8.1 Velocity and acceleration of a wheel sliding on a plane (Section 9.8).

The following shows a wheel $B$ of radius $R$ in contact with a horizontal road $N$.
Point $N_o$ is fixed on $N$.
Point $B_o$ is the wheel’s geometric center.
Point $B_1$ is fixed to $B$ at the wheel’s periphery.

Right-handed orthogonal unit vectors $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$ and $\hat{b}_x$, $\hat{b}_y$, $\hat{b}_z$ are fixed in $N$ and $B$, with
- $\hat{n}_x$ horizontally-right
- $\hat{n}_y$ vertically-upward
- $\hat{n}_z = \hat{b}_z$ parallel to $B$’s angular velocity in $N$
- $\hat{b}_y$ directed from $B_0$ to $B_1$

$B$’s translation in $N$ is characterized by $x$, the $\hat{n}_x$ measure of $B_o$’s position from $N_o$.
$B$’s rotation in $N$ is characterized by $\theta$, the angle from $\hat{n}_y$ to $\hat{b}_y$ with $-\hat{n}_z$ sense.

**Succinctly** answer each question in terms of $R$, $x$, $\dot{x}$, $\ddot{x}$, $\theta$, $\dot{\theta}$, $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$, $\hat{b}_x$, $\hat{b}_y$, $\hat{b}_z$.

(a) Determine $B$’s angular velocity in $N$ and $B$’s angular acceleration in $N$.
   Use definitions to calculate $B_o$’s velocity in $N$ and $B_1$’s velocity in $N$.
   **Result:**
   \[
   \begin{align*}
   N\omega^B & = \boxed{} \quad N\alpha^B = \boxed{} \\
   N\dot{\omega}^B_o & = \boxed{} \quad N\ddot{\omega}^B_1 = \boxed{\hat{n}_x + \hat{b}_x}
   \end{align*}
   \]

(b) Find $B_1$’s velocity and acceleration in $N$ at the **instant** when $B_1$ is in contact with $N$.
   **Result:**
   \[
   \begin{align*}
   N\dot{\omega}^B_{1\text{contact}} & = (\boxed{-} \boxed{0}) \hat{n}_x \\
   N\ddot{\omega}^B_{1\text{contact}} & = (\boxed{-} \boxed{0}) \hat{n}_x + \boxed{\hat{n}_y}
   \end{align*}
   \]

(c) Point $B_N$ designates the point of $B$ that is in contact with $N$ at each **instant**.
   Provide a formula relating $N\dot{\omega}^B_{N\text{contact}}$ to $N\dot{\omega}^B_0$ [this formula should not contain a derivative and uses the fact that $B_o$ and $B_N$ are both points of (fixed on) $B$]. Then, express $N\dot{\omega}^B_{N\text{contact}}$ in terms of $R$, $\dot{x}$, $\ddot{x}$.
   Similarly, provide a derivative-free formula relating $N\alpha^B_{N\text{contact}}$ to $N\alpha^B_0$ and then calculate $N\alpha^B_{N\text{in contact}}$.
   **Result:** (this is a quicker method for calculating the previous results)
   \[
   \begin{align*}
   N\dot{\omega}^B_{N\text{contact}} & = N\dot{\omega}^B_0 + \boxed{R} \times \boxed{R} \hat{n}_x = (\boxed{-} \boxed{0}) \hat{n}_x \\
   N\alpha^B_{N\text{contact}} & = N\alpha^B_0 + \boxed{R} \times \boxed{R} \hat{n}_x + \boxed{R} \hat{n}_y = (\ddot{x} - \dot{\theta} \hat{b}_y) \hat{n}_x + \hat{b}_y \dot{\theta} R \hat{n}_y
   \end{align*}
   \]

(d) Point $B_N$’s position vector from $N_o$ is **always** $\vec{r}^{B_N/N_o} = x \hat{n}_x$.
   Why is your previous result for $N\dot{\omega}^B_{N\text{in contact}}$ **not** the time-derivative in $N$ of $\dot{\hat{r}}^{B_N/N_o}$?
   Why is your previous result for $N\alpha^B_{N\text{in contact}}$ **not** the time-derivative in $N$ of $\dot{N}\vec{r}^{B_N}$?
   \[
   \begin{align*}
   N\dot{\omega}^B_{N\text{in contact}} & = (\ddot{x} - \dot{\theta} \hat{b}_y) \hat{n}_x \\
   N\alpha^B_{N\text{in contact}} & = (\ddot{x} - \dot{\theta} \hat{b}_y) \hat{n}_x + \hat{b}_y \dot{\theta} R \hat{n}_y
   \end{align*}
   \]

**Explain:**
8.2 FE/EIT Review – Velocity and acceleration of a wheel rolling on a plane (Section 9.8).

A thin wheel $B$ of radius $R$ remains in friction contact with a flat horizontal road $N$.

$B$ has a simple angular velocity parallel to $\hat{n}_z$ ($\hat{n}_z$ is perpendicular to the circular portion of $B$).

The point of $B$ in contact with $N$ at each instant is denoted $B_N$. The point of $N$ in contact with $B$ at each instant is $N_B$.

Answer questions below with symbols from Homework 8.1, (e.g., $R$, $\theta$, $\dot{\theta}$, $\ddot{\theta}$, $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$).

(a) Write the vector definition of rolling between $N$ and $B$. Use it to relate $\dot{x}$ to $R$, $\dot{\theta}$.

Result:

(b) If $B$ continuously rolls on $N$, can $\dot{x} = R \dot{\theta}$ be differentiated to calculate $\ddot{x} = R \ddot{\theta}$. Yes/No.

For continuous rolling, solve for $x(t)$ in terms of $\theta(t)$ and the initial value $x(0)$ (value of $x$ at $t = 0$).

Result: [use $\theta(0) = 0$]

(c) Calculate the following. Herein, regard $B_1$ as the point of $B$ at the top of the wheel ($\theta = 0$). Next, draw the velocities and accelerations on the wheel when $\ddot{\theta}$ is constant ($\ddot{\theta} = 0$).

Result: (Express results solely in terms of $R$, $\dot{\theta}$, $\ddot{\theta}$, $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$ - without $x$, $\dot{x}$, $\ddot{x}$.)

Answer the following. Assume continuous rolling or sliding. Assume non-zero motion of $B$ in $N$ (either $\dot{\theta} \neq 0$ or $\dot{x} \neq 0$).

- When $B$ rolls on $N$, the velocity of $B_N$ in $N$ must be zero. True/False
- When $B$ slides on $N$, the velocity of $B_N$ in $N$ must be zero. True/False
- When $B$ rolls on $N$, the acceleration of $B_N$ in $N$ can be zero. True/False
- When $B$ slides on $N$, the acceleration of $B_N$ in $N$ can be zero. True/False

† Prove your previous answer: If $\ddot{a}^{BN} = \ddot{a}$, can $B$ continuously slide on $N$? Determine $\dot{\theta}$, $\ddot{\theta}$, $\dot{x}$, $\ddot{x}$.

Result: $\dot{\theta}(t) = \ddot{\theta}(t) = \ddot{x}(t) = \dddot{x}(t) = \dddot{x}$ (Slides? Yes/No)