20.1 ♣ Concepts: Kinetic and potential energy for a system S of particles or bodies

Kinetic energy of a system S in a reference frame N always exists. **True/False**
Potential energy of a system S in a reference frame N always exists. **True/False**

20.2 ♣ FE/EIT Review – Bungee jumper conservation of energy $\boxed{\Delta K + \Delta U = 0}$

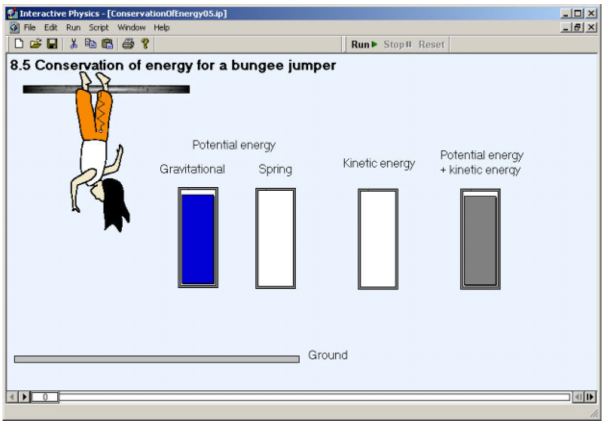
In ideal situations, energy can be converted from gravitational potential energy to spring potential energy and/or kinetic energy, and vice-versa, i.e., kinetic energy plus potential energy is "***conserved***" (constant). For example, the following bungee jumper is at **rest** on a platform before a jump. When she starts her jump, gravitational potential energy is converted into kinetic energy as she falls towards the river. At the bottom of her bounce, all her gravitational potential energy and kinetic energy have been converted to spring potential energy. When she first bounces back up, her spring potential energy starts converting to kinetic and gravitational potential energy. At the end of her upward bounce, all energy is converted back to potential energy.

$$\boxed{\begin{aligned} U_{\text{gravity}} &= m * g * h \\ U_{\text{spring}} &= \frac{1}{2} * k * s^2 \\ K &= \frac{1}{2} * m * v^2 \end{aligned}}$$

where, for a linear spring, k is the spring constant and s is the spring stretch.

$$\boxed{\Delta K + \Delta U = 0}$$

Conservation of mechanical energy



(a) The gravitational potential energy is largest when the jumper is at the **top/middle/bottom,** and smallest when she is at the **top/middle/bottom**.

(b) When the bungee jumper is at the top, there is no stretch in the bungee cord. Therefore, the spring potential energy is **smallest/largest**. At the bottom, the bungee cord is highly stretched, and the spring potential energy is **smallest/largest**.

(c) The kinetic energy seems to be highest when the jumper is at the **top/middle/bottom** of a bounce. At this point, her speed is **smallest/largest**.

(d) The sum of potential energy and kinetic energy **increases/decreases/remains the same**.

(e) Complete the following table. Each row represents a different bungee jumper height.

Gravitational potential energy (Joules)	Spring potential energy (Joules)	Kinetic energy (Joules)	Total (kinetic + potential) energy (Joules)	Bungee jumper height
1000	0	0		**top/bottom/in between**
356	415		1000	**top/bottom/in between**
	1000	0	1000	**top/bottom/in between**
276		200		**top/bottom/in between**

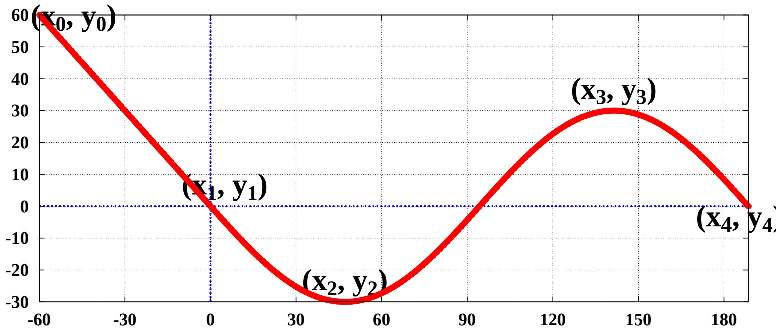
20.3 FE/EIT Review – Rollercoaster: $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ and conservation of mechanical energy

The shape of a roller-coaster track is approximated by a function $y(x)$ having two parts.

