed{}}{\boxed{}} + C$ $\quad\int t^8\,dt = \frac{\boxed{}}{\boxed{}} + \boxed{}$

$\int t^{\text{-}3}\,dt = \frac{\boxed{}}{\boxed{}} + C$ $\quad\int t^{\text{-}2}\,dt = \boxed{}$ $\quad\int t^{\text{-}1}\,dt = \boxed{}$

$\int \sin(t)\,dt = \boxed{}$ $\quad\int \cos(t)\,dt = \boxed{}$ $\quad\int e^t\,dt = \boxed{}$

$\int 5\,dt = \boxed{}$ $\quad\int 5/t\,dt = \boxed{}$ $\quad\int (5 + \frac{1}{t})\,dt = \boxed{} + \boxed{} + C$

5.16 Solve a 1^{st}-order ODE: Separate variables, integrate, initial value. (Section 1.7).



Solve $\frac{dv}{dt} = \text{-}9.8\ \frac{\text{m}}{\text{s}^2}$ with the initial value $v(t\!=\!0) = 33\ \frac{\text{m}}{\text{s}}$.

Result: $v(t) = \boxed{} + \boxed{}$ **Show work**

5.17 Solve a 2^{nd}-order ODE: Separate variables, integrate, initial value (twice). (Section 1.7).

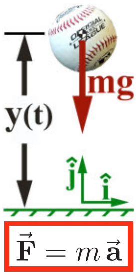
Solve $\frac{d^2y}{dt^2} = \text{-}9.8\ \frac{\text{m}}{\text{s}^2}$ with initial values $\dot{y}(t\!=\!0) = 33\ \frac{\text{m}}{\text{s}}$, $y(t\!=\!0) = 5$ m. **Show work**

Result: $y(t) = \boxed{} + \boxed{} + \boxed{}$ Hint: $\frac{d^2y}{dt^2} \triangleq \frac{d}{dt}\,(\frac{dy}{dt})$. Separate variables and integrate twice. Use both initial values.

Physics: Show $\frac{d^2y}{dt^2} = \text{-}9.8\ \frac{\text{m}}{\text{s}^2}$ results from using $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ for the baseball and simplifying.

Result:

$$\underbrace{\boxed{}}_{\vec{\mathbf{F}}} = \underbrace{\boxed{}}_{m\,\vec{\mathbf{a}}} \quad\Rightarrow\quad \boxed{}.$$



5.18 ♣ Solve a 3^{rd}-order ODE with mixed initial/boundary values. (Section 1.7).

Solve $\frac{d^3y}{dt^3} = 6$ with initial/boundary values $y(t\!=\!0) = 5$, $\dot{y}(t\!=\!0) = 0$, $y(t\!=\!3) = 50$.

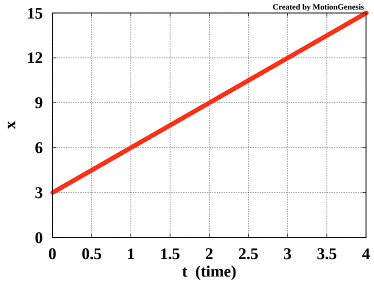
Result: $y(t) = \boxed{}$ Hint: $\frac{d^3y}{dt^3} \triangleq \frac{d}{dt}\,(\frac{d}{dt}\,(\frac{dy}{dt}))$. Then integrate three times.

5.19 ♣ Geometric interpretations of integrals and derivatives. (Section 1.7).

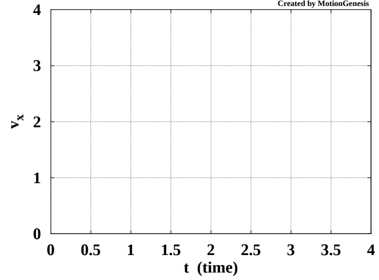
- Complete the blanks and graph the missing functions. Blanks should not have undetermined constants.

Hint: Synthesize information from each vertical column below. Constants of integration can be deduced from graphs. For example, for the 2^{nd} column, start at the bottom with $\frac{d^2x}{dt^2} = 0$ and work upward to determine $\frac{dx}{dt}$ and then $x(t)$.

