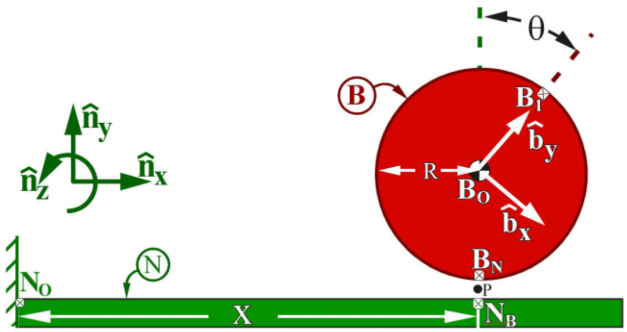
8.1 Velocity and acceleration of a wheel sliding on a plane (Section 9.8).

The following shows a wheel B of radius R in contact with a horizontal road N.
Point N_o is fixed on N.
Point B_o is the wheel's geometric center.
Point B_1 is fixed to B at the wheel's periphery.



Right-handed orthogonal unit vectors $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ and $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ are fixed in N and B, with

- $\widehat{\mathbf{n}}_x$ horizontally-right
- $\widehat{\mathbf{n}}_y$ vertically-upward
- $\widehat{\mathbf{n}}_z = \widehat{\mathbf{b}}_z$ parallel to B's angular velocity in N
- $\widehat{\mathbf{b}}_y$ directed from B_o to B_1

B's translation in N is characterized by x, the $\widehat{\mathbf{n}}_x$ measure of B_o's position from N_o.
B's rotation in N is characterized by θ, the angle from $\widehat{\mathbf{n}}_y$ to $\widehat{\mathbf{b}}_y$ with $-\widehat{\mathbf{n}}_z$ sense.

Succinctly answer each question in terms of R, x, $\dot{x}$, $\ddot{x}$, θ, $\dot{\theta}$, $\ddot{\theta}$, $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$, $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$.

(a) Determine B's angular velocity in N and B's angular acceleration in N.
Use **definitions** to calculate B_o's velocity in N and B_1's velocity in N.
Result: ${}^N\vec{\boldsymbol{\omega}}^B = \underline{\qquad}$ ${}^N\vec{\boldsymbol{\alpha}}^B = \underline{\qquad}$ ${}^N\vec{\mathbf{v}}^{B_o} = \underline{\qquad}$ ${}^N\vec{\mathbf{v}}^{B_1} = \underline{\qquad\qquad}$

(b) Find B_1's velocity in N at the <u>instant</u> when B_1 is in contact with N.
Result: (in terms of $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$) ${}^N\vec{\mathbf{v}}^{B_1}\Big|_{\text{contact}} = \underline{\qquad\qquad}$

(c) Point B_N designates the point <u>of</u> B that is in contact with N at each **instant**.
Provide a formula relating ${}^N\vec{\mathbf{v}}^{B_N}$ to ${}^N\vec{\mathbf{v}}^{B_o}$ [this formula should not contain a derivative and uses the fact that B_o and B_N are both points <u>of</u> (**fixed on**) B]. Then, express ${}^N\vec{\mathbf{v}}^{B_N}$ in terms of R, $\dot{x}$, $\dot{\theta}$.
Similarly, provide a derivative-free formula relating ${}^N\vec{\mathbf{a}}^{B_N}$ to ${}^N\vec{\mathbf{a}}^{B_o}$ and then calculate ${}^N\vec{\mathbf{a}}^{B_N}$.
Result:

$$ {}^N\vec{\mathbf{v}}^{B_N} \underset{(8.3)}{=} {}^N\vec{\mathbf{v}}^{B_o} + \underline{\qquad} \times \underline{\qquad} = (\underline{\quad} - \underline{\quad})\,\widehat{\mathbf{n}}_x $$

$$ {}^N\vec{\mathbf{a}}^{B_N} \underset{(8.4)}{=} {}^N\vec{\mathbf{a}}^{B_o} + \underline{\qquad} \times \underline{\qquad} + \underline{\qquad\qquad\qquad} = (\ddot{x} - R\,\ddot{\theta})\,\widehat{\mathbf{n}}_x + \dot{\theta}^2\,R\,\widehat{\mathbf{n}}_y $$

(d) Point B_N's position vector from N_o is ***always*** $\vec{\mathbf{r}}^{B_N/N_o} = x\,\widehat{\mathbf{n}}_x$.
Why is your previous result for ${}^N\vec{\mathbf{v}}^{B_N}$ <u>not</u> the time-derivative in N of $\vec{\mathbf{r}}^{B_N/N_o}$?
Why is your previous result for ${}^N\vec{\mathbf{a}}^{B_N}$ <u>not</u> the time-derivative in N of ${}^N\vec{\mathbf{v}}^{B_N}$?

$$ {}^N\vec{\mathbf{v}}^{B_N} = (\dot{x} - R\,\dot{\theta})\,\widehat{\mathbf{n}}_x \neq \frac{{}^N d\,\vec{\mathbf{r}}^{B_N/N_o}}{dt} $$

$$ {}^N\vec{\mathbf{a}}^{B_N} = (\ddot{x} - R\,\ddot{\theta})\,\widehat{\mathbf{n}}_x + \dot{\theta}^2\,R\,\widehat{\mathbf{n}}_y \neq \frac{{}^N d\,{}^N\vec{\mathbf{v}}^{B_N}}{dt} $$

