11.1 ♣ Velocity variables and degrees of freedom (Section 11.1).

Determine the minimum number of unknown ***velocity variables*** necessary to characterize the motion of the following systems in a reference frame N. Regard Q as a free-flying particle and A as a rigid body that is **free** to translate and rotate in 3D-space. Choose and **define** velocity variables that suffice to describe the motion (Note: The choice of velocity variables is **not unique**).

Optional: **Sketch** each system with names for each point/body.

System (Q, B, or A and B)	Degrees of freedom	Choice of velocity variables
Free-flying particle Q.	3	$v_x \;\; v_y \;\; v_z$ ${}^N\vec{\mathbf{v}}^Q = v_x\,\widehat{\mathbf{n}}_\mathrm{x} + v_y\,\widehat{\mathbf{n}}_\mathrm{y} + v_z\,\widehat{\mathbf{n}}_\mathrm{z}$
Particle Q moving in a slot (slot is parallel to a unit vector $\vec{\mathbf{n}}$).	1	v ${}^N\vec{\mathbf{v}}^Q = v\,\vec{\mathbf{n}}$
Free-flying rigid body B.	6	$\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y \;\; v_z$ ${}^N\vec{\boldsymbol{\omega}}^B = \omega_x\,\widehat{\mathbf{b}}_\mathrm{x} + \omega_y\,\widehat{\mathbf{b}}_\mathrm{y} + \omega_z\,\widehat{\mathbf{b}}_\mathrm{z}$ ${}^N\vec{\mathbf{v}}^{B_o} = v_x\,\widehat{\mathbf{n}}_\mathrm{x} + v_y\,\widehat{\mathbf{n}}_\mathrm{y} + v_z\,\widehat{\mathbf{n}}_\mathrm{z}$
Rigid body B connected to rigid body A by a revolute joint. (A connects to B at point A_B of A)	7	$\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y \;\; v_z \;\; \omega_B$ ${}^N\vec{\boldsymbol{\omega}}^A = \omega_x\,\widehat{\mathbf{a}}_\mathrm{x} + \omega_y\,\widehat{\mathbf{a}}_\mathrm{y} + \omega_z\,\widehat{\mathbf{a}}_\mathrm{z}$ ${}^N\vec{\mathbf{v}}^{A_B} = v_x\,\widehat{\mathbf{a}}_\mathrm{x} + v_y\,\widehat{\mathbf{a}}_\mathrm{y} + v_z\,\widehat{\mathbf{a}}_\mathrm{z}$ ${}^A\vec{\boldsymbol{\omega}}^B = \omega_B\,\widehat{\boldsymbol{\lambda}}$
Rigid body B connected to rigid body A by a rigid joint.	6	$\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y \;\; v_z$ ${}^N\vec{\boldsymbol{\omega}}^A = \omega_x\,\widehat{\mathbf{a}}_\mathrm{x} + \omega_y\,\widehat{\mathbf{a}}_\mathrm{y} + \omega_z\,\widehat{\mathbf{a}}_\mathrm{z}$ ${}^N\vec{\mathbf{v}}^{A_B} = v_x\,\widehat{\mathbf{n}}_\mathrm{x} + v_y\,\widehat{\mathbf{n}}_\mathrm{y} + v_z\,\widehat{\mathbf{n}}_\mathrm{z}$
Rigid body B connected to A by a ball-and-socket joint.	9	$\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y \;\; v_z \;\; \omega_x^B \;\; \omega_y^B \;\; \omega_z^B$ ${}^N\vec{\boldsymbol{\omega}}^A = \omega_x\,\widehat{\mathbf{a}}_\mathrm{x} + \omega_y\,\widehat{\mathbf{a}}_\mathrm{y} + \omega_z\,\widehat{\mathbf{a}}_\mathrm{z}$ ${}^N\vec{\mathbf{v}}^{A_B} = v_x\,\widehat{\mathbf{a}}_\mathrm{x} + v_y\,\widehat{\mathbf{a}}_\mathrm{y} + v_z\,\widehat{\mathbf{a}}_\mathrm{z}$ ${}^A\vec{\boldsymbol{\omega}}^B = \omega_x^B\,\widehat{\mathbf{b}}_\mathrm{x} + \omega_y^B\,\widehat{\mathbf{b}}_\mathrm{y} + \omega_z^B\,\widehat{\mathbf{b}}_\mathrm{z}$
Rigid body B connected to A by a revolute angular velocity motor. (A revolute angular velocity motor ***specifies*** B's angular velocity in A)	6	$\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y \;\; v_z$ ${}^N\vec{\boldsymbol{\omega}}^A = \omega_x\,\widehat{\mathbf{a}}_\mathrm{x} + \omega_y\,\widehat{\mathbf{a}}_\mathrm{y} + \omega_z\,\widehat{\mathbf{a}}_\mathrm{z}$ ${}^N\vec{\mathbf{v}}^{A_B} = v_x\,\widehat{\mathbf{n}}_\mathrm{x} + v_y\,\widehat{\mathbf{n}}_\mathrm{y} + v_z\,\widehat{\mathbf{n}}_\mathrm{z}$
Rectangular box B sliding on a flat rigid surface fixed in N. (B contacts N at a single vertex B_o of B)	5	$\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y$ ${}^N\vec{\mathbf{v}}^{B_o} = v_x\,\widehat{\mathbf{n}}_\mathrm{x} + v_y\,\widehat{\mathbf{n}}_\mathrm{y}$ ${}^N\vec{\boldsymbol{\omega}}^B = \omega_x\,\widehat{\mathbf{b}}_\mathrm{x} + \omega_y\,\widehat{\mathbf{b}}_\mathrm{y} + \omega_z\,\widehat{\mathbf{b}}_\mathrm{z}$
Rectangular box B sliding on a flat rigid surface fixed in N. (B contacts N on a single edge of B. The edge is parallel to $\widehat{\mathbf{b}}_\mathrm{x}$) (Flat surface is perpendicular to $\widehat{\mathbf{n}}_\mathrm{z}$)	4	$\omega_y \;\; \omega_z \;\; v_x \;\; v_y$ ${}^N\vec{\mathbf{v}}^{B_o} = v_x\,\widehat{\mathbf{n}}_\mathrm{x} + v_y\,\widehat{\mathbf{n}}_\mathrm{y}$ ${}^N\vec{\boldsymbol{\omega}}^B = \omega_y\,\widehat{\mathbf{b}}_\mathrm{x} + \omega_z\,\widehat{\mathbf{n}}_\mathrm{z}$
Rectangular box B sliding on a flat rigid surface fixed in N. (B contacts N on one surface of B)	3	$\omega_z \;\; v_x \;\; v_y$ ${}^N\vec{\mathbf{v}}^{B_o} = v_x\,\widehat{\mathbf{n}}_\mathrm{x} + v_y\,\widehat{\mathbf{n}}_\mathrm{y}$ ${}^N\vec{\boldsymbol{\omega}}^B = \omega_z\,\widehat{\mathbf{b}}_\mathrm{z}$