

Show work – except for ♣ fill-in-blanks.

**Power, work, potential energy, conservation of energy**

**16.1 ♣ Concepts: Kinetic and potential energy for a system  $S$  of particles or bodies**

Kinetic energy of a system  $S$  in a reference frame  $N$  always exists. **True/False**

Potential energy of a system  $S$  in a reference frame  $N$  always exists. **True/False**

**16.2 ♣ FE/EIT Review – Bungee jumper conservation of energy  $\Delta K + \Delta U = 0$**

In ideal situations, energy can be converted from gravitational potential energy to spring potential energy and/or kinetic energy, and vice-versa, i.e., kinetic energy plus potential energy is “*conserved*” (constant). For example, the following bungee jumper is at **rest** on a platform before a jump. When she starts her jump, gravitational potential energy is converted into kinetic energy as she falls towards the river. At the bottom of her bounce, all her gravitational potential energy and kinetic energy have been converted to spring potential energy. When she first bounces back up, her spring potential energy starts converting to kinetic and gravitational potential energy. At the end of her upward bounce, all energy is converted back to potential energy.

$$U_{\text{gravity}} = m * g * h$$

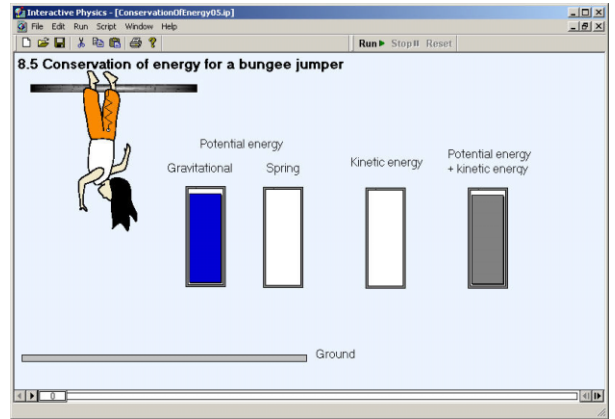
$$U_{\text{spring}} = \frac{1}{2} * k * s^2$$

$$K = \frac{1}{2} * m * v^2$$

where, for a linear spring,  $k$  is the spring constant and  $s$  is the spring stretch.

$$\Delta K + \Delta U = 0$$

*Conservation of mechanical energy*



- (a) The gravitational potential energy is largest when the jumper is at the **top/middle/bottom**, and smallest when she is at the **top/middle/bottom**.
- (b) When the bungee jumper is at the top, there is no stretch in the bungee cord. Therefore, the spring potential energy is **smallest/largest**. At the bottom, the bungee cord is highly stretched, and the spring potential energy is **smallest/largest**.
- (c) The kinetic energy seems to be highest when the jumper is at the **top/middle/bottom** of a bounce. At this point, her speed is **smallest/largest**.
- (d) The sum of potential energy and kinetic energy **increases/decreases/remains the same**.
- (e) Complete the following table. Each row represents a different bungee jumper height.

Gravitational potential energy (Joules)	Spring potential energy (Joules)	Kinetic energy (Joules)	Total (kinetic + potential) energy (Joules)	Bungee jumper height
1000	0	0		<b>top/bottom/in between</b>
356	415		1000	<b>top/bottom/in between</b>
	1000	0	1000	<b>top/bottom/in between</b>
276		200		<b>top/bottom/in between</b>

### 16.8 ♣ Power/energy-rate principle concepts

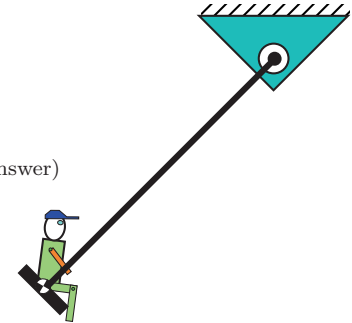
A force of 20 Newtons is to be briefly applied to a child on a swing (modeled as a particle on a 2 m rope). Determine the optimal time to push the child to change the swinging child's kinetic energy. The duration of this force is short compared to the swing's period of oscillation ( $\tau_{\text{period}} \approx \frac{2\pi}{\sqrt{g/L}} \approx \frac{2\pi}{\sqrt{9.8/2}} \approx 2.8$  sec).

