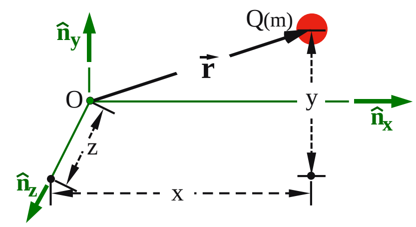
10.1 ♣ Concepts: What objects have a moment of inertia? (Section 12.1).

Consider the ***moment of inertia*** $I^{S/O}_{\hat{\mathbf{u}}\,\hat{\mathbf{u}}}$ of an object S about a point O for the unit vector $\hat{\mathbf{u}}$.
In general, for $I^{S/O}_{\hat{\mathbf{u}}\,\hat{\mathbf{u}}}$ to be a positive real number, S should be a (circle ***all*** appropriate objects):

Real number	Matrix	Set of points	Mass center of a rigid body
Vector	Point	Reference frame	Flexible body
3D orthogonal unit basis	Particle	Rigid body	System of particles and bodies

10.2 ♣ Formulas for a particle's moments and products of inertia (Sections 12.1 and 12.2).

The figure shows a particle Q of mass m and right-handed orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$.
Q's position vector from a point O is $x\,\hat{\mathbf{n}}_x + y\,\hat{\mathbf{n}}_y + z\,\hat{\mathbf{n}}_z$.



Express I_{xx} (Q's ***moment of inertia*** about O for $\hat{\mathbf{n}}_x$) in terms of some or all of m, x, y z. Similarly for I_{yy} and I_{zz}.

Express I_{xy} (Q's ***product of inertia*** about O for $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{n}}_y$) in terms of some or all of m, x, y z. Similarly for I_{xz} and I_{yz}.

Result:

$I_{xx} = \square\,(\square^2 + \square^2)$ $\quad I_{yy} = \square$ $\quad I_{zz} = \square$

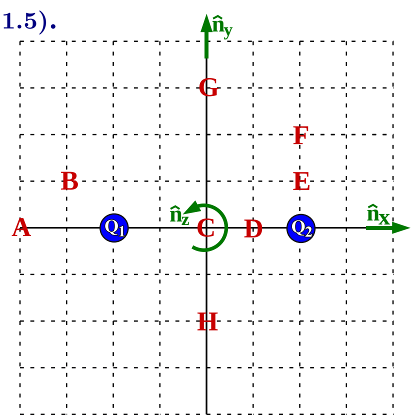
$I_{xy} = -\square\,\square\,\square$ $\quad I_{xz} = \square$ $\quad I_{yz} = \square$

Circa 1895, Gibbs invented the ***inertia dyadic*** as a **convenient "suitcase"** for holding moments and products of inertia. Write Q's inertia dyadic about O in terms of $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ and I_{ij} $(i, j = x, y, z)$. If needed, refer to Section 14.1.

$$\overset{\Rightarrow}{I} = I_{xx}\,\hat{\mathbf{n}}_x\,\hat{\mathbf{n}}_x + I_{xy}\,\hat{\mathbf{n}}_x\,\hat{\mathbf{n}}_y + \square\,\hat{\mathbf{n}}_x\,\hat{\mathbf{n}}_z + I_{xy}\,\hat{\mathbf{n}}_y\,\hat{\mathbf{n}}_x + \square\,\hat{\mathbf{n}}_y\,\hat{\mathbf{n}}_y + \square + \square + \square + \square$$

10.3 ♣ Parallel axis theorem and moments of inertia (Section 12.1.5).

The system S shown to the right consists of particles Q_1 and Q_2, each of mass m, in a plane perpendicular to the unit vector $\hat{\mathbf{n}}_z$.



The ***shift theorem*** (also called the ***parallel axis theorem***) shifts S's moment of inertia about S_{cm} (the ***mass center*** of S) for the unit vector $\hat{\mathbf{n}}_z$ to an arbitrary point P in the plane using

$$I^{S/P}_{zz} = I^{S/S_{cm}}_{zz} + m_S * d^2$$

where $I^{S/S_{cm}}_{zz}$ is the system's moment of inertia about S_{cm} for $\hat{\mathbf{n}}_z$, m_S is the mass of S, and d is the distance from S_{cm} to P.

Use the shift theorem to estimate the order of S's moment of inertia for the lines parallel to $\hat{\mathbf{n}}_z$ that pass through points A, B, C, D, E, F, G, and H, respectively. Note: Grid lines are equally spaced.

Result:

Smallest									Largest

Knowing each particle has mass $m = 1$ kg and the grid lines are spaced 1 m apart, calculate S's moment of inertia about A, B, C, D, E, F, G, and H, respectively.

