12.1 ♣ Sort from smallest mass unit to largest mass unit. (see Section 13.1)

1 oz_m	1 g	1 metric ton	1 kg	1 mg	1 U.S. ton	1 slug	1 lb_m

12.2 ♣ Concepts: What objects have kinetic energy or linear momentum?

${}^{N}K^{S}$, the ***kinetic energy*** of an object S in a reference frame N is to be determined.
Objects S that can have a non-zero kinetic energy are (circle ***all*** appropriate objects):

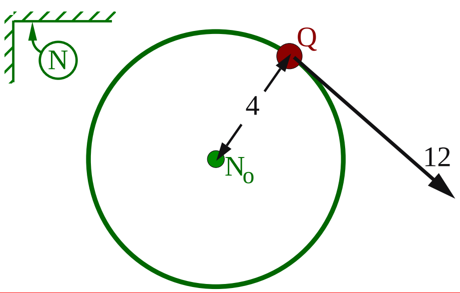
Real number	Matrix	Set of points	Mass center of a rigid body
Vector	Point	Reference frame	Flexible body
3D orthogonal unit basis	Particle	Rigid body	System of particles and bodies

Repeat for ${}^{N}\vec{L}^{S}$, the ***linear momentum*** of object S in reference frame N box appropriate objects.

12.3 ♣ Particle angular momentum concepts.

The following figures show a particle Q of mass 1 kg moving in a **plane** N. Point N_o is fixed in N. The figure on the left shows Q moving clockwise with speed 12 on a circle of radius 4 that is centered at N_o. The figure on the right shows Q moving with a speed of 12 on a horizontal line that is 4 from N_o. **Box** the following true statements about Q's ***angular momentum*** in N.

Q's angular momentum about N_o is $\vec{\mathbf{0}}$.
Q's angular momentum about N_o is not $\vec{\mathbf{0}}$.
Q's angular momentum about N_o is $\vec{\infty}$.
Q's angular momentum about N_o does not exist.



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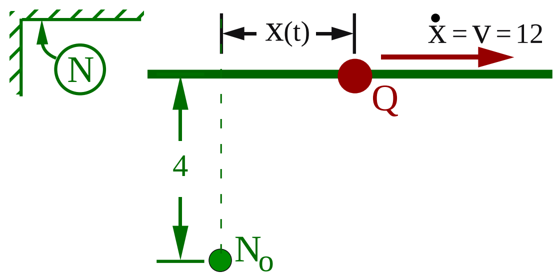
Q's angular momentum about N_o does not exist.



12.4 ♣ Optional: Just for fun. Culture, religion, science and "mass". (Sections 13.9, 13.7, 13.6)

Etymology of "mass"	Fill-in the blank
The "m" in $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$.	
Jewish Passover flat bread/cracker.	
Greek for flat bread.	
Latin for lump of dough.	
Spanish for lump of dough.	
Catholics eat bread at this Sunday event.	
Approximate number of atoms in 12 grams of carbon-12.	6 x 10
Estimated number of atoms in the visible universe.	1 x 10
Sub-atomic particle responsible for mass in animals, vegetables, and minerals.	
Most expensive science project in history to find sub-atomic particle with mass.	
Possible Earth-fatal object created by aforementioned science project.	

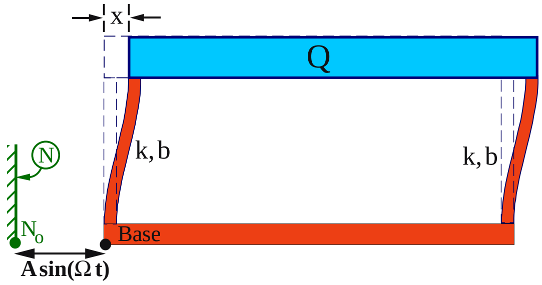
12.5 ♣ FE/EIT Review – Motion of a building in an earthquake.

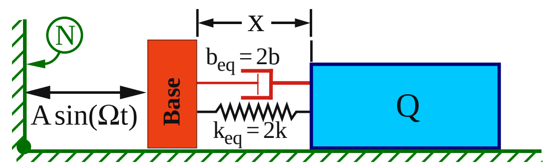
$\boxed{\vec{\mathbf{F}} \;\Rightarrow\; \vec{\mathbf{F}} = m\,\vec{\mathbf{a}} \;\Rightarrow\; \ddot{x}}$

A building moves due to an earthquake. The horizontally-right displacement of the building's base B is **modeled** as $A\,\sin(\Omega\,t)$ where the constant A is the magnitude of the ground's horizontal displacement and the constant Ω is the earthquake's frequency.