To best **increase** kinetic energy, push the child **forward** when (circle the best answer)

- The child just starts moving forward at the top of the swing
- The child is moving quickly forward at the bottom of the swing
- The child is moving quickly backward at the bottom of the swing
- The child just stops moving backward at the top of the swing
- Other (explain):

To best **decrease** kinetic energy, push the child **forward** when (circle the best answer)

- The child just starts moving forward at the top of the swing
- The child is moving quickly forward at the bottom of the swing
- The child is moving quickly backward at the bottom of the swing
- The child just stops moving backward at the top of the swing
- Other (explain):



**Reason:** Putting  in the direction of  increases energy most efficiently.

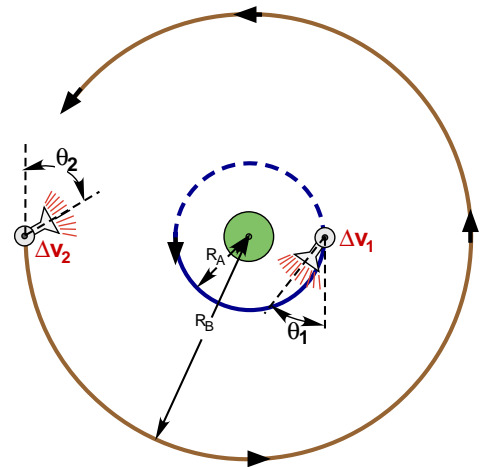
### 16.9 ♣ Power/energy-rate principle concepts: Minimum fuel-use orbit transfer

To thrust a satellite from low circular orbit about Earth to a higher circular orbit, an impulse is provided at two instants.

The first impulse can be directed radially outward, tangent to the satellite's circular orbit, or directed at some angle  $\theta_1$  from the satellite's orbital tangent. The first impulse puts the satellite into an elliptical orbit.

The second impulse is applied at apogee (when the satellite is furthest from Earth) and is directed at an angle  $\theta_2$  from the orbital tangent. The second impulse changes the orbit from elliptical to circular.

Using engineering insights, provide values for  $\theta_1$  and  $\theta_2$  that minimize the fuel required for this orbit transfer, a reason for choosing these values, and **roughly sketch** the trajectory.



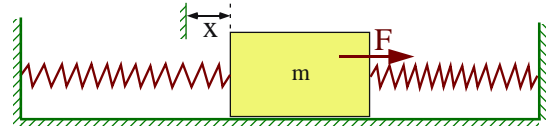
**Result:**  $\theta_1 =$  °       $\theta_2 =$  °

**Reason:** Putting  in the direction of  increases energy most efficiently.

Note: In 1925, Walter Hohmann described a minimum-fuel orbital maneuver (**Hohmann transfer orbit**) that uses two engine impulses to move a spacecraft between two coplanar circular orbits.

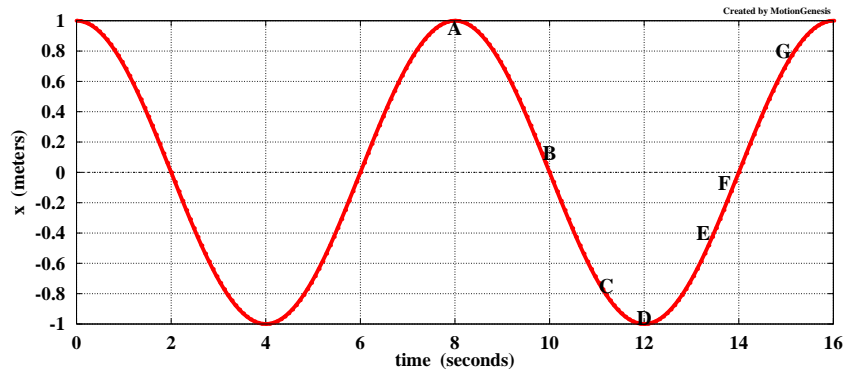
16.10 ♣ **Math & Physics: How to best apply force to increase vibration amplitude?**

A 100 kg rigid block translates back-and-forth between two walls along a straight horizontal frictionless slot. The block is attached to the walls with light linear springs.



A horizontally-right force  $F$  of magnitude 50 Newtons is to be applied to the block for 0.4 seconds. Referring to the following table and graph of  $x(t)$ , rank the most effective start time to apply  $F$  to increase the vibration amplitude of  $x$  (the block's horizontally-right displacement from equilibrium).

Start time	Effectiveness (Best = 1 Worst = 7)
A	<input type="checkbox"/>
B	<input type="checkbox"/>
C	<input type="checkbox"/>
D	<input type="checkbox"/>
E	<input type="checkbox"/>
F	<input type="checkbox"/>
G	<input type="checkbox"/>



**Explain:**