Result: (in $\text{kg}\,\text{m}^2$)

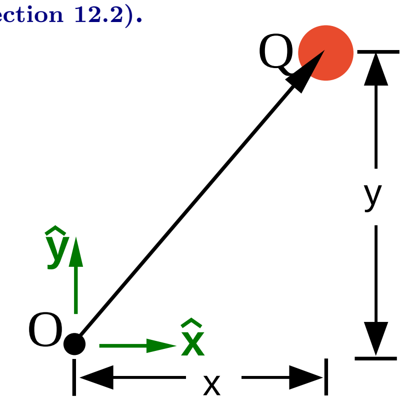
$I^{S/A}_{zz}$	$I^{S/B}_{zz}$	$I^{S/C}_{zz}$	$I^{S/D}_{zz}$	$I^{S/E}_{zz}$	$I^{S/F}_{zz}$	$I^{S/G}_{zz}$	$I^{S/H}_{zz}$
		8					

10.4 ♣ Sign conventions for products of inertia (Section 12.2.4).

There are two sign conventions (±) for products of inertia which often lead to errors **True/False**.

10.5 ♣ Calculations: Product of inertia for a single particle (Section 12.2).

The ***product of inertia*** of a single particle Q about a point O for the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ directions is calculated by the formula



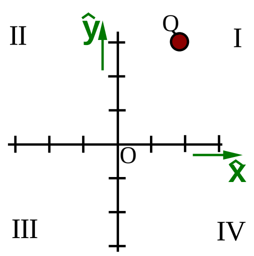
$$\mathrm{I_{xy}} = -m\, x\, y$$

- m is the mass of Q
- x is the $\hat{\mathbf{x}}$ measure of Q's position from O
- y is the $\hat{\mathbf{y}}$ measure of Q's position from O

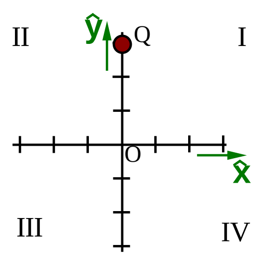
For example, if $m = 1$ kg, $x = 2$ m, and $y = 3$ m,

$$\mathrm{I_{xy}} = -(1\text{ kg})\,(2\text{ m})\,(3\text{ m}) = -6\text{ kg m}^2$$

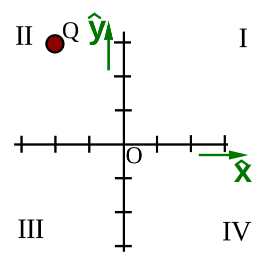
Knowing particle Q has a mass of 1 kg and each tick-mark represents 1 m, calculate Q's ***product of inertia*** $\mathrm{I_{xy}}$ about point O for each figure below.



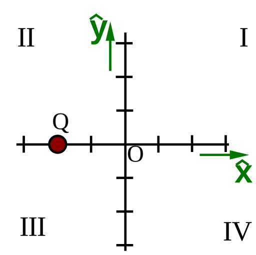
$\mathrm{I_{xy}} = -6$ kg m^2



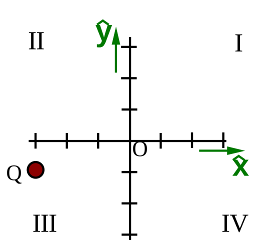
$\mathrm{I_{xy}} =$ ☐ kg m^2



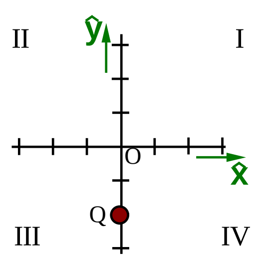
$\mathrm{I_{xy}} =$ ☐ kg m^2



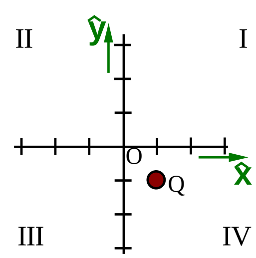
$\mathrm{I_{xy}} =$ ☐ kg m^2



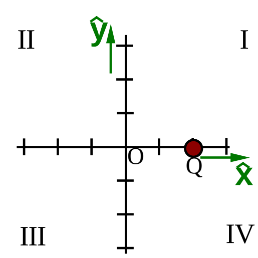
$\mathrm{I_{xy}} = -3$ kg m^2



$\mathrm{I_{xy}} =$ ☐ kg m^2



$\mathrm{I_{xy}} =$ ☐ kg m^2



$\mathrm{I_{xy}} =$ ☐ kg m^2

Circle the correct answer (negative, zero, or positive) for each statement about particle Q.

- When Q is in quadrant **I**, $\mathrm{I_{xy}}$ is **negative/zero/positive**.
- When Q is in quadrant **II**, $\mathrm{I_{xy}}$ is **negative/zero/positive**.
- When Q is in quadrant **III**, $\mathrm{I_{xy}}$ is **negative/zero/positive**.
- When Q is in quadrant **IV**, $\mathrm{I_{xy}}$ is **negative/zero/positive**.
- When Q is on a quadrant boundary, $\mathrm{I_{xy}}$ is **negative/zero/positive**.