Homework 9. Chapters 10, 11.

Show work – except for ♣♣♣♣♣♣♣♣♣♣♣ fill-in-blanks. Particles: Mass, momentum, energy, \( \mathbf{F} = m \mathbf{a} \).

9.1 ♣ Sort from smallest mass unit to largest mass unit. (see Section 11.1)

<table>
<thead>
<tr>
<th>1 oz(_m)</th>
<th>1 g</th>
<th>1 metric ton</th>
<th>1 kg</th>
<th>1 mg</th>
<th>1 U.S. ton</th>
<th>1 slug</th>
<th>1 lb(_m)</th>
</tr>
</thead>
</table>

9.2 ♣ Concepts: What objects have kinetic energy or linear momentum? 

\( ^N \mathbf{K}^S \), the kinetic energy of an object \( S \) in a reference frame \( N \) is to be determined.

Objects \( S \) that can have a non-zero kinetic energy are (circle all appropriate objects):

<table>
<thead>
<tr>
<th>Real number</th>
<th>Matrix</th>
<th>Set of points</th>
<th>Mass center of a rigid body</th>
<th>Vector</th>
<th>Point</th>
<th>Reference frame</th>
<th>Flexible body</th>
<th>3D orthogonal unit basis</th>
<th>Particle</th>
<th>Rigid body</th>
<th>System of particles and bodies</th>
</tr>
</thead>
</table>

Repeat for \( ^N \mathbf{L}^S \), the linear momentum of object \( S \) in reference frame \( N \) (box appropriate objects).

9.3 ♣ Particle angular momentum concepts.

The following figures show a particle \( Q \) of mass 1 kg moving in a plane \( N \). Point \( N_0 \) is fixed in \( N \). The figure on the left shows \( Q \) moving clockwise with speed 12 on a circle of radius 4 that is centered at \( N_0 \). The figure on the right shows \( Q \) moving with a speed of 12 on a horizontal line that is 4 from \( N_0 \). Box the following true statements about \( Q \)’s angular momentum in \( N \).

\( Q \)’s angular momentum about \( N_0 \) is \( \mathbf{0} \).
\( Q \)’s angular momentum about \( N_0 \) is not \( \mathbf{0} \).
\( Q \)’s angular momentum about \( N_0 \) is \( \vec{\infty} \).
\( Q \)’s angular momentum about \( N_0 \) does not exist.

9.4 ♣ Optional: Just for fun. Culture, religion, science and “mass”. (Sections , 11.7, 11.6)

<table>
<thead>
<tr>
<th>Etymology of “mass”</th>
<th>Fill-in the blank</th>
</tr>
</thead>
</table>
| The “m” in \( \mathbf{F} = m \mathbf{a} \). | |}
| Jewish Passover flat bread/cracker. | |}
| Greek for flat bread. | |}
| Latin for lump of dough. | |}
| Spanish for lump of dough. | |}
| Catholics eat bread at this Sunday event. | |}
| Approximate number of atoms in 12 grams of carbon-12. | \( 6 \times 10 \) |
| Estimated number of atoms in the visible universe. | \( 1 \times 10 \) |
| Sub-atomic particle responsible for mass in animals, vegetables, and minerals. | |}
| Most expensive science project in history to find sub-atomic particle with mass. | |}
| Possible Earth-fatal object created by aforementioned science project. | |}

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9.5 FE/EIT Review – Motion of a building in an earthquake.

A building moves due to an earthquake. The horizontally-right displacement of the building’s base \( B \) is modeled as \( A \sin(\Omega t) \) where the constant \( A \) is the magnitude of the ground’s horizontal displacement and the constant \( \Omega \) is the earthquake’s frequency.

The base motion causes the building’s roof \( Q \) of mass \( m \) to displace horizontally by \( x(t) \) from its base. The stiffness and material damping in each of the two columns that support the roof is modeled as a linear horizontal spring \( (k) \) and linear horizontal damper \( (b) \).

For this dynamic analysis, the system is modeled as shown right (with a spring of 0 natural length). It is helpful to introduce a horizontally-right unit vector \( \hat{n}_x \).

(a) Draw \( Q \)’s free-body diagram and determine the spring/damper force on \( Q \).
\[ \vec{F}_{Spring/Damper}^{FBD} = (−b_{eq} \dot{x} + −k_{eq} x) \hat{n}_x \]

(b) Form the relevant acceleration for \( \vec{F} = m \vec{a} \) (e.g., differentiate the relevant position vector/velocity).
Next, dot-product \( \vec{F} = m \vec{a} \) with \( \hat{n}_x \) to write a differential equation governing \( x(t) \).
\[ \ddot{x} + b_{eq} \dot{x} + k_{eq} x = m A \Omega^2 \sin(\Omega t) \]

(c) Graphed right is \( x(t) \) when:
\( \begin{align*} 
\text{(a)} & \quad \Omega = 0.8 \sqrt{\frac{k_{eq}}{m}} = 0.8 \omega_n \quad x_a(t) \\
\text{(b)} & \quad \Omega = 1.0 \sqrt{\frac{k_{eq}}{m}} = 1.0 \omega_n \quad x_b(t) \\
\text{(c)} & \quad \Omega = 1.2 \sqrt{\frac{k_{eq}}{m}} = 1.2 \omega_n \quad x_c(t) 
\end{align*} \)

Circle the \( \Omega \) that corresponds to the largest steady-state amplitude for \( x(t) \).

Note: These graphs use \( m = 5000 \text{ kg}, \quad b_{eq} = 1000 \frac{\text{N sec}}{\text{m}}, \quad k_{eq} = 20000 \frac{\text{N}}{\text{m}}, \quad A = 0.1 \text{ m} \).

(d) Physics \( (\vec{F} = m \vec{a}) \) gives the previous (boxed) equation in terms of positive constants \( m, b_{eq}, k_{eq} \). However, its mathematics is easier if that equation is rewritten (rearrange by dividing by \( m \)) in terms of the positive constants \( \zeta \) and \( \omega_n \) as shown in the boxed-equation below. Determine the building’s natural frequency \( \omega_n \) and damping ratio \( \zeta \) in terms of \( m, b_{eq}, k_{eq} \).
\[ \ddot{x} + 2 \zeta \omega_n \dot{x} + \omega_n^2 x = A \Omega^2 \sin(\Omega t) \quad \Rightarrow \quad \omega_n = \quad \zeta = \quad \]

\( \frac{\omega_n}{2} \)

(e) When \( \zeta = 0 \), the building vibrates at \( \omega_n \). When \( 0 \leq \zeta \leq 1 \) (common for many structures), the building vibrates at a damped natural frequency \( \omega_d \triangleq \omega_n \sqrt{1 − \zeta^2} \).
In general, damping slows things down and makes \( \omega_d < \omega_n \). True/False.