

Show work – except for ♣ fill-in-blanks.

Particles: Mass, momentum, energy,  $\vec{F} = m \vec{a}$ .

9.1 ♣ Sort from smallest mass unit to largest mass unit. (see Section 11.1)

1 oz<sub>m</sub>    1 g    1 metric ton    1 kg    1 mg    1 U.S. ton    1 slug    1 lb<sub>m</sub>

9.2 ♣ Concepts: What objects have kinetic energy or translational momentum?

${}^N K^S$ , the *kinetic energy* of an object  $S$  in a reference frame  $N$  is to be determined.

Objects  $S$  that can have a non-zero kinetic energy are (circle all appropriate objects):

Complex number	Point	Reference Frame	Center of mass of a set of particles
Vector	Set of Points	Rigid Body	Center of mass of a rigid body
Matrix	Particle	Flexible Body	Set of flexible bodies
Orthogonal basis	Set of Particles	Set of Rigid bodies	System of particles and bodies

Repeat for  ${}^N \vec{L}^S$ , the *translational momentum* of object  $S$  in reference frame  $N$

9.3 ♣ Particle angular momentum concepts.

The following figures show a particle  $Q$  of mass 1 kg moving in a **plane**  $N$ . Point  $N_o$  is fixed in  $N$ . The figure on the left shows  $Q$  moving clockwise with speed 12 on a circle of radius 4 that is centered at  $N_o$ . The figure on the right shows  $Q$  moving with a speed of 12 on a horizontal line that is 4 from  $N_o$ . **Box** the following true statements about  $Q$ 's *angular momentum* in  $N$ .

$Q$ 's angular momentum about  $N_o$  is  $\vec{0}$ .  
  $Q$ 's angular momentum about  $N_o$  is not  $\vec{0}$ .  
  $Q$ 's angular momentum about  $N_o$  is  $\vec{\omega}$ .  
  $Q$ 's angular momentum about  $N_o$  does not exist.

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  $Q$ 's angular momentum about  $N_o$  does not exist.

9.4 ♣ Optional: Just for fun. Culture, religion, science and “mass”. (Sections , 11.7, 11.6)

Etymology of “mass”	Fill-in the blank
The “m” in $\vec{F} = m \vec{a}$ .	<input type="text"/>
Jewish Passover flat bread/cracker.	<input type="text"/>
Greek for flat bread.	<input type="text"/>
Latin for lump of dough.	<input type="text"/>
Spanish for lump of dough.	<input type="text"/>
Catholics eat bread at this Sunday event.	<input type="text"/>
Approximate number of atoms in 12 grams of carbon-12.	$6 \times 10$ <input type="text"/>
Estimated number of atoms in the visible universe.	$1 \times 10$ <input type="text"/>
Sub-atomic particle responsible for mass in animals, vegetables, and minerals.	<input type="text"/>
Most expensive science project in history to find sub-atomic particle with mass.	<input type="text"/>
Possible Earth-fatal object created by aforementioned science project.	<input type="text"/>

9.5 FE/EIT Review – Motion of a building in an earthquake.

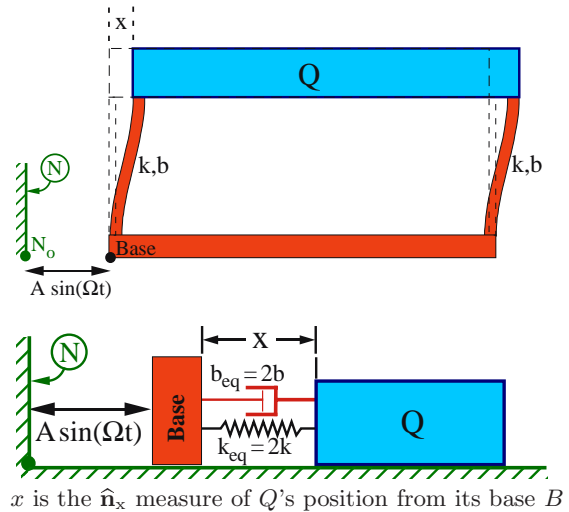
$$\vec{F} \Rightarrow \vec{F} = m \vec{a} \Rightarrow \ddot{x}$$

A building moves due to an earthquake. The horizontally-right displacement of the building's base  $B$  is modeled as  $A \sin(\Omega t)$  where the constant  $A$  is the magnitude of the ground's horizontal displacement and the constant  $\Omega$  is the earthquake's frequency.

The base motion causes the building's roof  $Q$  of mass  $m$  to displace horizontally by  $x(t)$  from its base.

