Show work – except for ♣ fill-in-blanks-problems. **Constraints (e.g., contact, rolling, and closed-chains/linkages)**

10.1 ♣ Velocity variables and degrees of freedom (Section 11.1).

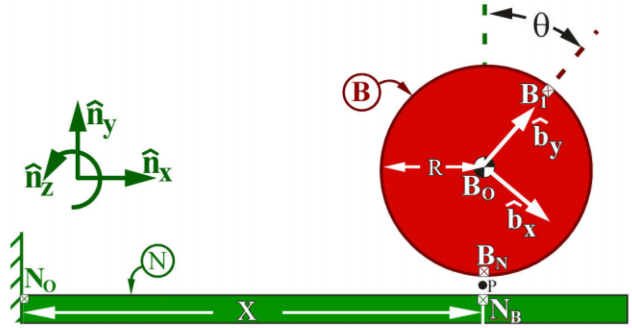
Determine the minimum number of unknown ***velocity variables*** necessary to characterize the motion of the following systems in a reference frame N. Regard Q as a free-flying particle and A as a rigid body that is **free** to translate and rotate in 3D-space. Choose and **define** velocity variables that suffice to describe the motion (Note: The choice of velocity variables is **not unique**).

Optional: Sketch each system with names for each point/body.

System (Q, B, or A and B)	Degrees of freedom	Choice of velocity variables
Free-flying particle Q.	3	$v_x \;\; v_y \;\; v_z$ ${}^N\vec{\mathbf{v}}^Q = v_x\,\widehat{\mathbf{n}}_x + v_y\,\widehat{\mathbf{n}}_y + v_z\,\widehat{\mathbf{n}}_z$
Particle Q moving in a slot (slot is parallel to a unit vector $\vec{\mathbf{n}}$).	1	v ${}^N\vec{\mathbf{v}}^Q = v\,\vec{\mathbf{n}}$
Free-flying rigid body B.	6	$\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y \;\; v_z$ ${}^N\vec{\boldsymbol{\omega}}^B = \omega_x\,\widehat{\mathbf{b}}_x + \omega_y\,\widehat{\mathbf{b}}_y + \omega_z\,\widehat{\mathbf{b}}_z$ ${}^N\vec{\mathbf{v}}^{B_o} = v_x\,\widehat{\mathbf{n}}_x + v_y\,\widehat{\mathbf{n}}_y + v_z\,\widehat{\mathbf{n}}_z$
Rigid body B connected to rigid body A by a revolute joint. (A connects to B at point A_B of A)	7	$\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y \;\; v_z \;\; \omega_B$ ${}^N\vec{\boldsymbol{\omega}}^A = \omega_x\,\widehat{\mathbf{a}}_x + \omega_y\,\widehat{\mathbf{a}}_y + \omega_z\,\widehat{\mathbf{a}}_z$ ${}^N\vec{\mathbf{v}}^{A_B} = v_x\,\widehat{\mathbf{a}}_x + v_y\,\widehat{\mathbf{a}}_y + v_z\,\widehat{\mathbf{a}}_z$ ${}^A\vec{\boldsymbol{\omega}}^B = \omega_B\,\widehat{\boldsymbol{\lambda}}$
Rigid body B connected to rigid body A by a rigid joint.	6	$\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y \;\; v_z$ ${}^N\vec{\boldsymbol{\omega}}^A = \omega_x\,\widehat{\mathbf{a}}_x + \omega_y\,\widehat{\mathbf{a}}_y + \omega_z\,\widehat{\mathbf{a}}_z$ ${}^N\vec{\mathbf{v}}^{A_B} = v_x\,\widehat{\mathbf{n}}_x + v_y\,\widehat{\mathbf{n}}_y + v_z\,\widehat{\mathbf{n}}_z$
Rigid body B connected to A by a ball-and-socket joint.	9	$\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y \;\; v_z \;\; \omega_x^B \;\; \omega_y^B \;\; \omega_z^B$ ${}^N\vec{\boldsymbol{\omega}}^A = \omega_x\,\widehat{\mathbf{a}}_x + \omega_y\,\widehat{\mathbf{a}}_y + \omega_z\,\widehat{\mathbf{a}}_z$ ${}^N\vec{\mathbf{v}}^{A_B} = v_x\,\widehat{\mathbf{a}}_x + v_y\,\widehat{\mathbf{a}}_y + v_z\,\widehat{\mathbf{a}}_z$ ${}^A\vec{\boldsymbol{\omega}}^B = \omega_x^B\,\widehat{\mathbf{b}}_x + \omega_y^B\,\widehat{\mathbf{b}}_y + \omega_z^B\,\widehat{\mathbf{b}}_z$
Rigid body B connected to A by a revolute angular velocity motor. (A revolute angular velocity motor ***specifies*** B's angular velocity in A)	6	$\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y \;\; v_z$ ${}^N\vec{\boldsymbol{\omega}}^A = \omega_x\,\widehat{\mathbf{a}}_x + \omega_y\,\widehat{\mathbf{a}}_y + \omega_z\,\widehat{\mathbf{a}}_z$ ${}^N\vec{\mathbf{v}}^{A_B} = v_x\,\widehat{\mathbf{n}}_x + v_y\,\widehat{\mathbf{n}}_y + v_z\,\widehat{\mathbf{n}}_z$
Rectangular box B sliding on a flat rigid surface fixed in N. (B contacts N at a single vertex B_o of B)	5	$\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y$ ${}^N\vec{\mathbf{v}}^{B_o} = v_x\,\widehat{\mathbf{n}}_x + v_y\,\widehat{\mathbf{n}}_y$ ${}^N\vec{\boldsymbol{\omega}}^B = \omega_x\,\widehat{\mathbf{b}}_x + \omega_y\,\widehat{\mathbf{b}}_y + \omega_z\,\widehat{\mathbf{b}}_z$
Rectangular box B sliding on a flat rigid surface fixed in N. (B contacts N on a single edge of B)	4	$\omega_y \;\; \omega_z \;\; v_x \;\; v_y$ ${}^N\vec{\mathbf{v}}^{B_o} = v_x\,\widehat{\mathbf{n}}_x + v_y\,\widehat{\mathbf{n}}_y$ ${}^N\vec{\boldsymbol{\omega}}^B = \omega_y\,\widehat{\mathbf{b}}_y + \omega_z\,\widehat{\mathbf{b}}_z$
Rectangular box B sliding on a flat rigid surface fixed in N. (B contacts N on one surface of B)	3	$\omega_z \;\; v_x \;\; v_y$ ${}^N\vec{\mathbf{v}}^{B_o} = v_x\,\widehat{\mathbf{n}}_x + v_y\,\widehat{\mathbf{n}}_y$ ${}^N\vec{\boldsymbol{\omega}}^B = \omega_z\,\widehat{\mathbf{b}}_z$