The base motion causes the building's roof Q of mass m to displace horizontally by $x(t)$ from its base. The stiffness and material damping in each of the two columns that support the roof is modeled as a linear horizontal spring (k) and linear horizontal damper (b).

For this dynamic analysis, the system is modeled as shown right (with a spring of 0 natural length). It is helpful to introduce a horizontally-right unit vector $\widehat{\mathbf{n}}_x$.





x is the $\widehat{\mathbf{n}}_x$ measure of Q's position from its base B

(a) Draw Q's ***free-body diagram*** and determine the spring/damper force on Q.
Result: $\vec{\mathbf{F}}_{\text{Spring/Damper}} \underset{\text{FBD}}{=} (-b_{\text{eq}}\,\dot{x} \;+\; -k_{\text{eq}}\,x)\,\widehat{\mathbf{n}}_x$

FBD of Q

(b) Form the relevant acceleration for $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ (e.g., differentiate the relevant position vector/velocity). Next, dot-product $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ with $\widehat{\mathbf{n}}_x$ to write a differential equation governing $x(t)$.
Result: [**Optional:** Verify results via Kane and/or Lagrange methods in Chapters 26, 27.]

$$\vec{\mathbf{a}} = [\;\;\; - \;\;\;]\,\widehat{\mathbf{n}}_x \quad\Rightarrow\quad \boxed{m\,\ddot{x} \;+\; b_{\text{eq}}\,\dot{x} \;+\; k_{\text{eq}}\,x \;=\; m\,A\,\Omega^2\,\sin(\Omega\,t)}$$

(c) Graphed right is $x(t)$ when:

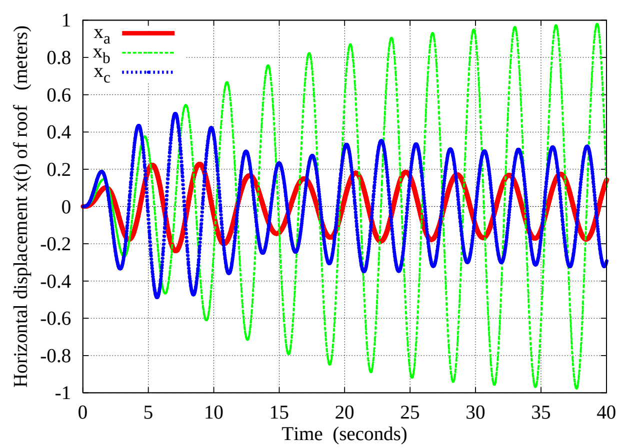
(a) $\Omega \;=\; 0.8\sqrt{\frac{k_{\text{eq}}}{m}} \;=\; 0.8\,\omega_n \qquad x_a(t)$

(b) $\Omega \;=\; 1.0\sqrt{\frac{k_{\text{eq}}}{m}} \;=\; 1.0\,\omega_n \qquad x_b(t)$

(c) $\Omega \;=\; 1.2\sqrt{\frac{k_{\text{eq}}}{m}} \;=\; 1.2\,\omega_n \qquad x_c(t)$

Circle the Ω that corresponds to the largest ***steady-state*** amplitude for $x(t)$.

Note: These graphs use $m \;=\; 5000$ kg, $b_{\text{eq}} \;=\; 1000\,\frac{\text{N*sec}}{\text{m}}$, $k_{\text{eq}} \;=\; 20000\,\frac{\text{N}}{\text{m}}$, $A \;=\; 0.1$ m.



(d) **Physics** ($\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$) gives the previous (boxed) equation in terms of positive constants m, b_{eq}, k_{eq}. However, its **mathematics** is easier if that equation is rewritten (rearrange by dividing by m) in terms of the positive constants ζ and ω_n as shown in the boxed-equation below. Determine the building's ***natural frequency*** ω_n and ***damping ratio*** ζ in terms of m, b_{eq}, k_{eq}.
Result:

$$\boxed{\ddot{x} \;+\; 2\,\zeta\,\omega_n\,\dot{x} \;+\; \omega_n^2\,x \;=\; A\,\Omega^2\,\sin(\Omega\,t)} \quad\Rightarrow\quad \omega_n = \sqrt{\frac{\;\;}{\;\;}} \qquad \zeta = \frac{\;\;}{2\sqrt{\;\;\;\;}}$$

(e) When $\zeta \;=\; 0$, the building vibrates at ω_n. When $0 \le \zeta \le 1$ (common for many structures), the building vibrates at a ***damped natural frequency*** $\omega_d \triangleq \omega_n\,\sqrt{1-\zeta^2}$.
In general, damping slows things down and makes $\omega_d \;<\; \omega_n$. **True/False**.