

Show work – except for ♣ fill-in-blanks.

Particles: Mass, momentum, energy, $\vec{F} = m\vec{a}$.

12.1 ♣ Sort from smallest mass unit to largest mass unit. (see Section 13.1)

1 oz_m 1 g 1 metric ton 1 kg 1 mg 1 U.S. ton 1 slug 1 lb_m

12.2 ♣ Concepts: What objects have kinetic energy or translational momentum?

${}^N K^S$, the *kinetic energy* of an object S in a reference frame N is to be determined.

Objects S that can have a non-zero kinetic energy are (circle all appropriate objects):

Complex number	Point	Reference Frame	Center of mass of a set of particles
Vector	Set of Points	Rigid Body	Center of mass of a rigid body
Matrix	Particle	Flexible Body	Set of flexible bodies
Orthogonal basis	Set of Particles	Set of Rigid bodies	System of particles and bodies

Repeat for ${}^N \vec{L}^S$, the *translational momentum* of object S in reference frame N

12.3 ♣ Particle angular momentum concepts.

The following figures show a particle Q of mass 1 kg moving in a **plane** N . Point N_o is fixed in N . The figure on the left shows Q moving clockwise with speed 12 on a circle of radius 4 that is centered at N_o . The figure on the right shows Q moving with a speed of 12 on a horizontal line that is 4 from N_o . **Box** the following true statements about Q 's *angular momentum* in N .

Q 's angular momentum about N_o is $\vec{0}$.
 Q 's angular momentum about N_o is not $\vec{0}$.
 Q 's angular momentum about N_o is $\vec{\omega}$.
 Q 's angular momentum about N_o does not exist.

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 Q 's angular momentum about N_o does not exist.

12.4 ♣ Optional: Just for fun. Culture, religion, science and “mass”. (Sections 13.9, 13.7, 13.6)

Etymology of “mass”	Fill-in the blank
The “m” in $\vec{F} = m\vec{a}$.	<input type="text"/>
Jewish Passover flat bread/cracker.	<input type="text"/>
Greek for flat bread.	<input type="text"/>
Latin for lump of dough.	<input type="text"/>
Spanish for lump of dough.	<input type="text"/>
Catholics eat bread at this Sunday event.	<input type="text"/>
Approximate number of atoms in 12 grams of carbon-12.	6×10 <input type="text"/>
Estimated number of atoms in the visible universe.	1×10 <input type="text"/>
Sub-atomic particle responsible for mass in animals, vegetables, and minerals.	<input type="text"/>
Most expensive science project in history to find sub-atomic particle with mass.	<input type="text"/>
Possible Earth-fatal object created by aforementioned science project.	<input type="text"/>

12.5 FE/EIT Review – Motion of a building in an earthquake.

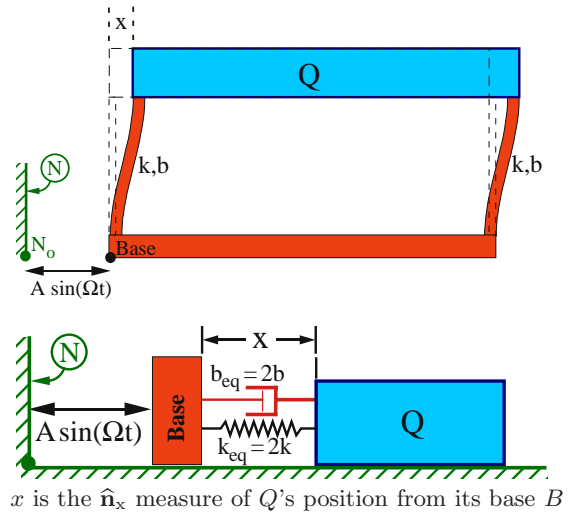
$$\vec{F} \Rightarrow \vec{F} = m \vec{a} \Rightarrow \ddot{x}$$

A building moves due to an earthquake. The horizontally-right displacement of the building's base B is **modeled** as $A \sin(\Omega t)$ where the constant A is the magnitude of the ground's horizontal displacement and the constant Ω is the earthquake's frequency.

The base motion causes the building's roof Q of mass m to displace horizontally by $x(t)$ from its base.

The stiffness and material damping in each of the two columns that support the roof is modeled as a linear horizontal spring (k) and linear horizontal damper (b).