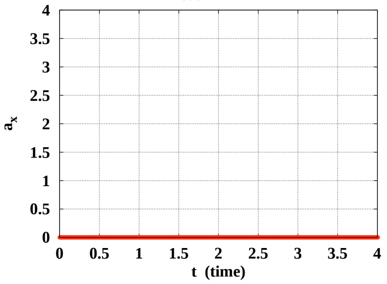
$x \;=\; \boxed{} + \boxed{}$



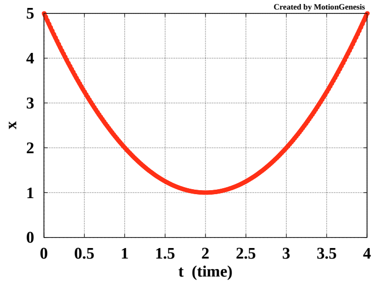
$v_x \triangleq \frac{dx}{dt} \;=\; \boxed{}$



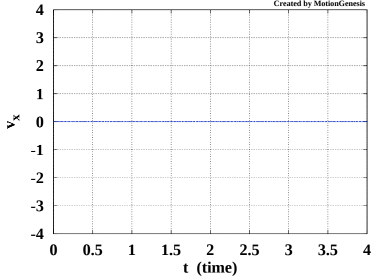
$a_x \triangleq \frac{d^2x}{dt^2} \;=\; \boxed{}$



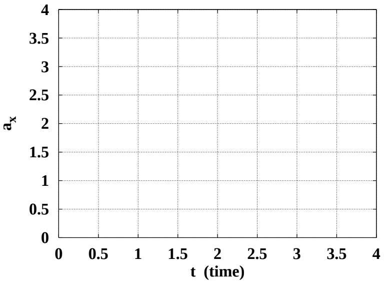
$x \;=\; \boxed{} - \boxed{} + \boxed{}$



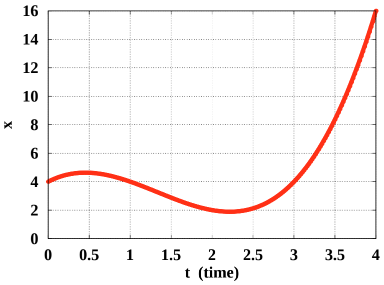
$v_x \triangleq \frac{dx}{dt} \;=\; \boxed{} - \boxed{}$



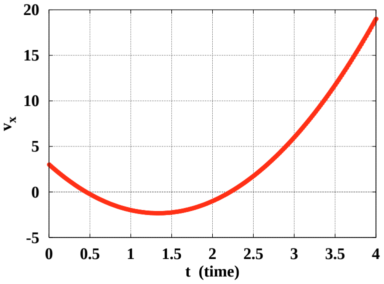
$a_x \triangleq \frac{d^2x}{dt^2} \;=\; 2$



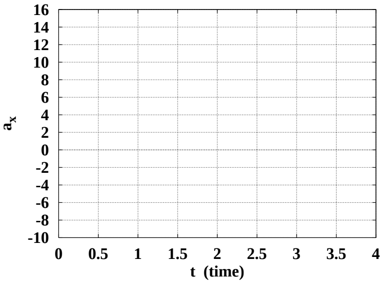
$x \;=\; \boxed{} - \boxed{} + \boxed{} + \boxed{}$



$v_x \triangleq \frac{dx}{dt} \;=\; 3\,t^2 \;-\; 8\,t \;+\; 3$



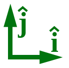
$a_x \triangleq \frac{d^2x}{dt^2} \;=\; \boxed{} - \boxed{}$

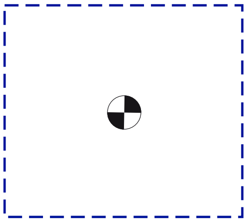


$\boxed{\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}}$

- A rocket-sled/rider is modeled as a particle of mass m whose motion is affected by thrust, normal, and gravity forces. **Draw** its ***free-body diagram*** and write the net force $\vec{\mathbf{F}}_{\text{Net}}$ in terms of scalars F_{T}, F_{n}, $m\,g$ (associated with thrust, normal force, gravity force) and the unit vectors $\widehat{\mathbf{i}}$ and $\widehat{\mathbf{j}}$.

Result: $\vec{\mathbf{F}}_{\text{Net}} \;=\; \boxed{}\,\widehat{\mathbf{i}} + (\boxed{})\,\widehat{\mathbf{j}}$





Thrust $\vec{\mathbf{F}}_{\text{T}} \;=\; \boxed{}\,\widehat{\mathbf{i}}$

Normal $\vec{\mathbf{F}}_{\text{n}} \;=\; \boxed{F_{\text{n}}}\,\boxed{}$

Gravity $\vec{\mathbf{F}}_g \;=\; \boxed{}\,\widehat{\mathbf{j}}$

$\vec{\mathbf{F}}_{\text{Net}} \;=\; \vec{\mathbf{F}}_{\text{T}} + \vec{\mathbf{F}}_{\text{n}} + \vec{\mathbf{F}}_g$

- Set $\vec{\mathbf{F}}_{\text{Net}} = m\,\vec{\mathbf{a}}$, form scalar equations, solve for $\ddot{x}$, F_{n}.

