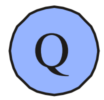
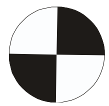
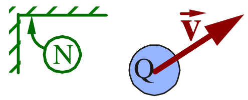
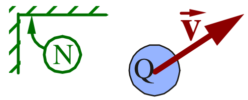
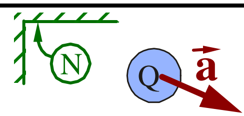
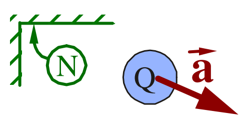
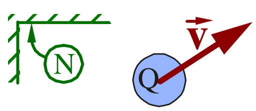
Particle vocabulary: Concept, Calculate, Context

Word	Picture	Symbol & Definition	Relationship to Force/Moment	When useful
Mass		m_Q or m^Q Standard 1 kg mass. Avagadro's number.	$\mathbf{F}^Q = m^Q \; {}^N\mathbf{a}^Q$	Statics, dynamics, gravity, momentum, energy, inertia,
Center of mass		Scm	$\mathbf{F}^S = m^S \; {}^N\mathbf{a}^{Scm}$	Statics, dynamics, gravity, momentum, energy, inertia,
Kinetic Energy				Conservation of energy, work/energy principle, power/energy-rate principle, Lagrange mechanics.
Translational Momentum				Collisions and explosions. Conservation of translational momentum. Translational momentum principle.
Angular Momentum (moment of momentum)				Collisions and explosions. Conservation of angular momentum. Angular momentum principle.
Below this line: Advanced Dynamics Only				
Effective Force				D'Alembert and Kane mechanics. Relationship to translational momentum.
Moment of Effective Force				D'Alembert's and Kane mechanics. Relationship to angular momentum.
Generalized Effective Force				Kane's mechanics. Relationship to Lagrange mechanics.
Generalized Momentum				Collisions and explosions. Lagrange and Kane impact mechanics.

11.1 ♣ Sort from smallest mass unit to largest mass unit. (see Section 13.1)

1 oz_m	1 g	1 metric ton	1 kg	1 mg	1 U.S. ton	1 slug	1 lb_m
1 mg	1 g	1 oz	1 lb_m	1 kg	1 slug	1 metric ton	1 U.S. ton

11.2 ♣ Concepts: What objects have kinetic energy or translational momentum?
${}^{N}K^{S}$, the ***kinetic energy*** of an object S in a reference frame N is to be determined.
Objects S that can have a non-zero kinetic energy are (circle ***all*** appropriate objects):

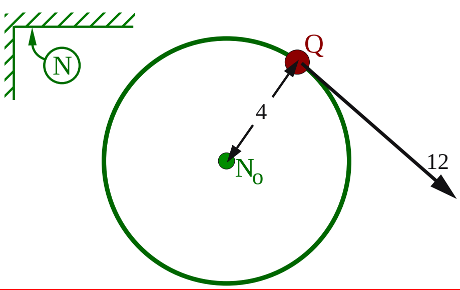
Complex number	Point	Reference Frame	Center of mass of a set of particles
Vector	Set of Points	Rigid Body	Center of mass of a rigid body
Matrix	Particle	Flexible Body	Set of flexible bodies
Orthogonal basis	Set of Particles	Set of Rigid bodies	System of particles and bodies

Repeat for ${}^{N}\vec{\mathrm{L}}^{S}$, the ***translational momentum*** of object S in reference frame N box appropriate objects.

11.3 ♣ Angular momentum concepts.

The following figures show a particle Q of mass 1 kg moving in a **plane** N. Point N_o is fixed in N. The figure on the left shows Q moving clockwise with speed 12 on a circle of radius 4 that is centered at N_o. The figure on the right shows Q moving with a speed of 12 on a horizontal line that is 4 from N_o. **Box** the following true statements about Q's ***angular momentum*** in N.

Q's angular momentum about N_o is $\vec{\mathbf{0}}$.
Q's angular momentum about N_o is not $\vec{\mathbf{0}}$.
Q's angular momentum about N_o is $\vec{\infty}$.
Q's angular momentum about N_o does not exist.



