

Show work – except for ♣ fill-in-blanks.

**Moments, torques, and static equilibrium.**

**17.1 ♣ Concepts: Define and draw the moment of a force**

Write the *definition* for the moment of force  $\vec{F}^Q$  applied to point  $Q$  about point  $O$ . Draw a sketch with *each* part of your definition clearly labeled.



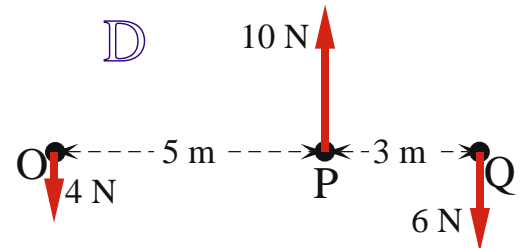
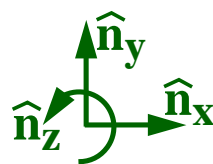
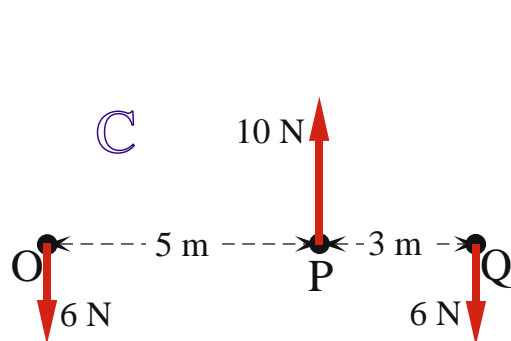
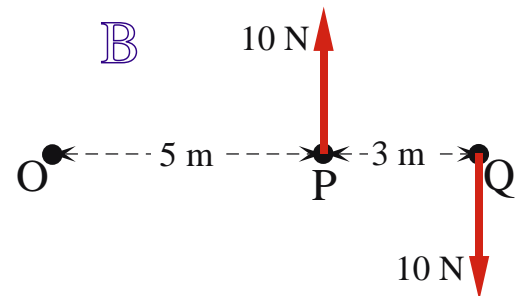
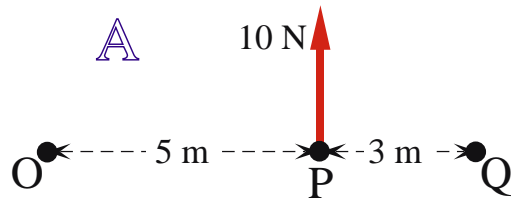
Result:

$$\vec{M}^{\vec{F}^Q/O} \triangleq \text{[ ]} \times \text{[ ]}$$

**17.2 ♣ Moment vs. torque (refer to Section 19.5)**

Consider the various sets  $S$  of forces, their resultants  $\vec{F}^S$ , and moments about points  $O$ ,  $P$ , and  $Q$ . This example shows how to easily determine whether a moment is a torque.<sup>1</sup>

$S$	$\vec{F}^S$	$\vec{M}^{S/O}$	$\vec{M}^{S/P}$	$\vec{M}^{S/Q}$	$\vec{M}^{S/O} \stackrel{?}{=} \vec{M}^{S/P} \stackrel{?}{=} \vec{M}^{S/Q}$	Moment is torque?
A	$10 \hat{n}_y$	$50 \hat{n}_z$	$\vec{0}$	[ ]	Yes/No	Yes/No
B	[ ]	[ ]	[ ]	[ ]	Yes/No	Yes/No
C	[ ]	[ ]	[ ]	[ ]	Yes/No	Yes/No
D	[ ]	[ ]	[ ]	[ ]	Yes/No	Yes/No



**17.3 ♣ Moment and torque concepts**

75% All torques are moments.

True/False

61% All moments are torques.

True/False

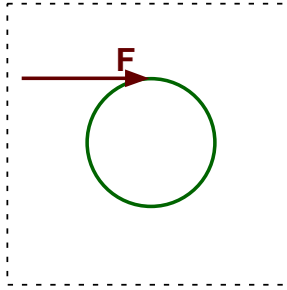
61% The moment of a couple about a point  $O$  is equal to the moment of the couple about any other point  $P$  True/False

<sup>1</sup>Since  $\vec{T}^S \triangleq \vec{M}^{S/O}$  if  $\vec{F}^S = \vec{0}$  (point  $O$  is *any* point), the *moment* is a *torque* if  $\vec{F}^S = \vec{0}$  (it is that simple).