Explain: $\underline{\qquad\qquad\qquad\qquad\qquad\qquad}$

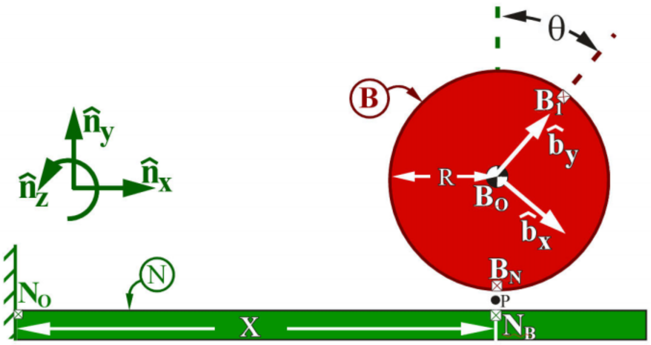
8.2 FE/EIT Review – Velocity and acceleration of a wheel rolling on a plane (Section 9.8).

A thin wheel B of radius R remains in contact with a flat horizontal road N.

B has a simple angular velocity parallel to $\widehat{\mathbf{n}}_z$ ($\widehat{\mathbf{n}}_z$ is perpendicular to the circular portion of B).

The point **of** B in contact with N at each ***instant*** is denoted B_N. The point **of** N in contact with B at each ***instant*** is N_B.

Answer questions below with symbols from Homework 8.1, (e.g., R, θ, $\dot{\theta}$, $\ddot{\theta}$, $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$).



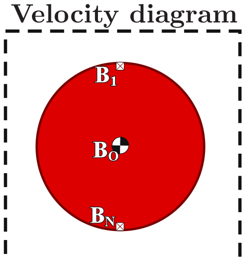
(a) Write the vector definition of ***rolling*** between N and B. Use it to relate $\dot{x}$ to R and $\dot{\theta}$.
Result: $\boxed{\quad} \underset{\text{(rolling)}}{\triangleq} \boxed{\quad} \quad \Rightarrow \quad \dot{x} = \boxed{\quad}$

(b) If B **continuously** rolls on N, can $\dot{x} = R\,\dot{\theta}$ be differentiated to calculate $\ddot{x} = R\,\ddot{\theta}$. **Yes/No**.
For continuous rolling, solve for $x(t)$ in terms of $\theta(t)$ and the initial value $x(0)$ (value of x at $t = 0$).
Result: $x(t) = x(0) + \boxed{\quad}$

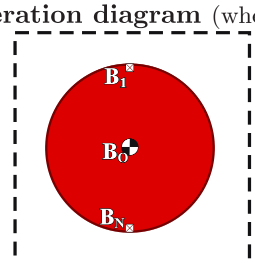
(c) Calculate the following. Herein, regard B_1 as the point of B at the top of the wheel ($\theta = 0$).
Next, **draw** the velocities and accelerations on the wheel when $\dot{\theta}$ is constant ($\ddot{\theta} = 0$).
Result: (Express results **solely** in terms of R, $\dot{\theta}$, $\ddot{\theta}$, $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ - **without** x, $\dot{x}$, $\ddot{x}$.)

Velocity	${}^N\vec{\mathbf{v}}^{B_N} = \boxed{\quad}$	${}^N\vec{\mathbf{v}}^{B_o} = \boxed{\quad}$	${}^N\vec{\mathbf{v}}^{B_1} = \boxed{\quad}$
Acceleration	${}^N\vec{\mathbf{a}}^{B_N} = R\,\dot{\theta}^2\,\widehat{\mathbf{n}}_y$	${}^N\vec{\mathbf{a}}^{B_o} = \boxed{\quad}$	${}^N\vec{\mathbf{a}}^{B_1} = \boxed{\quad}\,\widehat{\mathbf{n}}_x + \boxed{\quad}\,\widehat{\mathbf{n}}_y$

Velocity diagram



Acceleration diagram (when $\ddot{\theta} = 0$)



Answer the following. Assume rolling and sliding imply non-zero motion of B in N (i.e., non-zero $\dot{\theta}$ and/or $\dot{x}$).

- When B ***rolls*** on N, the velocity of B_N in N ***must be*** zero. **True/False**
- When B ***slides*** on N, the velocity of B_N in N ***must be*** zero. **True/False**
- When B ***rolls*** on N, the acceleration of B_N in N ***can be*** zero. **True/False**
- When B ***slides*** on N, the acceleration of B_N in N ***can be*** zero. **True/False**

If B continuously ***slides*** on N and ${}^N\vec{\mathbf{a}}^{B_N} = \vec{\mathbf{0}}$, determine the values/conditions on $\dot{x}$, $\ddot{x}$, $\dot{\theta}$, $\ddot{\theta}$.
Result: $\dot{x}(t) = \boxed{\quad}$ $\quad \ddot{x}(t) = \boxed{\quad}$ $\quad \dot{\theta}(t) = \boxed{\quad}$ $\quad \ddot{\theta}(t) = \boxed{\quad}$