Q's angular momentum about N_o is $\vec{\mathbf{0}}$.
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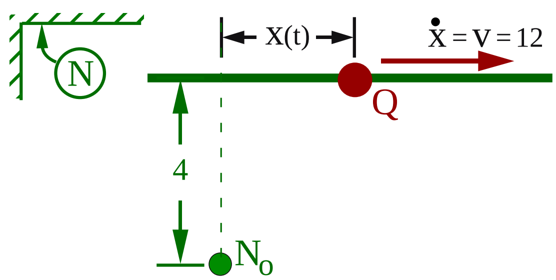
Q's angular momentum about N_o is $\vec{\infty}$.
Q's angular momentum about N_o does not exist.



11.4 ♣ Optional: Just for fun. Culture, religion, science and "mass". (Sections 13.9, 13.7, 13.6)

Etymology of "mass"	Fill-in the blank
The "m" in $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$.	mass
Jewish Passover flat bread/cracker.	matzah
Greek for flat bread.	maza
Latin for lump of dough.	massa
Spanish for lump of dough.	masa
Catholics eat bread at this Sunday event.	Mass
Approximate number of atoms in 12 grams of carbon-12.	6×10^{23}
Estimated number of atoms in the visible universe.	1×10^{80}
Sub-atomic particle responsible for mass in animals, vegetables, and minerals.	Higgs boson
Most expensive science project in history to find sub-atomic particle with mass.	Hadron collider
Possible Earth-fatal object created by aforementioned science project.	black hole

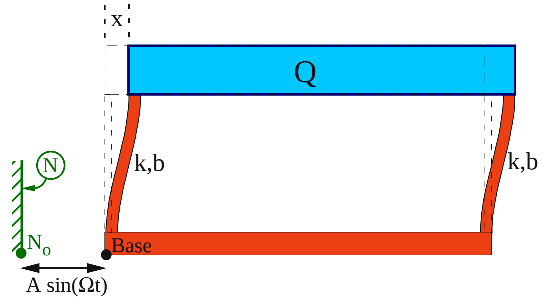
11.5 FE/EIT Review – Motion of a building in an earthquake.

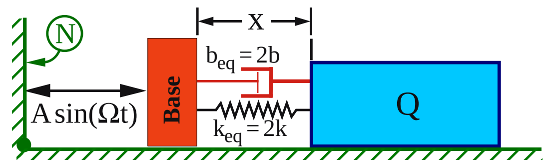
$\vec{\mathbf{F}} \;\Rightarrow\; \vec{\mathbf{F}} = m\,\vec{\mathbf{a}} \;\Rightarrow\; \ddot{x}$

A building moves due to an earthquake. The horizontally-right displacement of the building's base B is **modeled** as $A \sin(\Omega\, t)$ where the constant A is the magnitude of the ground's horizontal displacement and the constant Ω is the earthquake's frequency.

The base motion causes the building's roof Q of mass m to displace horizontally by $x(t)$ from its base.
The stiffness and material damping in each of the two columns that support the roof is modeled as a linear horizontal spring (k) and linear horizontal damper (b).

For this dynamic analysis, the system is modeled as shown right (with a spring of 0 natural length). It is helpful to introduce a horizontally-right unit vector $\widehat{\mathbf{n}}_{\mathrm{x}}$.





x is the $\widehat{\mathbf{n}}_{\mathrm{x}}$ measure of Q's position from its base B

(a) Draw Q's ***free-body diagram*** and determine the spring/damper force on Q.
Result: $\vec{\mathbf{F}}_{\text{Spring/Damper}} \underset{\textbf{FBD}}{=} (-b_{\text{eq}}\,\dot{x} \;+\; -k_{\text{eq}}\,x)\,\widehat{\mathbf{n}}_{\mathrm{x}}$

FBD of Q

(b) Form the relevant acceleration for $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ (e.g., differentiate the relevant position vector/velocity).
Next, dot-product $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ with $\widehat{\mathbf{n}}_{\mathrm{x}}$ to write a differential equation governing $x(t)$.
Result: $\vec{\mathbf{a}} = [\;\ddot{x} \;-\; A\,\Omega^2 \sin(\Omega\, t)\;]\,\widehat{\mathbf{n}}_{\mathrm{x}} \quad\Rightarrow\quad m\,\ddot{x} \;+\; b_{\text{eq}}\,\dot{x} \;+\; k_{\text{eq}}\,x \;=\; m\,A\,\Omega^2 \sin(\Omega\, t)$

(c) Graphed right is $x(t)$ when:

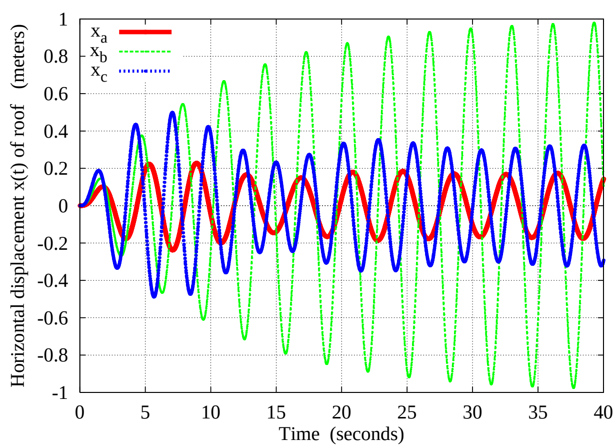
(a) $\Omega \;=\; 0.8\sqrt{\frac{k_{\text{eq}}}{m}} \;=\; 0.8\,\omega_n \qquad x_a(t)$

(b) $\Omega \;=\; 1.0\sqrt{\frac{k_{\text{eq}}}{m}} \;=\; 1.0\,\omega_n \qquad x_b(t)$

(c) $\Omega \;=\; 1.2\sqrt{\frac{k_{\text{eq}}}{m}} \;=\; 1.2\,\omega_n \qquad x_c(t)$

Circle the Ω that corresponds to the largest ***steady-state*** amplitude for $x(t)$.

Note: These graphs use $m \;=\; 5000$ kg,
$b_{\text{eq}} \;=\; 1000\;\frac{\text{N*sec}}{\text{m}}$, $k_{\text{eq}} \;=\; 20000\;\frac{\text{N}}{\text{m}}$, $A \;=\; 0.1$ m.



(d) **Physics** ($\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$) gives the previous (boxed) equation in terms of positive constants m, b_{eq}, k_{eq}.
However, its **mathematics** is easier if that equation is rewritten (rearrange by dividing by m) in terms of the positive constants ζ and ω_n as shown in the boxed-equation below. Determine the building's ***natural frequency*** ω_n and ***damping ratio*** ζ in terms of m, b_{eq}, k_{eq}.
Result:

$$\ddot{x} \;+\; 2\,\zeta\,\omega_n\,\dot{x} \;+\; \omega_n^2\,x \;=\; A\,\Omega^2 \sin(\Omega\, t) \quad\Rightarrow\quad \omega_n = \sqrt{\frac{k_{\text{eq}}}{m}} \qquad \zeta = \frac{b_{\text{eq}}}{2\,\sqrt{m\,k_{\text{eq}}}}$$

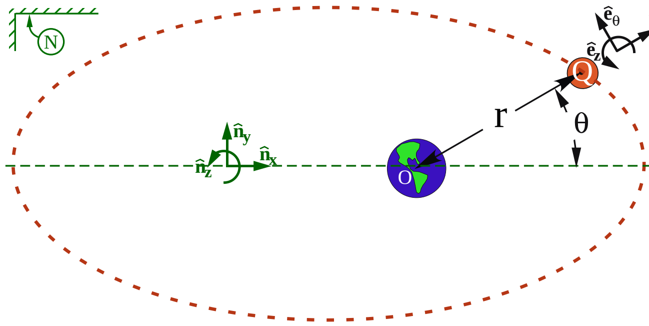
(e) When $\zeta \;=\; 0$, the building vibrates at ω_n. When $0 \le \zeta \le 1$ (common for many structures), the building vibrates at a ***damped natural frequency*** $\omega_d \triangleq \omega_n\,\sqrt{1-\zeta^2}$.
In general, damping slows things down and makes $\omega_d \;<\; \omega_n$. **True**/**False**.

11.6 FE/EIT Review – Momentum, energy, and orbital mechanics. $\vec{\mathbf{F}} \;\Rightarrow\; \vec{\mathbf{F}} = m\,\vec{\mathbf{a}} \;\Rightarrow\; \ddot{r},\ \ddot{\theta}$

The following figure shows a satellite Q (modeled as a particle of mass m) in an elliptical orbit around Earth. Earth is modeled as a particle O fixed in a Newtonian reference frame N.

Draw right-handed orthogonal unit vectors $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ fixed in N with $\widehat{\mathbf{n}}_x$ horizontally-right and parallel to the ellipse's major diameter, $\widehat{\mathbf{n}}_y$ vertically-upward and parallel to the ellipse's minor diameter, and $\widehat{\mathbf{n}}_z$ perpendicular to the ellipse's plane.

