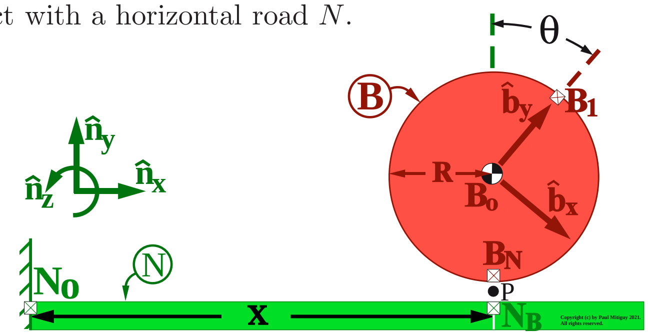
10.1 Velocity and acceleration of a wheel sliding on a plane (Section 11.12).

The figure shows a wheel B of radius R in contact with a horizontal road N.
Point N_o is fixed on N.
Point B_o is the wheel's geometric center.
Point B_1 is fixed to B at the wheel's periphery.



Right-handed orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ and $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$ are fixed in N and B, with

- $\hat{\mathbf{n}}_x$ horizontally-right, $\hat{\mathbf{n}}_y$ vertically-upward.
- $\hat{\mathbf{n}}_z = \hat{\mathbf{b}}_z$ parallel to B's angular velocity in N.
- $\hat{\mathbf{b}}_y$ directed from B_o to B_1.

B's translation in N is characterized by x, the $\hat{\mathbf{n}}_x$ measure of B_o's position from N_o.
B's rotation in N is characterized by θ, the angle from $\hat{\mathbf{n}}_y$ to $\hat{\mathbf{b}}_y$ with -$\hat{\mathbf{n}}_z$ sense.

(a) Determine B's angular velocity in N and B's angular acceleration in N.
Result: $\quad {}^N\vec{\boldsymbol{\omega}}^B = \square\,\hat{\mathbf{n}}_z \qquad {}^N\vec{\boldsymbol{\alpha}}^B = -\ddot{\theta}\,\hat{\mathbf{n}}_z$

(b) Form B_o's position from N_o and B_1's position from N_o. Use **definitions** to calculate B_o's velocity and acceleration in N and B_1's velocity and acceleration in N.
Result:
$${}^{N_o}\vec{\mathbf{r}}^{B_o} = \square\,\hat{\mathbf{n}}_x + \square\,\hat{\mathbf{n}}_y \qquad {}^N\vec{\mathbf{v}}^{B_o} = \square\,\hat{\mathbf{n}}_x \qquad {}^N\vec{\mathbf{a}}^{B_o} = \square\,\hat{\mathbf{n}}_x$$
$${}^{N_o}\vec{\mathbf{r}}^{B_1} = \square\,\hat{\mathbf{n}}_x + \square\,\hat{\mathbf{n}}_y + \square\,\hat{\mathbf{b}}_y \qquad {}^N\vec{\mathbf{v}}^{B_1} = \square\,\hat{\mathbf{n}}_x + \square\,\hat{\mathbf{b}}_x \qquad {}^N\vec{\mathbf{a}}^{B_1} = \square\,\hat{\mathbf{n}}_x + \square\,\hat{\mathbf{b}}_x - \square\,\hat{\mathbf{b}}_y$$

(c) Find B_1's velocity and acceleration in N at the **instant** when B_1 is in contact with N.
Result:
$${}^N\vec{\mathbf{v}}^{B_1}\Big|_{\text{contact}} = (\square - \square)\,\hat{\mathbf{n}}_x \qquad {}^N\vec{\mathbf{a}}^{B_1}\Big|_{\text{contact}} = (\square - \square)\,\hat{\mathbf{n}}_x + \square\,\hat{\mathbf{n}}_y$$

(d) Point B_N designates the point **of** B that is in contact with N at each **instant**.
Relate ${}^N\vec{\mathbf{v}}^{B_N}$ to ${}^N\vec{\mathbf{v}}^{B_o}$ with a formula that does not contain a derivative and uses the fact that both B_o and B_N are points **of** (**fixed on**) B. Then, express ${}^N\vec{\mathbf{v}}^{B_N}$ in terms of R, $\dot{x}$, $\dot{\theta}$.
Similarly, provide a derivative-free formula relating ${}^N\vec{\mathbf{a}}^{B_N}$ to ${}^N\vec{\mathbf{a}}^{B_o}$ and then calculate ${}^N\vec{\mathbf{a}}^{B_N}$.
Result: [Hint: Eqns (10.3, 10.4) are **efficient** formulas for velocity/acceleration of two-points fixed on a body].