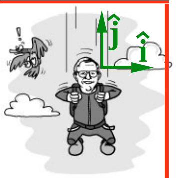
Result:

$$\underbrace{\boxed{}\,\widehat{\mathbf{i}} + (\boxed{})\,\widehat{\mathbf{j}}}_{\vec{\mathbf{F}}_{\text{Net}}} \;=\; \underbrace{m\,\ddot{x}\,\widehat{\mathbf{i}}}_{m\,\vec{\mathbf{a}}} \quad\Rightarrow\quad \ddot{x} = \frac{F_{\text{T}}}{\boxed{}} \qquad F_{\text{n}} = m\,\boxed{}$$

- Given $m = 100$ kg, $F_{\text{T}} = 800$ Newton, $x(t\!=\!0) = 7$ m, $\dot{x}(t\!=\!0) = 0\ \frac{\text{m}}{\text{s}}$, show $\boxed{x(t) \;=\; 4\,t^2 \;+\; 7}$.

5.20 ♣ FE/EIT: $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ for a sky-diver and rocket-sled. (complete the blanks, graphs, etc).

A sky-diver (modeled as a particle Q of mass m) free-falls for 4 seconds after leaving a stationary helicopter from a height $y(0) = 200$ m above Earth (y is positive-upward).



FBD: Draw Q's ***free-body diagram*** and write the net force on the sky-diver (assume gravity is the only relevant force).

Result: $\vec{\mathbf{F}}_{\text{Net}} = \boxed{\quad}\,\hat{\mathbf{j}}$

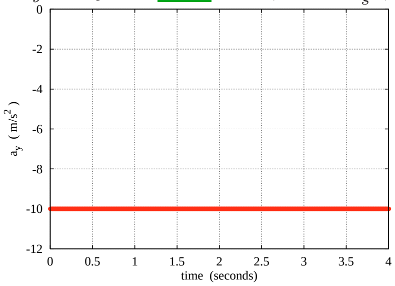
Sketch particle Q, Earth's surface N, a point N_o on N, $y(t)$, $y(0)$, and the helicopter. Form $\vec{\mathbf{r}}$, the position vector from N_o to Q. Differentiate $\vec{\mathbf{r}}$ to form Q's velocity $\vec{\mathbf{v}}$ and acceleration $\vec{\mathbf{a}}$.

Result: $\vec{\mathbf{r}} = \boxed{\quad}$ $\quad \vec{\mathbf{v}} = \boxed{\quad}\,\hat{\mathbf{j}}$ $\quad \vec{\mathbf{a}} = \boxed{\quad}\,\hat{\mathbf{j}}$ in terms of y $\dot{y}$, $\ddot{y}$

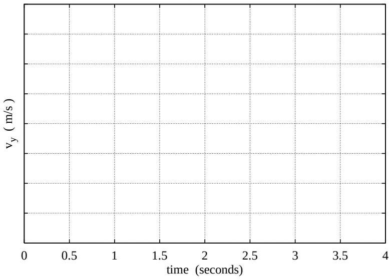
Set $\vec{\mathbf{F}}_{\text{Net}} = m\,\vec{\mathbf{a}}$, form scalar equation, solve for $\ddot{y}$.

$$\overbrace{\boxed{\quad}\,\hat{\mathbf{j}}}^{\vec{\mathbf{F}}_{\text{Net}}} = \overbrace{\boxed{\quad}\,\hat{\mathbf{j}}}^{m\,\vec{\mathbf{a}}} \quad\Rightarrow\quad \ddot{y} = \boxed{\quad} \qquad (g \approx 10\ \tfrac{m}{s^2})$$

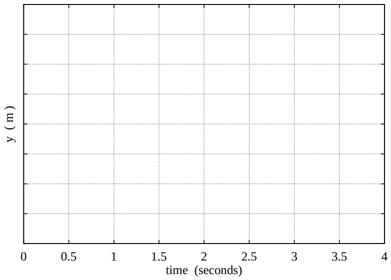
$a_y \;=\; \ddot{y} \;=\; \boxed{-10}$ (units of $\frac{m}{s^2}$)



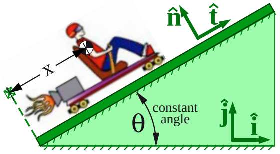
$v_y \;=\; \dot{y} \;=\; \boxed{\quad}$ (units of $\frac{m}{s}$)



$y \;=\; \boxed{\quad} \;+\; \boxed{\quad}$ (units of meters)



A rocket-sled/rider (modeled as a particle Q of mass m) is thrust along smooth rails with a force F_T. The variable x measure's the sled's position along the inclined rails. Initially, $x = 5$ m and $\dot{x} = 0\ \frac{\text{m}}{\text{s}}$. Unit vector $\hat{\mathbf{t}}$ is tangent to the rails. Unit vector $\hat{\mathbf{n}}$ is normal to the rails.