Draw right-handed orthogonal unit vectors $\widehat{\mathbf{e}}_r$, $\widehat{\mathbf{e}}_\theta$, $\widehat{\mathbf{e}}_z$, with $\widehat{\mathbf{e}}_r$ from O to Q and $\widehat{\mathbf{e}}_z = \widehat{\mathbf{n}}_z$.



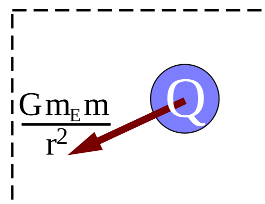
Quantity	Symbol	Type	Value
Universal gravitational constant	G	Constant	6.67×10^{-11} N∗m²/kg²
Mass of Earth O	m^E	Constant	5.97×10^{24} kg
Mass of Q	m	Constant	200 kg
Angle between line OQ and long axis of the ellipse	θ	Variable	Initial value
Distance between O and Q	r	Variable	Initial value

(a) Form Q's ***translational momentum***, ***angular momentum*** about O, and ***kinetic energy*** in N.
Result: (in terms of symbols in the previous table, their time-derivatives, and $\widehat{\mathbf{e}}_r$, $\widehat{\mathbf{e}}_\theta$, $\widehat{\mathbf{e}}_z$).

$$^N\vec{\mathbf{L}}^Q = m\,(\dot{r}\,\widehat{\mathbf{e}}_r + \dot{\theta}\,r\,\widehat{\mathbf{e}}_\theta) \qquad ^N\vec{\mathbf{H}}^{Q/O} = m\,r^2\,\dot{\theta}\,\widehat{\mathbf{e}}_z \qquad ^NK^Q = \frac{1}{2}\,m\,[\dot{r}^2 + (\dot{\theta}\,r)^2]$$

(b) **Draw** Q's free-body diagram and determine the resultant force on Q.
Dot $\vec{\mathbf{F}}^Q = m\,^N\vec{\mathbf{a}}^Q$ with "clever" unit vectors and solve for $\ddot{r}$ and $\ddot{\theta}$.
Result:

$$\vec{\mathbf{F}}^Q = \frac{-G\,m^E\,m}{r^2}\,\widehat{\mathbf{e}}_r \qquad \ddot{r} = r\,\dot{\theta}^2 - \frac{G\,m^E}{r^2} \qquad \ddot{\theta} = \frac{-2\,\dot{\theta}\,\dot{r}}{r}$$



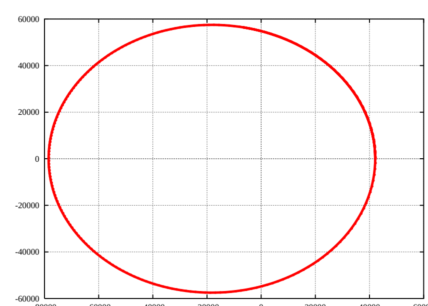
(c) Form an expression for Q's mechanical energy in N (sum of kinetic energy and potential energy U).
Result: ***Conservation of mechanical energy*** $\quad \text{KePe} = \frac{1}{2}\,m\left[\dot{r}^2 + (\dot{\theta}\,r)^2\right] - \frac{G\,m\,m^E}{r}$

(d) **Optional:** Starting with the ***angular momentum principle*** in equation (22.4), form an expression that stays constant during Q's motion in N ***(conservation of angular momentum)***.
Result: $H_{\text{Constant}} \underset{(22.9)}{=} m\,r^2\,\dot{\theta}$ (Section 22.8)

(e) **Draw** a line L tangent to the ellipse at Q and perpendicular to $\widehat{\mathbf{n}}_z$.

In general,	$^N\vec{\mathbf{v}}^Q$ (Q's velocity in N) is parallel to $\widehat{\mathbf{e}}_\theta$.	True/**[False]**
In general,	$\widehat{\mathbf{e}}_\theta$ is parallel to L.	True/**[False]**
In general,	$^N\vec{\mathbf{v}}^Q$ is parallel to L.	**[True]**/False

(f) Shown right is a plot of Q's orbital trajectory when the differential equations from part (6b) are solved with the initial values of part (**??**). Query the Internet (or a textbook, instructor, colleague, etc.) for the next answers.



- Clearly mark Earth's location on the plot.
- The Earth is located at (circle one):
  the center of the ellipse/[a focus of the ellipse].