Dynamic Celt
Product of inertia affects spin, pitch, and roll rotational motions

From earliest recorded history, children have skipped stones and spun tops. The challenge in physics is to provide scientific explanations for their curious motions. The concept of "***product of inertia***" and its effect on ***rotational motion*** is difficult to understand. Designed for science instruction, the ***Dynamic Celt*** (also called a ***rattleback*** or ***wobblestone***) provides visual, tactile, and auditory insights into the cause and effect of product of inertia. Beyond its educational appeal, the Dynamic Celt is a static/kinetic friction-verification test for professional motion software.[1]

Spin motions of a Dynamic Celt

To investigate the effect of product of inertia on ***spin*** motions, ensure the Dynamic Celt looks like the one shown to the right (**zero** product of inertia). Place the Dynamic Celt on a flat, hard surface and spin it so it rotates **clockwise**. After observing its motion, spin it **counter-clockwise** and circle the best description of the resulting motion.

Product of inertia: **Zero**

Initial motion	Best description of resulting motion	
Clockwise spin	Simple spin	Spin reversal and wobble
Counter-clockwise spin	Simple spin	Spin reversal and wobble

Now, adjust the bar so that it has a **negative** product of inertia (like the figure to the right – with the bar ≈ 20° from the Celt's long-axis). Place the Dynamic Celt on a flat, hard surface and spin it so it rotates **clockwise**. After observing its motion, spin it **counter-clockwise**.

Product of inertia: **Negative**

Initial motion	Best description of resulting motion	
Clockwise spin	Simple spin	Spin reversal and wobble
Counter-clockwise spin	Simple spin	Spin reversal and wobble

Next, adjust the bar so that it has a **positive** product of inertia (like the figure to the right). Spin the Dynamic Celt so it rotates **clockwise**. After observing its motion, spin it **counter-clockwise**.

Product of inertia: **Positive**

Initial motion	Best description of resulting motion	
Clockwise spin	Simple spin	Spin reversal and wobble
Counter-clockwise spin	Simple spin	Spin reversal and wobble

Adjust the bar so it makes a 5° angle with the Celt's long-axis (bar is nearly aligned with the Celt's long axis). Spin the Celt **counter-clockwise**. Readjust the bar-angle to 45° and repeat.

The bar-angle that creates the largest amplitude wobble is ⟦ 5° ⟧ / 45° (circle one).

 Homework 10: Moments/products of inertia

Pitch motions of a Dynamic Celt

To investigate the effect of product of inertia on **pitch** motions, ensure the Dynamic Celt looks like the one shown to the right (**zero** product of inertia) and place it on a flat, hard surface. Touch its end so it rocks up and down.

Best description of resulting motion		
Mostly pitch	Pitch and clockwise spin	Pitch and counter-clockwise spin

Product of inertia: **Zero**

Now, adjust the bar so that it has a **negative** product of inertia. Touch its end so it rocks up and down.

Best description of resulting motion		
Mostly pitch	Pitch and clockwise spin	Pitch and counter-clockwise spin

Product of inertia: **Negative**

Next, adjust the bar so that it has a **positive** product of inertia. Touch its end so it rocks up and down.

Best description of resulting motion		
Mostly pitch	Pitch and clockwise spin	Pitch and counter-clockwise spin

Product of inertia: **Positive**

Roll motions of a Dynamic Celt

To investigate the effect of product of inertia on **roll** motions, ensure the Dynamic Celt looks like the one shown to the right (zero product of inertia) and place it on a flat, hard surface. Press its side so it **strongly** rolls from side to side.

Best description of resulting motion		
Mostly pitch	Pitch and clockwise spin	Pitch and counter-clockwise spin
Mostly roll	Roll and weak clockwise spin	Roll and weak counter-clockwise spin

Product of inertia: **Zero**

Now, adjust the bar so that it has a **negative** product of inertia. Press its side so it rolls from side to side.

Best description of resulting motion		
Mostly pitch	Pitch and clockwise spin	Pitch and counter-clockwise spin
Mostly roll	Roll and weak clockwise spin	Roll and weak counter-clockwise spin

Product of inertia: **Negative**

Next, adjust the bar so that it has a **positive** product of inertia. Press its side so it rolls from side to side.

Best description of resulting motion		
Mostly pitch	Pitch and clockwise spin	Pitch and counter-clockwise spin
Mostly roll	Roll and weak clockwise spin	Roll and weak counter-clockwise spin

Product of inertia: **Positive**

Summary of spin, pitch, and roll preferences

Consider a Dynamic Celt with a **negative** product of inertia.
For each initial motion, circle the final **spin** direction.

Initial motion	Preferred spin direction	
Spin	[Clockwise]	Counter-clockwise
Pitch	[Clockwise]	Counter-clockwise
Roll	Clockwise	[Counter-clockwise]

Product of inertia: **Negative**

Consider a Dynamic Celt with a **positive** product of inertia.
For each initial motion, circle the final **spin** direction.

Initial motion	Preferred spin direction	
Spin	Clockwise	[Counter-clockwise]
Pitch	Clockwise	[Counter-clockwise]
Roll	[Clockwise]	Counter-clockwise

Product of inertia: **Positive**

How it works?

For more than a century, the Dynamic Celt has attracted the attention of eminent physicists. Most agree that its curious behavior results from three things:

- Friction at the point of contact between Celt and surface

- A curved surface with two different radii of curvature

- Principal axes of curvature that are not aligned with the principal axes of inertia (i.e., there is a non-zero ***product of inertia*** associated with the principal axes of curvature).

This dynamic Celt was designed to have a perfect ellipsoidly-shaped bottom with easily-identifiable principal axes of curvature. The principal inertia axes are adjusted by the bar (adjusting the bar does not change the Celt's mass or mass center location). (Note: Many Celt have grossly imperfect non-ellipsoidal geometry.)

Although detailed mathematical analyses accurately predict the Celt's behavior, there is still great difficultly in understanding the mathematics in physical terms. Quoting Mont Hubbard (a prominent scientist who published detailed experimental and theoretical work on the Celt):

> "**I don't intuitively understand it**".

If you find the Dynamic Celt's motion perplexing, you are in good company. In terms of energy, physicists explain the Celt's behavior as energy exchange between spin, pitching oscillations, and rolling oscillations. In terms of forces, several play a role in the Celt's motion:[2]

- Normal and friction contact forces on the Dynamic Celt

- Local gravity forces

- Centrifugal forces and related forces due to acceleration

The author's best answer to "why does the Dynamic Celt move like this" is $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$.

$\bar{\mathbf{F}} = m\,\bar{\mathbf{a}}$
www.MotionGenesis.com

The physics, math, and simulation of this dynamic celt is at
www.MotionGenesis.com ⇒ Get Started ⇒ Rattleback (dynamic celt).

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[2]Humans seem to have a better intuitive understanding of the cause and effect of contact and distance forces (e.g., friction and gravity) than the centrifugal and Coriolis "forces" associated with acceleration.