

13.4 ♣ **Calculations: Product of inertia for a single particle** (Section 14.2).

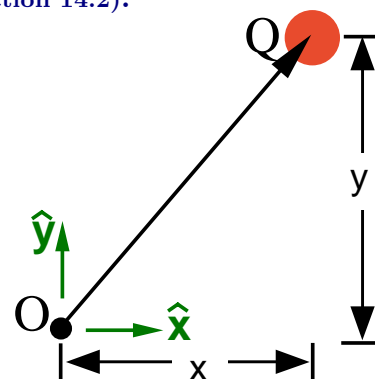
The *product of inertia* of a single particle Q about a point O for the \hat{x} and \hat{y} directions is calculated by the formula

$$I_{xy} = -mxy$$

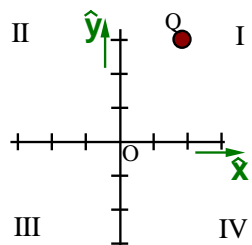
- m is the mass of Q
- x is the \hat{x} measure of Q 's position vector from O
- y is the \hat{y} measure of Q 's position vector from O

For example, if $m = 1$ kg, $x = 2$ m, and $y = 3$ m,

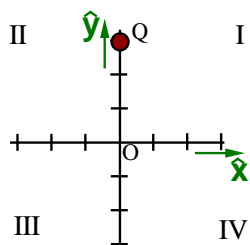
$$I_{xy} = -(1 \text{ kg})(2 \text{ m})(3 \text{ m}) = -6 \text{ kg m}^2$$



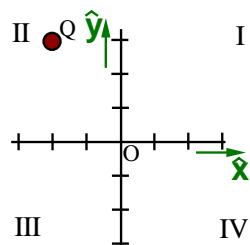
Knowing particle Q has a mass of 1 kg and each tick-mark represents 1 m, calculate Q 's *product of inertia* I_{xy} about point O for each figure below.



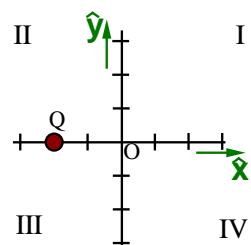
$$I_{xy} = -6 \text{ kg m}^2$$



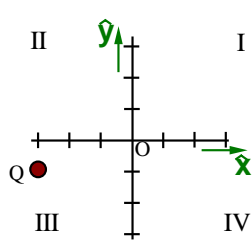
$$I_{xy} = \square \text{ kg m}^2$$



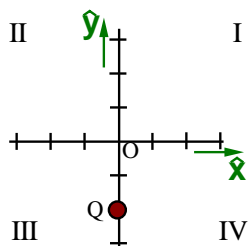
$$I_{xy} = \square \text{ kg m}^2$$



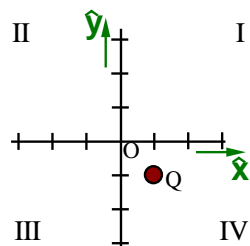
$$I_{xy} = \square \text{ kg m}^2$$



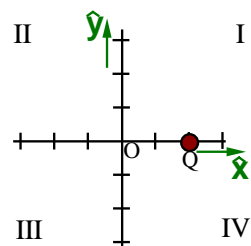
$$I_{xy} = -3 \text{ kg m}^2$$



$$I_{xy} = \square \text{ kg m}^2$$



$$I_{xy} = \square \text{ kg m}^2$$



$$I_{xy} = \square \text{ kg m}^2$$

Circle the correct answer (negative, zero, or positive) for each statement about particle Q .

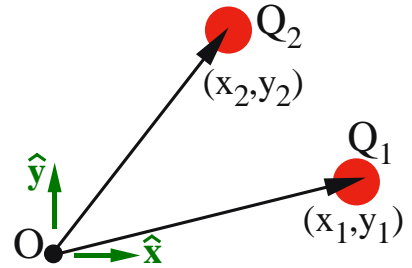
- When Q is in quadrant **I**, I_{xy} is **negative/zero/positive**.
- When Q is in quadrant **II**, I_{xy} is **negative/zero/positive**.
- When Q is in quadrant **III**, I_{xy} is **negative/zero/positive**.
- When Q is in quadrant **IV**, I_{xy} is **negative/zero/positive**.
- When Q is on a quadrant boundary, I_{xy} is **negative/zero/positive**.