The stiffness and material damping in each of the two columns that support the roof is modeled as a linear horizontal spring ( $k$ ) and linear horizontal damper ( $b$ ).

For this dynamic analysis, the system is modeled as shown right (with a spring of 0 natural length). It is helpful to introduce a horizontally-right unit vector  $\hat{n}_x$ .



- (a) Draw  $Q$ 's **free-body diagram** and determine the spring/damper force on  $Q$ .

**Result:**  $\vec{F}_{\text{Spring/Damper}} = (-b_{\text{eq}} \dot{x} + -k_{\text{eq}} x) \hat{n}_x$



- (b) Form the relevant acceleration for  $\vec{F} = m \vec{a}$  (e.g., differentiate the relevant position vector/velocity). Next, dot-product  $\vec{F} = m \vec{a}$  with  $\hat{n}_x$  to write a differential equation governing  $x(t)$ .

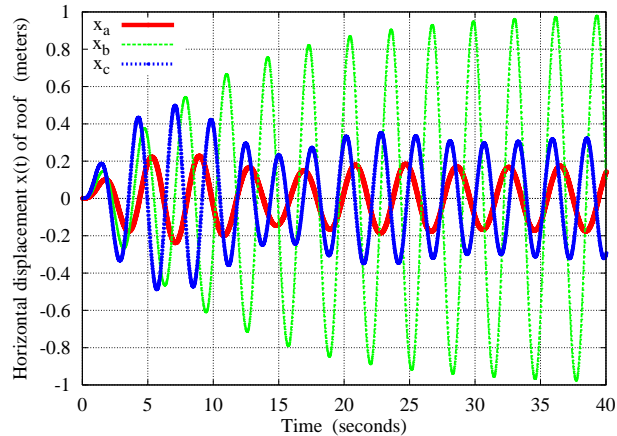
**Result:**  $m \ddot{x} + b_{\text{eq}} \dot{x} + k_{\text{eq}} x = m A \Omega^2 \sin(\Omega t)$

- (c) Graphed right is  $x(t)$  when:

- (a)  $\Omega = 0.8 \sqrt{\frac{k_{\text{eq}}}{m}} = 0.8 \omega_n \quad x_a(t)$
- (b)  $\Omega = 1.0 \sqrt{\frac{k_{\text{eq}}}{m}} = 1.0 \omega_n \quad x_b(t)$
- (c)  $\Omega = 1.2 \sqrt{\frac{k_{\text{eq}}}{m}} = 1.2 \omega_n \quad x_c(t)$

Circle the  $\Omega$  that corresponds to the largest **steady-state** amplitude for  $x(t)$ .

Note: These graphs use  $m = 5000 \text{ kg}$ ,  
 $b_{\text{eq}} = 1000 \frac{\text{N} \cdot \text{sec}}{\text{m}}$ ,  $k_{\text{eq}} = 20000 \frac{\text{N}}{\text{m}}$ ,  $A = 0.1 \text{ m}$ .



- (d) **Physics** ( $\vec{F} = m \vec{a}$ ) gives the previous (boxed) equation in terms of positive constants  $m$ ,  $b_{\text{eq}}$ ,  $k_{\text{eq}}$ . However, its **mathematics** is easier if that equation is rewritten (rearrange by dividing by  $m$ ) in terms of the positive constants  $\zeta$  and  $\omega_n$  as shown in the boxed-equation below. Determine the building's **natural frequency**  $\omega_n$  and **damping ratio**  $\zeta$  in terms of  $m$ ,  $b_{\text{eq}}$ ,  $k_{\text{eq}}$ .

**Result:**  $\ddot{x} + 2\zeta \omega_n \dot{x} + \omega_n^2 x = A \Omega^2 \sin(\Omega t) \Rightarrow \omega_n = \sqrt{\frac{k_{\text{eq}}}{m}} \quad \zeta = \frac{b_{\text{eq}}}{2\sqrt{m k_{\text{eq}}}}$

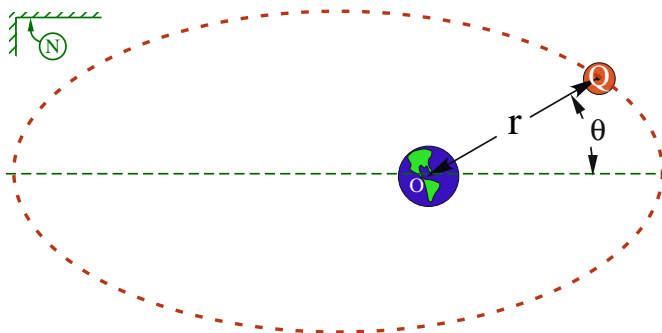
- (e) When  $\zeta = 0$ , the building vibrates at  $\omega_n$ . When  $0 \leq \zeta \leq 1$  (common for many structures), the building vibrates at a **damped natural frequency**  $\omega_d \triangleq \omega_n \sqrt{1 - \zeta^2}$ . In general, damping slows things down and makes  $\omega_d < \omega_n$ . **True/False**.

