## 11.1 ♣ Velocity variables and degrees of freedom (Section 11.1).

Determine the minimum number of unknown ***velocity variables*** necessary to characterize the motion of the following systems in a reference frame $N$. Regard $Q$ as a free-flying particle and $A$ as a rigid body that is **free** to translate and rotate in 3D-space. Choose and **define** velocity variables that suffice to describe the motion (Note: The choice of velocity variables is **not unique**).

**Optional: Sketch** each system with names for each point/body.

| System ($Q$, $B$, or $A$ and $B$) | Degrees of freedom | Choice of velocity variables |
|---|---|---|
| Free-flying particle $Q$. | 3 | $v_x \;\; v_y \;\; v_z$ <br> ${}^N\vec{\mathbf{v}}^Q = v_x\,\widehat{\mathbf{n}}_\mathrm{x} + v_y\,\widehat{\mathbf{n}}_\mathrm{y} + v_z\,\widehat{\mathbf{n}}_\mathrm{z}$ |
| Particle $Q$ moving in a slot (slot is parallel to a unit vector $\vec{\mathbf{n}}$). | | |
| Free-flying rigid body $B$. | 6 | $\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y \;\; v_z$ <br> ${}^N\vec{\boldsymbol{\omega}}^B = \omega_x\,\widehat{\mathbf{b}}_\mathrm{x} + \omega_y\,\widehat{\mathbf{b}}_\mathrm{y} + \omega_z\,\widehat{\mathbf{b}}_\mathrm{z}$ <br> ${}^N\vec{\mathbf{v}}^{B_o} = v_x\,\widehat{\mathbf{n}}_\mathrm{x} + v_y\,\widehat{\mathbf{n}}_\mathrm{y} + v_z\,\widehat{\mathbf{n}}_\mathrm{z}$ |
| Rigid body $B$ connected to rigid body $A$ by a revolute joint. <br> ($A$ connects to $B$ at point $A_B$ of $A$) | 7 | $\omega_x \;\; \omega_y \;\; \omega_z \;\; v_x \;\; v_y \;\; v_z \;\; \omega_B$ <br> ${}^N\vec{\boldsymbol{\omega}}^A = \omega_x\,\widehat{\mathbf{a}}_\mathrm{x} + \omega_y\,\widehat{\mathbf{a}}_\mathrm{y} + \omega_z\,\widehat{\mathbf{a}}_\mathrm{z}$ <br> ${}^N\vec{\mathbf{v}}^{A_B} = v_x\,\widehat{\mathbf{a}}_\mathrm{x} + v_y\,\widehat{\mathbf{a}}_\mathrm{y} + v_z\,\widehat{\mathbf{a}}_\mathrm{z}$ <br> ${}^A\vec{\boldsymbol{\omega}}^B = \omega_B\,\widehat{\boldsymbol{\lambda}}$ |
| Rigid body $B$ connected to rigid body $A$ by a rigid joint. | | |
| Rigid body $B$ connected to $A$ by a ball-and-socket joint. | | |
| Rigid body $B$ connected to $A$ by a revolute angular velocity motor. <br> (A revolute angular velocity motor ***specifies*** $B$'s angular velocity in $A$) | | |
| Rectangular box $B$ sliding on a flat rigid surface fixed in $N$. <br> ($B$ contacts $N$ at a single vertex $B_o$ of $B$) | | |
| Rectangular box $B$ sliding on a flat rigid surface fixed in $N$. <br> ($B$ contacts $N$ on a single edge of $B$. The edge is parallel to $\widehat{\mathbf{b}}_\mathrm{x}$) <br> (Flat surface is perpendicular to $\widehat{\mathbf{n}}_\mathrm{z}$) | | |
| Rectangular box $B$ sliding on a flat rigid surface fixed in $N$. <br> ($B$ contacts $N$ on one surface of $B$) | | |