13.5 ♣ Calculations: Product of inertia for a system of particles (Section 14.2).

The *product of inertia* of a system of particles is simply the sum of the products of inertias of each of the individual particles. For example, the product of inertia of particles Q_1 and Q_2 about point O for the \hat{x} and \hat{y} directions is calculated by the formula

$$I_{xy} = \sum_{i=1}^2 -m_i x_i y_i$$

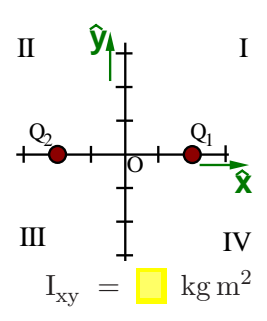
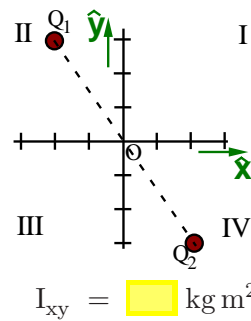
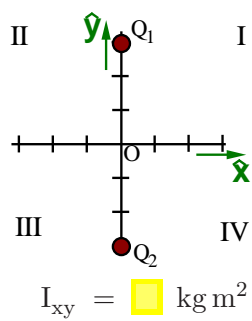
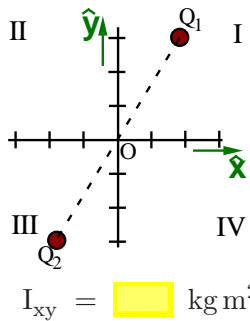
- m_i is the mass of particle Q_i ($i=1, 2$)
- x_i is the \hat{x} measure of Q_i 's position vector from O
- y_i is the \hat{y} measure of Q_i 's position vector from O



For example, if $m_1 = 1$ kg, $x_1 = 3$ m, $y_1 = 1$ m, and $m_2 = 2$ kg, $x_2 = 2$ m, $y_2 = 3$ m,

$$I_{xy} = -m_1 x_1 y_1 + -m_2 x_2 y_2 = -(1 \text{ kg})(3 \text{ m})(1 \text{ m}) + -(2 \text{ kg})(2 \text{ m})(3 \text{ m}) = -15 \text{ kg m}^2$$

Knowing each particle has a mass of 1 kg and each tick-mark represents 1 m, calculate the system's *product of inertia* I_{xy} about point O for each figure below.



Circle the correct answer (negative, zero, or positive) for each of the following statements.

- When the particles are in quadrants **I** and **III**, I_{xy} is **negative/zero/positive**.
- When the particles are in quadrants **II** and **IV**, I_{xy} is **negative/zero/positive**.
- When the particles are on quadrant boundaries, I_{xy} is **negative/zero/positive**.

13.6 ♣ Concepts: Products of inertia of propellers (Section 14.2.2).

The following shows four uniform-density objects. For each object, consider I_{xy} the product of inertia of the object for lines that pass through point O and are parallel to \hat{n}_x and \hat{n}_y . For each object, *visually determine* whether the product of inertia is *negative*, *zero*, or *positive*.