10.2 Velocity and acceleration of a wheel sliding on a plane (Section 11.12).

The following shows a wheel B of radius R in contact with a horizontal road N.
Point N_o is fixed on N.
Point B_o is the wheel's geometric center.
Point B_1 is fixed to B at the wheel's periphery.



Right-handed orthogonal unit vectors $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ and $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ are fixed in N and B, with

- $\widehat{\mathbf{n}}_x$ horizontally-right
- $\widehat{\mathbf{n}}_y$ vertically-upward
- $\widehat{\mathbf{n}}_z = \widehat{\mathbf{b}}_z$ parallel to B's angular velocity in N
- $\widehat{\mathbf{b}}_y$ directed from B_o to B_1

B's translation in N is characterized by x, the $\widehat{\mathbf{n}}_x$ measure of B_o's position from N_o.
B's rotation in N is characterized by θ, the angle from $\widehat{\mathbf{n}}_y$ to $\widehat{\mathbf{b}}_y$ with $-\widehat{\mathbf{n}}_z$ sense.

Succinctly answer each question in terms of $R, x, \dot{x}, \ddot{x}, \theta, \dot{\theta}, \ddot{\theta}, \widehat{\mathbf{n}}_x, \widehat{\mathbf{n}}_y, \widehat{\mathbf{n}}_z, \widehat{\mathbf{b}}_x, \widehat{\mathbf{b}}_y, \widehat{\mathbf{b}}_z$.