For this dynamic analysis, the system is modeled as shown right (with a spring of 0 natural length). It is helpful to introduce a horizontally-right unit vector \hat{n}_x .



- (a) Draw Q 's **free-body diagram** and determine the spring/damper force on Q .

Result: $\vec{F}_{\text{Spring/Damper}} = (-b_{\text{eq}} \dot{x} - k_{\text{eq}} x) \hat{n}_x$

FBD of Q

- (b) Form the relevant acceleration for $\vec{F} = m \vec{a}$ (e.g., differentiate the relevant position vector/velocity). Next, dot-product $\vec{F} = m \vec{a}$ with \hat{n}_x to write a differential equation governing $x(t)$.

Result: [Optional: Verify results via Kane and/or Lagrange methods in Chapters 26, 27.]

$$\vec{a} = [\text{yellow box} - \text{yellow box}] \hat{n}_x \Rightarrow m \ddot{x} + b_{\text{eq}} \dot{x} + k_{\text{eq}} x = m A \Omega^2 \sin(\Omega t)$$

- (c) Graphed right is $x(t)$ when:

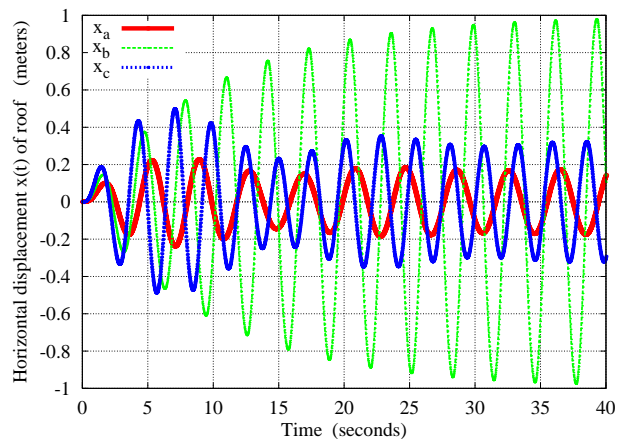
(a) $\Omega = 0.8 \sqrt{\frac{k_{\text{eq}}}{m}} = 0.8 \omega_n \quad x_a(t)$

(b) $\Omega = 1.0 \sqrt{\frac{k_{\text{eq}}}{m}} = 1.0 \omega_n \quad x_b(t)$

(c) $\Omega = 1.2 \sqrt{\frac{k_{\text{eq}}}{m}} = 1.2 \omega_n \quad x_c(t)$

Circle the Ω that corresponds to the largest **steady-state** amplitude for $x(t)$.

Note: These graphs use $m = 5000 \text{ kg}$,
 $b_{\text{eq}} = 1000 \frac{\text{N} \cdot \text{sec}}{\text{m}}$, $k_{\text{eq}} = 20000 \frac{\text{N}}{\text{m}}$, $A = 0.1 \text{ m}$.



- (d) **Physics** ($\vec{F} = m \vec{a}$) gives the previous (boxed) equation in terms of positive constants m , b_{eq} , k_{eq} . However, its **mathematics** is easier if that equation is rewritten (rearrange by dividing by m) in terms of the positive constants ζ and ω_n as shown in the boxed-equation below. Determine the building's **natural frequency** ω_n and **damping ratio** ζ in terms of m , b_{eq} , k_{eq} .

Result:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = A\Omega^2\sin(\Omega t) \Rightarrow \omega_n = \sqrt{\frac{\text{yellow box}}{\text{yellow box}}} \quad \zeta = \frac{\text{yellow box}}{2 \text{ yellow box}}$$

- (e) When $\zeta = 0$, the building vibrates at ω_n . When $0 \leq \zeta \leq 1$ (common for many structures), the building vibrates at a **damped natural frequency** $\omega_d \triangleq \omega_n \sqrt{1 - \zeta^2}$. In general, damping slows things down and makes $\omega_d < \omega_n$. **True/False.**