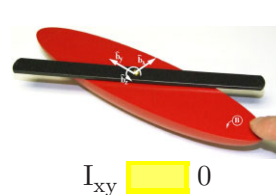
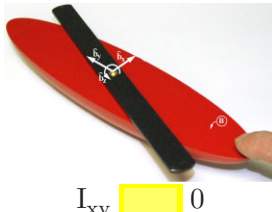
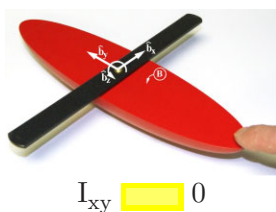
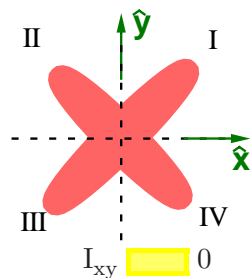
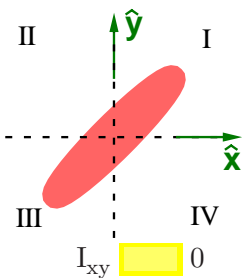
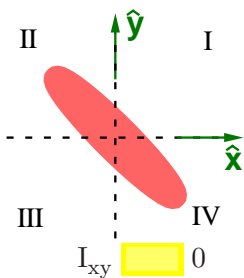
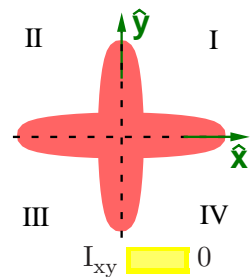
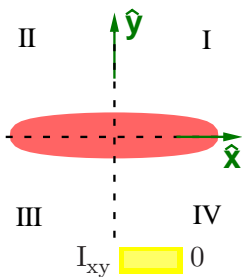
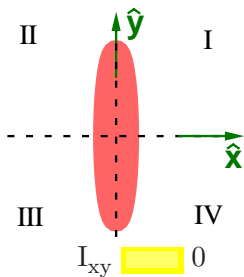
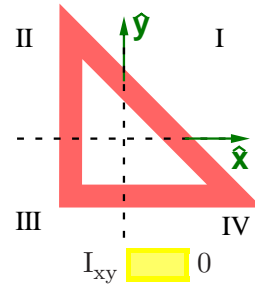
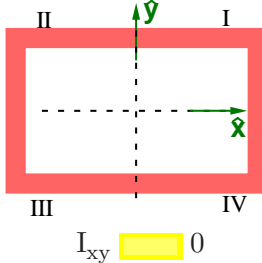
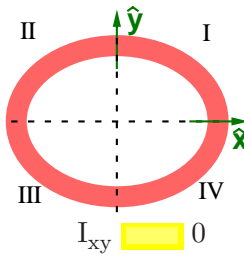
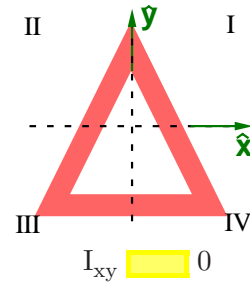
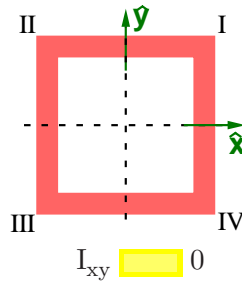
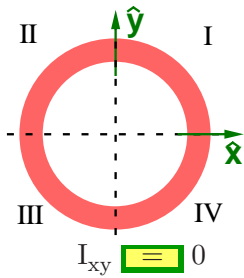
1. Propeller with one blade along the positive y-axis: **zero**
 2. Propeller with one blade along the positive x-axis: **negative**
 3. Propeller with one blade along the negative x-axis: **negative**
 4. Propeller with three blades symmetric about the y-axis: **zero**
 5. Propeller with three blades symmetric about the x-axis: **[]**
 6. Propeller with four blades symmetric about both axes: **[]**
 7. Propeller with five blades symmetric about both axes: **[]**
 8. Propeller with three blades symmetric about the y-axis: **Guess**

13.7 ♣ Concepts: Products of inertia – what is I_{xy} ? (Section 14.2.2).

Product of inertia is a measure of the symmetry of mass distribution in two directions about a point. To investigate this concept, use your geometrical insights (not equations) to determine which of the following uniform-density objects have a negative, zero, or positive **product of inertia** I_{xy} .

Visually sum the mass distribution for I_{xy} in quadrants **II** and **IV** and compare that to the mass distribution in quadrants **I** and **III**. Complete each blank below with $<$ or $=$ or $>$.