(a) Determine B's angular velocity in N and B's angular acceleration in N.
Use **definitions** to calculate B_o's velocity in N and B_1's velocity in N.
Result:

$${}^N\vec{\boldsymbol{\omega}}^B = -\dot{\theta}\,\widehat{\mathbf{n}}_z \qquad {}^N\vec{\boldsymbol{\alpha}}^B = -\ddot{\theta}\,\widehat{\mathbf{n}}_z \qquad {}^N\vec{\mathbf{v}}^{B_o} = \dot{x}\,\widehat{\mathbf{n}}_x \qquad {}^N\vec{\mathbf{v}}^{B_1} = \dot{x}\,\widehat{\mathbf{n}}_x + R\,\dot{\theta}\,\widehat{\mathbf{b}}_x$$

(b) Find B_1's velocity in N at the **instant** when B_1 is in contact with N.
Result: (in terms of $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$)

$${}^N\vec{\mathbf{v}}^{B_1}\Big|_{\text{contact}} = (\dot{x} - R\,\dot{\theta})\,\widehat{\mathbf{n}}_x$$

(c) Point B_N designates the point **of** B that is in contact with N at each **instant**.
Provide a formula relating ${}^N\vec{\mathbf{v}}^{B_N}$ to ${}^N\vec{\mathbf{v}}^{B_o}$ [this formula should not contain a derivative and uses the fact that B_o and B_N are both points **of** (**fixed on**) B]. Then, express ${}^N\vec{\mathbf{v}}^{B_N}$ in terms of R, $\dot{x}$, $\dot{\theta}$.
Similarly, provide a derivative-free formula relating ${}^N\vec{\mathbf{a}}^{B_N}$ to ${}^N\vec{\mathbf{a}}^{B_o}$ and then calculate ${}^N\vec{\mathbf{a}}^{B_N}$.
Result:

$${}^N\vec{\mathbf{v}}^{B_N} \underset{(10.3)}{=} {}^N\vec{\mathbf{v}}^{B_o} + {}^N\vec{\boldsymbol{\omega}}^B \times \vec{\mathbf{r}}^{B_N/B_o} = (\dot{x} - R\,\dot{\theta})\,\widehat{\mathbf{n}}_x$$

$${}^N\vec{\mathbf{a}}^{B_N} \underset{(10.4)}{=} {}^N\vec{\mathbf{a}}^{B_o} + {}^N\vec{\boldsymbol{\alpha}}^B \times \vec{\mathbf{r}}^{B_N/B_o} + {}^N\vec{\boldsymbol{\omega}}^B \times ({}^N\vec{\boldsymbol{\omega}}^B \times \vec{\mathbf{r}}^{B_N/B_o}) = (\ddot{x} - R\,\ddot{\theta})\,\widehat{\mathbf{n}}_x + \dot{\theta}^2\,R\,\widehat{\mathbf{n}}_y$$

(d) Point B_N's position vector from N_o is ***always*** $\vec{\mathbf{r}}^{B_N/N_o} = x\,\widehat{\mathbf{n}}_x$.
Why is your previous result for ${}^N\vec{\mathbf{v}}^{B_N}$ **not** the time-derivative in N of $\vec{\mathbf{r}}^{B_N/N_o}$?
Why is your previous result for ${}^N\vec{\mathbf{a}}^{B_N}$ **not** the time-derivative in N of ${}^N\vec{\mathbf{v}}^{B_N}$?

$${}^N\vec{\mathbf{v}}^{B_N} = (\dot{x} - R\,\dot{\theta})\,\widehat{\mathbf{n}}_x \neq \frac{{}^N d\,\vec{\mathbf{r}}^{B_N/N_o}}{dt}$$

$${}^N\vec{\mathbf{a}}^{B_N} = (\ddot{x} - R\,\ddot{\theta})\,\widehat{\mathbf{n}}_x + \dot{\theta}^2\,R\,\widehat{\mathbf{n}}_y \neq \frac{{}^N d\,{}^N\vec{\mathbf{v}}^{B_N}}{dt}$$

Explain: Point B_N is the point **of** B in contact with N. Point B_1 is only B_N for an **instant**.
The time-derivative of B_1's position vector at this **instant** ***does not*** produce its velocity!
If B maintains contact with N, point B_N is continuously being ***renamed*** – which is strange, but useful.

(e)

• The symbols B_N and B_1 sometimes designate the same point.	**True**/False
• Point B_1 is always the name of the same (one) specific point of B.	**True**/False
• Point B_N is always the name of the same (one) specific point of B.	True/**False**
• Point B_N is ***continuously renamed*** as B rotates in N.	**True**/False
• When B_1 is in contact with N, ${}^N\vec{\mathbf{v}}^{B_1}\Big\vert_{\text{contact}}$ has the same meaning as ${}^N\vec{\mathbf{v}}^{B_N}$.	**True**/False