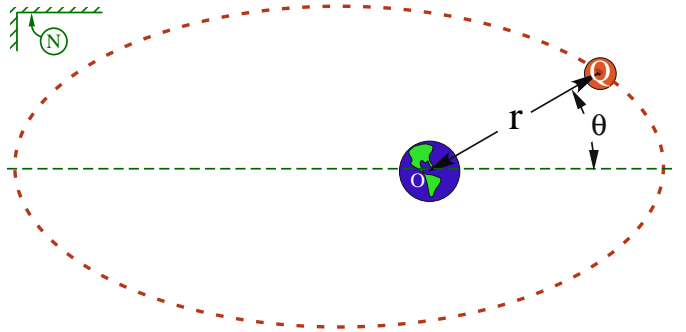
12.6 FE/EIT Review – Momentum, energy, and orbital mechanics.

$$\vec{F} \Rightarrow \vec{F} = m\vec{a} \Rightarrow \ddot{r}, \ddot{\theta}$$

The following figure shows a satellite Q (modeled as a particle of mass m) in an elliptical orbit around Earth. Earth is modeled as a particle O fixed in a Newtonian reference frame N .

Draw right-handed orthogonal unit vectors $\hat{n}_x, \hat{n}_y, \hat{n}_z$ fixed in N with \hat{n}_x horizontally-right and parallel to the ellipse's major diameter, \hat{n}_y vertically-upward and parallel to the ellipse's minor diameter, and \hat{n}_z perpendicular to the ellipse's plane.

Draw right-handed orthogonal unit vectors $\hat{e}_r, \hat{e}_\theta, \hat{e}_z$, with \hat{e}_r from O to Q and $\hat{e}_z = \hat{n}_z$.



Quantity	Symbol	Type	Value
<i>Universal gravitational constant</i>	G	Constant	$6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$
Mass of Earth O	m^E	Constant	$5.97 \times 10^{24} \text{ kg}$
Mass of Q	m	Constant	200 kg
Angle between line OQ and long axis of the ellipse	θ	Variable	Initial value
Distance between O and Q	r	Variable	Initial value

- (a) Form Q 's *translational momentum*, *angular momentum* about O , and *kinetic energy* in N .

Result: (in terms of symbols in the previous table, their time-derivatives, and $\hat{e}_r, \hat{e}_\theta, \hat{e}_z$).

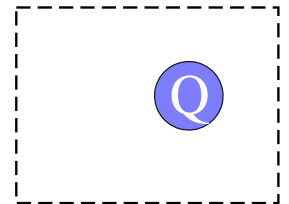
$${}^N\vec{L}^Q = \text{[]} \quad {}^N\vec{H}^{Q/O} = \text{[]} \quad {}^NK^Q = \text{[]}$$

- (b) **Draw** Q 's free-body diagram and determine the resultant force on Q .

Dot $\vec{F}^Q = m {}^N\vec{a}^Q$ with “clever” unit vectors and solve for \ddot{r} and $\ddot{\theta}$.

Result:

$$\vec{F}^Q = \text{[]} \quad \ddot{r} = \text{[]} \quad \ddot{\theta} = \text{[]}$$



- (c) Form an expression for Q 's mechanical energy in N (sum of kinetic energy and potential energy U).

Result: *Conservation of mechanical energy* $\text{KePe} = \frac{1}{2} m [\dot{r}^2 + (\dot{\theta} r)^2] - \frac{G m m^E}{r}$

- (d) Starting with the *angular momentum principle* in equation (22.4), form an expression that stays constant during Q 's motion in N (*conservation of angular momentum*).

Result: $H_{\text{Constant}} = \text{[]}$ (Section 22.7)

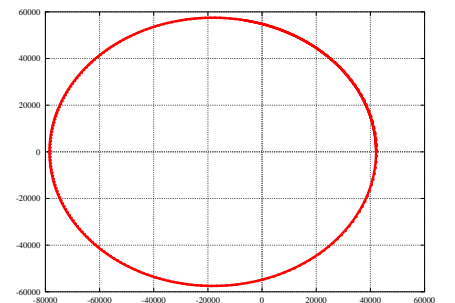
- (e) **Draw** a line L tangent to the ellipse at Q and perpendicular to \hat{n}_z .

In general, ${}^N\vec{v}^Q$ (Q 's velocity in N) is parallel to \hat{e}_θ .	True/False
In general, \hat{e}_θ is parallel to L .	True/False
In general, ${}^N\vec{v}^Q$ is parallel to L .	True/False

- (f) Shown right is a plot of Q 's orbital trajectory when the differential equations from part (6b) are solved with the initial values of part (6i). Query the Internet (or a textbook, instructor, colleague, etc.) for the next answers.