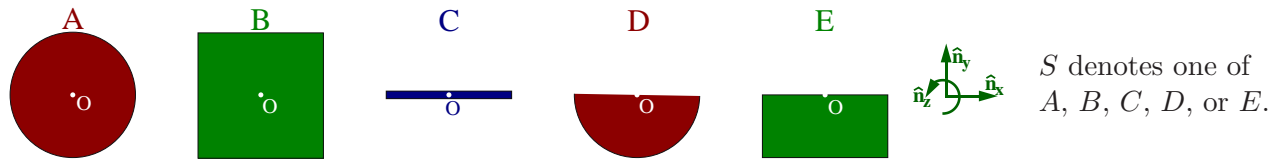
- If mass distribution in quadrants **II** + **IV** is greater than in quadrants **I** + **III**, $I_{xy} > 0$.
- If mass distribution in quadrants **II** + **IV** is smaller than in quadrants **I** + **III**, $I_{xy} < 0$.
- If mass distribution in quadrants **II** + **IV** is equal to quadrants **I** + **III**, $I_{xy} = 0$.



Purchase rattleback at www.arbor-sci.com. Explained: <http://www.youtube.com/watch?v=0RyLV-Fsl4A>

13.8 ♣ **Conceptual understanding of moments and products of inertia** (Sections 14.1.2 and 14.2.2).

Objects $A, B, C, D,$ and E are all flat planar objects with uniform density and the *same* mass. The circle and semi-circle's diameter, square and rectangle's width, and thin rod's length are *equal*.



36% Consider $I_{zz}^{S/O}$, S 's moment of inertia about the line passing through point O and parallel to \hat{n}_z . Knowing moment of inertia is mass * distance², use visual estimates to list the objects in ascending order of $I_{zz}^{S/O}$. If two objects have the same value of $I_{zz}^{S/O}$, group them together.

Result:

Smallest						Largest
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28% Consider $I_{zz}^{S/S_{cm}}$, S 's moment of inertia about the line passing through S_{cm} (the mass center of S) and parallel to \hat{n}_z . Use visual estimates to list the objects in ascending order of $I_{zz}^{S/S_{cm}}$. Note: A and E have nearly equal $I_{zz}^{S/S_{cm}}$. The textbook's inertia appendix helps resolve their difference.

Result:

Smallest	D (given)					Largest
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23% Consider $I_{xy}^{S/O}$, S 's product of inertia for point O and unit vectors \hat{n}_x and \hat{n}_y . For each object, visually determine if $I_{xy}^{S/O}$ is negative (-), zero (0), or positive (+).

Result:

A	B	C	D	E
- 0 +	- 0 +	- 0 +	- 0 +	- 0 +

13.9 ♣ **Assembling inertia dyadics from the textbook's inertia appendix** (Sections 16.1 and 16.5).

Referring to the following figures and the textbook's inertia appendix, assemble $\vec{\mathbf{I}}^{B/B_{cm}}$ (B 's inertia dyadic about its center of mass B_{cm}) for the unit vectors $\hat{b}_x, \hat{b}_y, \hat{b}_z$. Express results in matrix form.

$$\begin{bmatrix} \text{yellow} & \text{yellow} & 0 \\ \text{yellow} & \frac{1}{4}ma^2 & 0 \\ 0 & 0 & \text{yellow} \end{bmatrix}_{\hat{b}_{xyz}}$$

$$\begin{bmatrix} \text{yellow} & \text{yellow} & \text{yellow} \\ \text{yellow} & \text{yellow} & \text{yellow} \\ \text{yellow} & \text{yellow} & \text{yellow} \end{bmatrix}_{\hat{b}_{xyz}}$$

$$\begin{bmatrix} \text{yellow} & \text{yellow} & \text{yellow} \\ \text{yellow} & \text{yellow} & \text{yellow} \\ \text{yellow} & \text{yellow} & \text{yellow} \end{bmatrix}_{\hat{b}_{xyz}}$$

$$\begin{bmatrix} \text{yellow} & \text{yellow} & \text{yellow} \\ \text{yellow} & \text{yellow} & \text{yellow} \\ \text{yellow} & \text{yellow} & \text{yellow} \end{bmatrix}_{\hat{b}_{xyz}}$$