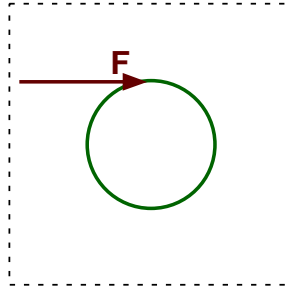
### 17.4 ♣ Drawing couples

Each figure below shows a single force  $\vec{F}$  applied tangentially to a point on the periphery of a circle. Complete each figure by drawing couples consisting of **2**, **3**, and **4** forces, respectively, so:

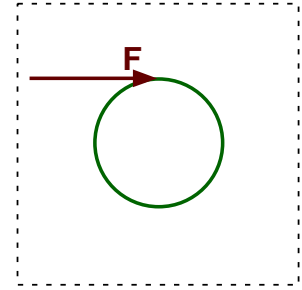
- Each force has magnitude  $|\vec{F}|$  and is applied at distinct points on the circle's periphery
- Each force is directed **tangent** to the circle's periphery
- The set of forces create a couple with non-zero torque



Couple with **2** forces



Couple with **3** forces

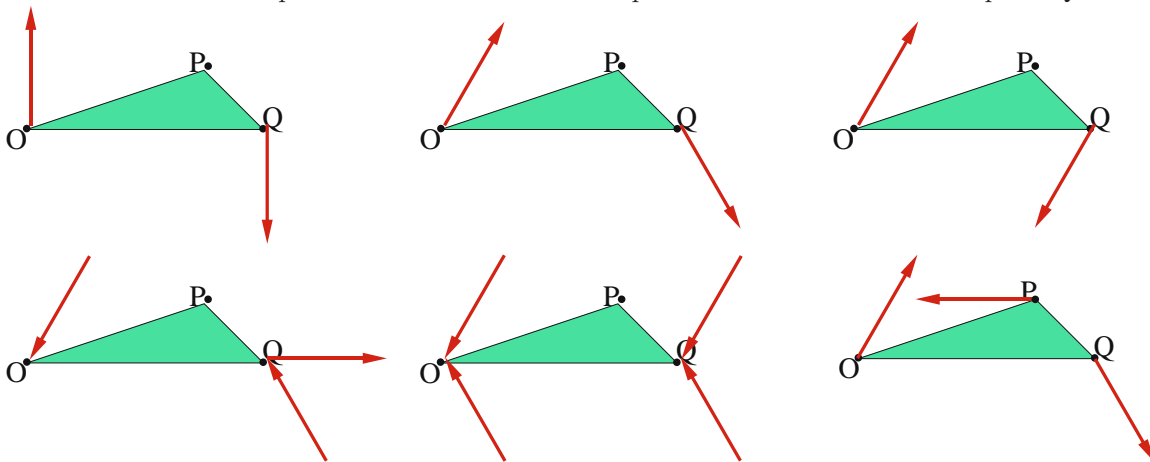


Couple with **4** forces

### 17.5 ♣ Moments of forces about various points

Consider the six figures below, each which contain a set of forces. Circle the figure(s) in which the moment of its set of forces about points  $O$ ,  $P$ , and  $Q$  all are equal, i.e.,

$$\text{Moment around point } O = \text{Moment around point } P = \text{Moment around point } Q$$



Note: All forces have the same magnitude. Forces that are not horizontal or vertical are  $30^\circ$  from vertical.

### 17.6 ♣ Forces, moments, and lines of action

**Draw** a non-zero force  $\vec{F}^P$  on point  $P$  and a non-zero force  $\vec{F}^Q$  on point  $Q$  so:

- $\vec{F}^P = \vec{F}^Q$  (force on  $P$  has the same magnitude and direction as the force on  $Q$ )
- $\vec{M}^{\vec{F}^P/O}$  (moment of  $\vec{F}^P$  about point  $O$ ) is **equal** to  $\vec{M}^{\vec{F}^Q/O}$  (moment of  $\vec{F}^Q$  about  $O$ ).

Repeat on the right-figure below, except ensure  $\vec{F}^P$  and  $\vec{F}^Q$  produce **unequal** moments about  $O$ .

<p style="text-align: center;"><math>\vec{F}^P = \vec{F}^Q \quad \vec{M}^{\vec{F}^P/O} = \vec{M}^{\vec{F}^Q/O}</math></p> <div style="border: 1px dashed black; padding: 20px; text-align: center;"> </div> <p><math>\vec{F}^P</math> has the same line of action as <math>\vec{F}^Q</math>.</p>	<p style="text-align: center;"><math>\vec{F}^P = \vec{F}^Q \quad \vec{M}^{\vec{F}^P/O} \neq \vec{M}^{\vec{F}^Q/O}</math></p> <div style="border: 1px dashed black; padding: 20px; text-align: center;"> </div> <p><math>\vec{F}^P</math> has a different line of action than <math>\vec{F}^Q</math>.</p>
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