- Clearly mark Earth's location on the plot.
- The Earth is located at (circle one):

the center of the ellipse/a focus of the ellipse.



- (g) Form **Lagrange's equations** for generalized coordinates r and θ and solve for \ddot{r} and $\ddot{\theta}$.

Result: Note: U is given above and $K = {}^N K^Q$. **Lagrange's equations** are described in Chapter 27.

$$\begin{aligned} -\frac{\partial U}{\partial r} & \stackrel{(27.1)}{=} \frac{d}{dt} \left(\frac{\partial K}{\partial \dot{r}} \right) - \frac{\partial K}{\partial r} & \Rightarrow \ddot{r} & = \text{[Yellow Box]} \\ -\frac{\partial U}{\partial \theta} & \stackrel{(27.1)}{=} \frac{d}{dt} \left(\frac{\partial K}{\partial \dot{\theta}} \right) - \frac{\partial K}{\partial \theta} & \Rightarrow \ddot{\theta} & = \text{[Yellow Box]} \end{aligned}$$

Just calculate!

Lagrange's equations result in the same \ddot{r} and $\ddot{\theta}$ as part (6b). **True/False.**

- (h) Form **Kane's equation of motion** for the generalized speeds \dot{r} and $\dot{\theta}$ and solve for \ddot{r} and $\ddot{\theta}$.

Result: Note: **Kane's equations** are described in Chapter 26.

$$\begin{aligned} \mathcal{F}_{\dot{r}} & = -\frac{\partial V}{\partial r} \quad {}^N \mathcal{F}_{\dot{r}}^Q = \frac{\partial {}^N \vec{v}^Q}{\partial \dot{r}} \cdot m {}^N \vec{a}^Q & \Rightarrow \mathcal{F}_{\dot{r}} & \stackrel{(26.1)}{=} {}^N \mathcal{F}_{\dot{r}}^Q & \Rightarrow \ddot{r} & = \text{[Yellow Box]} \\ \mathcal{F}_{\dot{\theta}} & = -\frac{\partial V}{\partial \theta} \quad {}^N \mathcal{F}_{\dot{\theta}}^Q = \frac{\partial {}^N \vec{v}^Q}{\partial \dot{\theta}} \cdot m {}^N \vec{a}^Q & \Rightarrow \mathcal{F}_{\dot{\theta}} & \stackrel{(26.1)}{=} {}^N \mathcal{F}_{\dot{\theta}}^Q & \Rightarrow \ddot{\theta} & = \text{[Yellow Box]} \end{aligned}$$

Just calculate!

Kane's equations result in the same \ddot{r} and $\ddot{\theta}$ as part (6b). **True/False.**

- (i) **Optional:** Use a computer to solve the differential equations for $\theta(t)$ and $r(t)$ with the initial values (i.e, values at $t = 0$) of θ , $\dot{\theta}$, r , and \dot{r} shown below. Use a numerical integration step (tStep) of 0.2 hours. Plot y vs x for $0 \leq t \leq 48$ hours.

Result: Submit your computational files (e.g., MotionGenesis, MATLAB[®]) and plot of y vs. x .

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Sample MotionGenesis commands
Constant G = 6.67E-11 N*m^2/kg^2, mE = 5.97E24 kg, m = 200 kg
Variable r'', theta''
r'' = r * theta'^2 - G * mE / r^2
theta'' = -2 * theta' * r' / r
Input theta = 0 degrees, theta' = 0.3 rad/hour, r = 42108 km, r' = 0 km/hour
Input tFinal = 48 hours, tStep = 0.2 hours, absError = 1.0E-7
OutputPlot x = r*cos(theta); y = r*sin(theta)
Output t hours, theta degrees, r km, x km, y km, HConstant kg*m^2/sec, KePe Joules

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- (j) **Optional:** Verify each value in the following chart.

Quantity	Value
$H_{constant}$	$2.955 \times 10^{13} \text{ kg}\cdot\text{m}^2/\text{sec}$
Sum of kinetic and potential energy (<i>KePe</i>)	$-6.606 \times 10^8 \text{ Joules}$
Number of hours to complete one orbit	41 hours
Minimum distance between O and Q	42108 km
Maximum distance between O and Q	78500 km