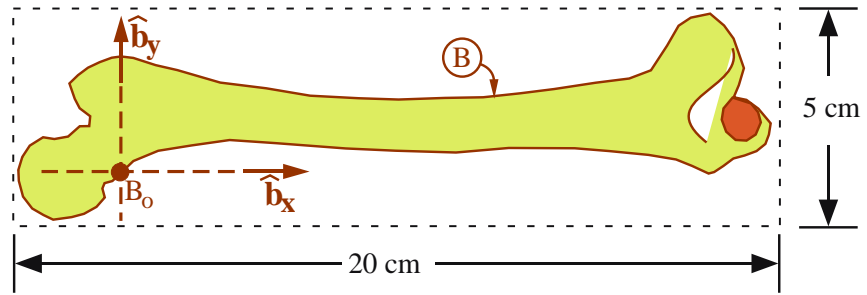
$$\text{yellow} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{\hat{b}_{xyz}} = \text{yellow} \vec{\mathbf{1}}$$

13.10 ♣ **Biomechanics: Estimating mass distribution properties of a bone** (Sections 14.1 and 14.2).

The following figure shows a relatively **thin**, uniform-density, rigid bone B . Right-handed orthogonal unit vectors $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$ are fixed in B . A point B_o of B is midway though the bone ($\vec{\mathbf{r}}^{B_{cm}/B_o} \cdot \hat{\mathbf{b}}_z = 0$).

Estimate and **draw** the location of B_{cm} (B 's center of mass) on the figure.

Hint: Draw horizontal (parallel to $\hat{\mathbf{b}}_x$) and vertical (parallel to $\hat{\mathbf{b}}_y$) lines passing through B_o . Similarly for B_{cm} .



Knowing the 1 kg bone fits snugly in a 20 cm by 5 cm by 1 cm box, **visually estimate** the following values. Note: Other than 0.0 which is used 4 times, use each of the following values **once** in your table.

-13 -2 0.0 1.0 3.0 30 31 86 89

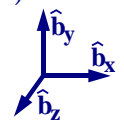
Description	Symbol	Approximate value
$\hat{\mathbf{b}}_x$ measure of B_{cm} 's position vector from B_o	x	7.5 cm
$\hat{\mathbf{b}}_y$ measure of B_{cm} 's position vector from B_o	y	1.5 cm
B 's moment of inertia about B_{cm} for $\hat{\mathbf{b}}_x$	$I_{xx}^{B/B_{cm}}$	<input type="text"/> kg cm ²
B 's moment of inertia about B_{cm} for $\hat{\mathbf{b}}_y$	$I_{yy}^{B/B_{cm}}$	<input type="text"/> kg cm ²
B 's moment of inertia about B_{cm} for $\hat{\mathbf{b}}_z$	$I_{zz}^{B/B_{cm}}$	<input type="text"/> kg cm ²
B 's product of inertia about B_{cm} for $\hat{\mathbf{b}}_x$ and $\hat{\mathbf{b}}_y$	$I_{xy}^{B/B_{cm}}$	<input type="text"/> kg cm ²
B 's product of inertia about B_{cm} for $\hat{\mathbf{b}}_x$ and $\hat{\mathbf{b}}_z$	$I_{xz}^{B/B_{cm}}$	<input type="text"/> kg cm ²
B 's product of inertia about B_{cm} for $\hat{\mathbf{b}}_y$ and $\hat{\mathbf{b}}_z$	$I_{yz}^{B/B_{cm}}$	<input type="text"/> kg cm ²
B 's moment of inertia about B_o for $\hat{\mathbf{b}}_x$	I_{xx}^{B/B_o}	<input type="text"/> kg cm ²
B 's moment of inertia about B_o for $\hat{\mathbf{b}}_y$	I_{yy}^{B/B_o}	<input type="text"/> kg cm ²
B 's moment of inertia about B_o for $\hat{\mathbf{b}}_z$	I_{zz}^{B/B_o}	<input type="text"/> kg cm ²
B 's product of inertia about B_o for $\hat{\mathbf{b}}_x$ and $\hat{\mathbf{b}}_y$	I_{xy}^{B/B_o}	<input type="text"/> kg cm ²
B 's product of inertia about B_o for $\hat{\mathbf{b}}_x$ and $\hat{\mathbf{b}}_z$	I_{xz}^{B/B_o}	<input type="text"/> kg cm ²
B 's product of inertia about B_o for $\hat{\mathbf{b}}_y$ and $\hat{\mathbf{b}}_z$	I_{yz}^{B/B_o}	<input type="text"/> kg cm ²