$${}^N\vec{\mathbf{v}}^{B_N} \underset{(10.3)}{=} {}^N\vec{\mathbf{v}}^{B_o} + \square \times \square = (\square - \square)\,\hat{\mathbf{n}}_x$$ **(derivative may not mean what you think)**

$${}^N\vec{\mathbf{a}}^{B_N} \underset{(10.4)}{=} {}^N\vec{\mathbf{a}}^{B_o} + \square \times \square + \square = (\ddot{x} - R\,\ddot{\theta})\,\hat{\mathbf{n}}_x + \dot{\theta}^2\,R\,\hat{\mathbf{n}}_y$$

(e) Point B_N's position vector from N_o is ***always*** ${}^{N_o}\vec{\mathbf{r}}^{B_N} = x\,\hat{\mathbf{n}}_x$.
Explain why $\quad {}^N\vec{\mathbf{v}}^{B_N} \neq \dfrac{{}^N d\,({}^{N_o}\vec{\mathbf{r}}^{B_N} = x\,\hat{\mathbf{n}}_x)}{dt}$ and ${}^N\vec{\mathbf{a}}^{B_N} \neq \dfrac{{}^N d\;{}^N\vec{\mathbf{v}}^{B_N}}{dt}$

Explain:

(f)

• The symbols B_N and B_1 sometimes designate the same point.	**True/False**
• Point B_1 is always the name of the same (one) specific point of B.	**True/False**
• Point B_N is always the name of the same (one) specific point of B.	**True/False**
• Point B_N is ***continuously renamed*** as B rotates in N.	**True/False**
• When B_1 is in contact with N, ${}^N\vec{\mathbf{v}}^{B_1}\Big\vert_{\text{contact}}$ has the same meaning as ${}^N\vec{\mathbf{v}}^{B_N}$.	**True/False**

(g) Calculate N_o's velocity and acceleration in B.

Result: $\quad {}^{B}\vec{\mathbf{v}}^{N_o} = (\square - \square)\,\hat{\mathbf{n}}_x - \square\,\hat{\mathbf{n}}_y$

$${}^{B}\vec{\mathbf{a}}^{N_o} = (x\,\dot\theta^2 + R\,\ddot\theta - \ddot x)\,\hat{\mathbf{n}}_x + (R\,\dot\theta^2 - 2\,\dot\theta\,\dot x - x\,\ddot\theta)\,\hat{\mathbf{n}}_y$$

(h) Point N_B designates the point <u>of</u> N that is in contact with B at each instant. Determine N_B's velocity and acceleration in B.
Result: $\quad {}^{B}\vec{\mathbf{v}}^{N_B} = (\square - \square)\,\hat{\mathbf{n}}_x \qquad {}^{B}\vec{\mathbf{a}}^{N_B} = (R\,\ddot\theta - \ddot x)\,\hat{\mathbf{n}}_x + (R\,\dot\theta^2 - 2\,\dot\theta\,\dot x)\,\hat{\mathbf{n}}_y$

(i) The ***path point*** P is the point in space that continuously traces out contact between B and N. Determine P's velocity and acceleration in N and P's velocity and acceleration in B.
Result: $\quad {}^{N}\vec{\mathbf{v}}^{P} = \square\,\hat{\mathbf{n}}_x \qquad {}^{N}\vec{\mathbf{a}}^{P} = \ddot x\,\hat{\mathbf{n}}_x$

$${}^{B}\vec{\mathbf{v}}^{P} = \square\,\hat{\mathbf{n}}_x \qquad {}^{B}\vec{\mathbf{a}}^{P} = R\,\ddot\theta\,\hat{\mathbf{n}}_x + R\,\dot\theta^2\,\hat{\mathbf{n}}_y$$

As defined, the existence of ***path point*** P depends on **contact** between B and N.
If contact ceases, point P ceases to exist.