The first part is a straight line connecting ($x = -60$, $y = 60$) to (0, 0).

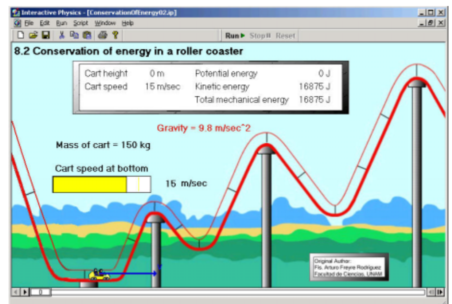
The second part is a sine curve with amplitude 30 m and wave-length ($60\,\pi \approx 188.5$) m.

$$y(x) = \begin{cases} -x & x < 0 \\ -30\,\sin\left(\frac{x}{30}\right) & x \geq 0 \end{cases}$$



A roller-coaster car is released from **rest** at ($x = -60$, $y = 60$). Knowing the track is smooth (frictionless), Earth's gravitational acceleration $g \approx 10\ \frac{\text{m}}{\text{s}^2}$, and the roller-coaster car with occupants is modeled as a particle with mass $m = 100$ kg, calculate the roller-coaster speed at each location $x_0 \ldots x_4$ (3^+ digits). Determine the time t_1 when the roller-coaster car is at x_1.

† **Optional:** Determine times $t_2 \ldots t_4$.



Location		Frictionless	
x (m)	y (m)	time t (sec)	Speed (m/s)
$x_0 = -60$	$y_0 = 60$	$t_0 = 0$	$v_0 = 0$
$x_1 = 0$	$y_1 = 0$	$t_1 \approx$	$v_1 =$
$x_2 = 15\,\pi$	$y_2 = -30$	$t_2 \approx 6.4$	$v_2 =$
$x_3 = 45\,\pi$	$y_3 = 30$	$t_3 \approx 9.8$	$v_3 =$
$x_4 = 60\,\pi$	$y_4 = 0$	$t_4 \approx 11.8$	$v_4 = \sqrt{1200} \approx 34.6$

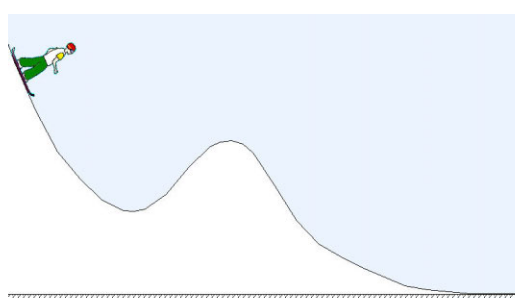
Note: To find $t_2 \ldots t_4$, it is helpful to run a simulation that outputs $\frac{x}{\pi}$ and $t + 4.898979$.

†† **Optional:** Determine the same quantities using a coefficient of kinetic friction $\mu_k = 0.1$.

20.4 Snowboarder: Conservation of energy to normal forces $\boxed{\Delta K + \Delta U = 0 \quad \Rightarrow \quad \vec{\mathbf{v}} \quad \Rightarrow \quad \vec{\mathbf{F}}}$

In ideal situations, potential energy can be converted to kinetic energy and vice versa. i.e., kinetic energy plus potential energy is "***conserved***" (constant). For example, a snowboarder gains potential energy as he is chair-lifted to the top of the mountain. As he goes down a slope, his potential energy decreases and his speed and kinetic increase.

$$\boxed{\begin{aligned} U_{\text{gravity}} &= m * g * h \\ K &= \frac{1}{2} * m * v^2 \end{aligned}}$$



When there is negligible air-resistance, friction, etc., the change of the snowboarder's kinetic plus potential energy from an "initial" state to a " final" state is zero.

$$\boxed{\Delta K + \Delta U = 0}$$

Conservation of mechanical energy

Model the snowboarder as a particle of mass m that starts from **rest** from a height h_i above the bottom of the hill.

(a) Express the snowboarder's speed v_f at a height h_f above the bottom of the hill (in terms of g, h_i, h_f). Using $g \approx 10\ \frac{\text{m}}{\text{s}^2}$, calculate the snowboarder's speed when boarding down a 30° slope for a distance 160 m (along the slope). Report results in both $\frac{\text{m}}{\text{s}}$ and mph ($1\ \frac{\text{m}}{\text{s}} \approx 2.2$ mph).