Hint: Start with products of inertia, then smallest moments of inertia, then largest moments of inertia.
Hint: Draw a side-view of the bone to help estimate the two I_{yz} . Draw a top-view to estimate the two I_{xz} .

13.11 ♣ **Dyadics and dot-products with orthogonal unit vectors $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$** (Section 15.1).

Non-symmetric dyadic: $\vec{\vec{\mathbf{N}}} = \hat{\mathbf{b}}_x \hat{\mathbf{b}}_x + 2 \hat{\mathbf{b}}_y \hat{\mathbf{b}}_y + 6 \hat{\mathbf{b}}_y \hat{\mathbf{b}}_z + 3 \hat{\mathbf{b}}_z \hat{\mathbf{b}}_z$

Symmetric dyadic: $\vec{\vec{\mathbf{S}}} = \hat{\mathbf{b}}_x \hat{\mathbf{b}}_x + 2 \hat{\mathbf{b}}_y \hat{\mathbf{b}}_y + 6 \hat{\mathbf{b}}_y \hat{\mathbf{b}}_z + 6 \hat{\mathbf{b}}_z \hat{\mathbf{b}}_y + 3 \hat{\mathbf{b}}_z \hat{\mathbf{b}}_z$



<i>Non-symmetric dyadic</i>	<i>Symmetric dyadic</i>
$\vec{\vec{\mathbf{N}}} \cdot (\hat{\mathbf{b}}_x + \hat{\mathbf{b}}_y) = \boxed{\hat{\mathbf{b}}_x} + \boxed{2 \hat{\mathbf{b}}_y}$	$\vec{\vec{\mathbf{S}}} \cdot (\hat{\mathbf{b}}_x + \hat{\mathbf{b}}_y) = \boxed{\phantom{\hat{\mathbf{b}}_x}} + \boxed{\phantom{2 \hat{\mathbf{b}}_y}} + \boxed{\phantom{\hat{\mathbf{b}}_z}}$
$(\hat{\mathbf{b}}_x + \hat{\mathbf{b}}_y) \cdot \vec{\vec{\mathbf{N}}} = \boxed{\phantom{\hat{\mathbf{b}}_x}} + \boxed{\phantom{2 \hat{\mathbf{b}}_y}} + \boxed{\phantom{\hat{\mathbf{b}}_z}}$	$(\hat{\mathbf{b}}_x + \hat{\mathbf{b}}_y) \cdot \vec{\vec{\mathbf{S}}} = \boxed{\phantom{\hat{\mathbf{b}}_x}} + \boxed{\phantom{2 \hat{\mathbf{b}}_y}} + \boxed{\phantom{\hat{\mathbf{b}}_z}}$
$\vec{\vec{\mathbf{N}}} \cdot \text{any Vector} = \text{any Vector} \cdot \vec{\vec{\mathbf{N}}}$ True/False	$\vec{\vec{\mathbf{S}}} \cdot \text{any Vector} = \text{any Vector} \cdot \vec{\vec{\mathbf{S}}}$ True/False