(j)

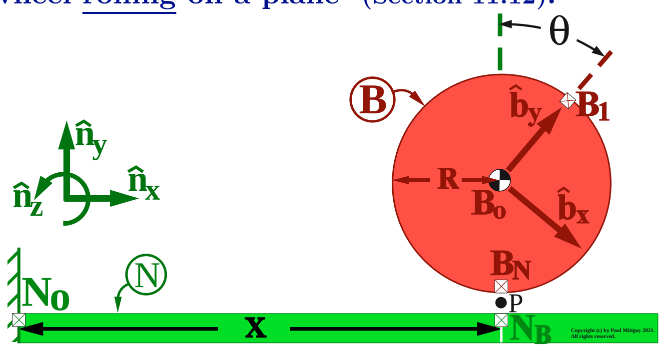
• Points B_N and N_B	**never/sometimes/always**	designate the same point.
• Points B_N and N_B are	**never/sometimes/always**	coincident (co-located).
• Points B_N and P	**never/sometimes/always**	designate the same point.
• Points B_N and P are	**never/sometimes/always**	coincident (co-located).

10.2 FE/EIT – Velocity and acceleration of a wheel <u>rolling</u> on a plane (Section 11.12).

Due to friction, a thin wheel B of radius R rolls on a flat horizontal road N.

The point <u>of</u> B in contact with N at each ***instant*** is denoted B_N. The point <u>of</u> N in contact with B at each ***instant*** is N_B.

Answer questions with symbols from Hw 10.1 (R, θ, $\dot\theta$, $\ddot\theta$, $\dot x$, $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$). Note: ${}^{N}\vec{\boldsymbol{\omega}}^{B} = -\dot\theta\,\hat{\mathbf{n}}_z$ ($\hat{\mathbf{n}}_z$ is perpendicular to the circular portion of B).



(a) Write the vector definition of ***rolling*** between N and B. Use it to relate $\dot x$ to R, $\dot\theta$.
Result: $\quad {}^{\square}\vec{\mathbf{v}}^{\square} \overset{\Delta}{=} {}^{\square}\vec{\mathbf{v}}^{\square}$ (**rolling**) $\quad \Rightarrow$ show work $\quad \dot x = \square$

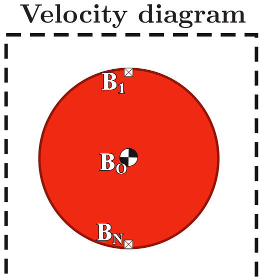
(b) If B <u>**continuously**</u> rolls on N, can $\dot x = R\,\dot\theta$ be differentiated to calculate $\ddot x = R\,\ddot\theta$. **Yes/No**.
For continuous rolling, solve for $x(t)$ in terms of $\theta(t)$ and the initial value $x(0)$ (value of x at $t = 0$).
Result: [use $\theta(0) = 0$] $\quad x(t) = x(0) + \square$

(c) Calculate the following. Herein, regard B_1 as the point of B at the top of the wheel ($\theta = 0$). Next, **draw** the velocities and accelerations on the wheel when $\dot\theta$ is constant ($\ddot\theta = 0$).
Result: [Hint: Eqns (10.3, 10.4) are **efficient** formulas for velocity/acceleration of two-points fixed on a body].

Velocity diagram

In terms of R, $\dot\theta$, $\ddot\theta$.

$${}^{N}\vec{\mathbf{v}}^{B_1} = \square\,\hat{\mathbf{n}}_x$$
$${}^{N}\vec{\mathbf{v}}^{B_o} = \square\,\hat{\mathbf{n}}_x$$
$${}^{N}\vec{\mathbf{v}}^{B_N} = \square$$



Acceleration diagram (when $\ddot\theta = 0$)

In terms of R, $\dot\theta$, $\ddot\theta$.

$${}^{N}\vec{\mathbf{a}}^{B_1} = \square\,\hat{\mathbf{n}}_x - \square\,\hat{\mathbf{n}}_y$$
$${}^{N}\vec{\mathbf{a}}^{B_o} = \square\,\hat{\mathbf{n}}_x$$
$${}^{N}\vec{\mathbf{a}}^{B_N} = R\,\dot\theta^2\,\hat{\mathbf{n}}_y$$