9.6 FE/EIT Review – Momentum, energy, and orbital mechanics.  $\vec{F} \Rightarrow \vec{F} = m\vec{a} \Rightarrow \ddot{r}, \ddot{\theta}$

The following figure shows a satellite  $Q$  (modeled as a particle of mass  $m$ ) in an elliptical orbit around Earth. Earth is modeled as a particle  $O$  fixed in a Newtonian reference frame  $N$ .

**Draw** right-handed orthogonal unit vectors  $\hat{n}_x, \hat{n}_y, \hat{n}_z$  fixed in  $N$  with  $\hat{n}_x$  horizontally-right and parallel to the ellipse's major diameter,  $\hat{n}_y$  vertically-upward and parallel to the ellipse's minor diameter, and  $\hat{n}_z$  perpendicular to the ellipse's plane.

**Draw** right-handed orthogonal unit vectors  $\hat{e}_r, \hat{e}_\theta, \hat{e}_z$ , with  $\hat{e}_r$  from  $O$  to  $Q$  and  $\hat{e}_z = \hat{n}_z$ .



Quantity	Symbol	Type	Value
<b>Universal gravitational constant</b>	$G$	Constant	$6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$
Mass of Earth $O$	$m^E$	Constant	$5.97 \times 10^{24} \text{ kg}$
Mass of $Q$	$m$	Constant	200 kg
Angle between line $OQ$ and long axis of the ellipse	$\theta$	Variable	Initial value
Distance between $O$ and $Q$	$r$	Variable	Initial value

- (a) Form  $Q$ 's **translational momentum**, **angular momentum** about  $O$ , and **kinetic energy** in  $N$ .

**Result:** (in terms of symbols in the previous table, their time-derivatives, and  $\hat{e}_r, \hat{e}_\theta, \hat{e}_z$ ).

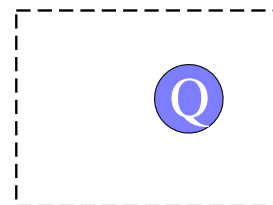
${}^N\vec{L}^Q =$         ${}^N\vec{H}^{Q/O} =$         ${}^NK^Q =$

- (b) **Draw**  $Q$ 's free-body diagram and determine the resultant force on  $Q$ .

Dot  $\vec{F}^Q = m {}^N\vec{a}^Q$  with “clever” unit vectors and solve for  $\ddot{r}$  and  $\ddot{\theta}$ .

**Result:**

$\vec{F}^Q = \frac{-G m^E m}{r^2} \hat{e}_r$        $\ddot{r} =$         $\ddot{\theta} =$



- (c) Form an expression for  $Q$ 's mechanical energy in  $N$  (sum of kinetic energy and potential energy  $U$ ).

**Result:** **Conservation of mechanical energy**       $\text{KePe} = \frac{1}{2} m [\dot{r}^2 + (\dot{\theta} r)^2] - \frac{G m m^E}{r}$

- (d) **Optional:** Starting with the **angular momentum principle** in equation (20.4), form an expression that stays constant during  $Q$ 's motion in  $N$  (**conservation of angular momentum**).

**Result:**  $H_{\text{Constant}} =$   (Section 20.8)

- (e) **Draw** a line  $L$  tangent to the ellipse at  $Q$  and perpendicular to  $\hat{n}_z$ .

In general, ${}^N\vec{v}^Q$ ( $Q$ 's velocity in $N$ ) is parallel to $\hat{e}_\theta$ .	<b>True/False</b>
In general, $\hat{e}_\theta$ is parallel to $L$ .	<b>True/False</b>
In general, ${}^N\vec{v}^Q$ is parallel to $L$ .	<b>True/False</b>

- (f) Shown right is a plot of  $Q$ 's orbital trajectory when the differential equations from part (6b) are solved. Query the Internet (or a textbook, instructor, colleague, etc.) for the next answers.

- Clearly mark Earth's location on the plot.
- The Earth is located at (circle one):  
**the center of the ellipse/a focus of the ellipse.**