FBD: Draw the forces and write the net force on the rocket-sled/rider.

Result: $\vec{\mathbf{F}}_{\text{Net}} = \boxed{\quad}\,\hat{\mathbf{t}} + \boxed{\quad}\,\hat{\mathbf{n}} - \boxed{\quad}\,\hat{\mathbf{j}}$

Form Q's position vector, velocity, and acceleration (in terms of x, $\dot{x}$, $\ddot{x}$).

Result: $\vec{\mathbf{r}} = \boxed{\quad}\,\hat{\mathbf{t}}$ $\quad \vec{\mathbf{v}} = \boxed{\quad}\,\hat{\mathbf{t}}$ $\quad \vec{\mathbf{a}} = \boxed{\quad}\,\hat{\mathbf{t}}$

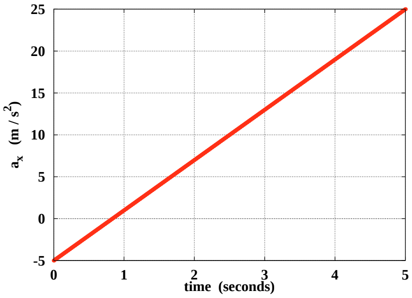
Set $\vec{\mathbf{F}}_{\text{Net}} = m\,\vec{\mathbf{a}}$, form scalar equations, solve for $\ddot{x}$, F_n.

$$\overbrace{[\boxed{\quad} - m\,g\,\boxed{\quad}]\,\hat{\mathbf{t}} + [F_n - m\,g\,\boxed{\quad}]\,\hat{\mathbf{n}}}^{\vec{\mathbf{F}}_{\text{Net}}} \;=\; \overbrace{m\,\ddot{x}\,\hat{\mathbf{t}}}^{m\,\vec{\mathbf{a}}}$$

$$\ddot{x} = \frac{\boxed{\quad}}{\boxed{\quad}} - \boxed{\quad}\,\sin(\theta) \qquad F_n = \boxed{\quad}\,\cos(\theta)$$

F_n measures normal force.

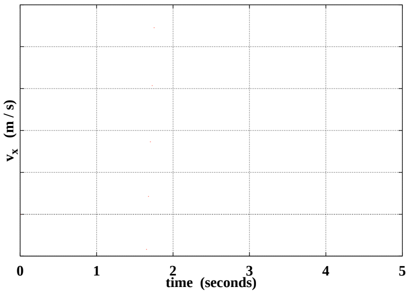
$a_x \;=\; \dot{v}_x \;=\; \ddot{x} \;=\; \boxed{6}\,t \;-\; \boxed{5}$ (units of $\frac{m}{s^2}$)



Determine $\ddot{x}$ when:

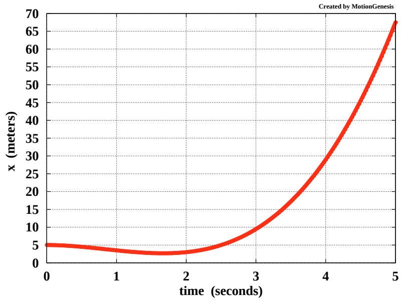
$\theta \;=\; 30°$
$m \;=\; 100$ kg
$g \;\approx\; 10\ \frac{\text{m}}{\text{s}^2}$
$F_T \;=\; (600\ \frac{\text{N}}{\text{s}}) * t.$

$v_x \;=\; \dot{x} \;=\; \boxed{\quad} \;-\; \boxed{\quad}$ (units of $\frac{m}{s}$)



Integrate to form $\dot{x}$. Use the initial value (at time $t = 0$) of $\dot{x}(0) = 0\ \frac{\text{m}}{\text{s}}$ to determine the constant of integration.

$x \;=\; \boxed{\quad} \;-\; \boxed{\quad}\,t^2 \;+\; \boxed{\quad}$ (units of meters)



Integrate to form x. Use the initial value (at time $t = 0$) of $x(0) = 5$ m to determine the constant of integration.