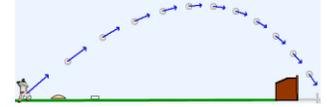


21.1.1 MG road-map: Projectile motion (2D)

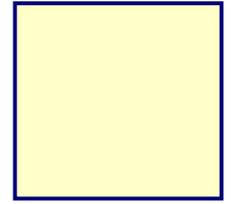
A baseball (particle Q) flies over Earth N (a Newtonian reference frame). Aerodynamic forces on the baseball are modeled as $-b\vec{v}$ (\vec{v} is Q 's velocity in N).

\hat{n}_x is horizontally-right, \hat{n}_y is vertically-upward, and N_o is home-plate (point fixed in N).



MG road-map for projectile motion x and y (\hat{n}_x, \hat{n}_y measures of Q 's position vector from N_o)

Variable	Translate/Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation
x	Translate	\hat{n}_x	Q	Draw	Not applicable	$\hat{n}_x \cdot (\vec{F}^Q)_{(20.1)} = m^Q N \vec{a}^Q$
y	Translate	\hat{n}_y	Q	Draw	Not applicable	$\hat{n}_y \cdot (\vec{F}^Q)_{(20.1)} = m^Q N \vec{a}^Q$



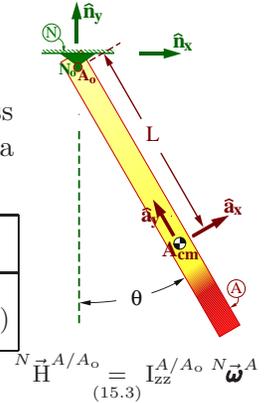
Draw FBD

Solution and simulation link at www.MotionGenesis.com ⇒ [Textbooks](#) ⇒ [Resources](#).

21.1.2 MG road-map: Rigid body pendulum (2D)

A non-uniform density rigid rod A is attached at point A_o of A by a frictionless revolute/pin joint to Earth N (Newtonian reference frame). The rod swings with a “pendulum angle” θ in a vertical plane that is perpendicular to unit vector \hat{a}_z .

Variable	Translate/Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation
θ	Rotate	$\hat{a}_z = \hat{n}_z$	A	Draw	A_o	$\hat{a}_z \cdot (\vec{M}^{A/A_o})_{(20.4)} = \frac{N d^N \vec{H}^{A/A_o}}{dt}$

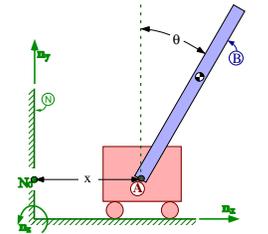


Solution and simulation link at www.MotionGenesis.com ⇒ [Textbooks](#) ⇒ [Resources](#).

$${}^N \vec{H}^{A/A_o} = I_{zz}^{A/A_o} {}^N \vec{\omega}^A \quad (15.3)$$

21.1.3 MG road-map: Inverted pendulum on cart (x and θ) (2D)

A rigid rod B is pinned to a massive cart A (modeled as a particle) that translates horizontally in a Newtonian reference frame N . The cart's position vector from a point N_o fixed in N is $x\hat{n}_x$ (\hat{n}_x is horizontally-right). B 's swinging motion in N is in a vertical plane perpendicular to \hat{n}_z (a unit vector fixed in both B and N).



Variable	Translate/Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation
x	Translate	\hat{n}_x	A, B	Draw	Not applicable	$\hat{n}_x \cdot (\vec{F}^S)_{(20.1)} = m^S * N \vec{a}^{S_{cm}}$
θ	Rotate	$\hat{b}_z = \hat{n}_z$	B	Draw	A	$\hat{b}_z \cdot (\vec{M}^{B/A})_{(20.4)} = \frac{N d^N \vec{H}^{B/A}}{dt} + \dots$

Hw 15.7 and Chapter 25 complete these calculations.

Note: $m^S * N \vec{a}^{S_{cm}} \stackrel{(11.3)}{=} m^A * N \vec{a}^A + m^B * N \vec{a}^{B_{cm}}$ and $\frac{N d^N \vec{H}^{B/A}}{dt} + \dots \stackrel{(20.5)}{=} I_{zz}^{B/A} * N \vec{\alpha}^B + m^B * \vec{r}^{B_{cm}/A} \times N \vec{a}^A$.

21.1.4 MG road-map: Rotating rigid body (3D)

Shown right is a rotating rigid body B (e.g., tennis racquet, spacecraft, or aircraft) in a Newtonian frame N . B 's angular velocity in N is $\omega_x \hat{\mathbf{b}}_x + \omega_y \hat{\mathbf{b}}_y + \omega_z \hat{\mathbf{b}}_z$ ($\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$ are unit vectors fixed in B).

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	<i>MG road-map equation</i>
ω_x	Rotate	$\hat{\mathbf{b}}_x$	B	Draw	B_{cm}	$\hat{\mathbf{b}}_x \cdot \left(\vec{\mathbf{M}}^{B/B_{cm}} = \frac{{}^N d {}^N \vec{\mathbf{H}}^{B/B_{cm}}}{dt} \right)$ (20.4)
ω_y	Rotate	$\hat{\mathbf{b}}_y$	B	Draw	B_{cm}	$\hat{\mathbf{b}}_y \cdot \left(\vec{\mathbf{M}}^{B/B_{cm}} = \frac{{}^N d {}^N \vec{\mathbf{H}}^{B/B_{cm}}}{dt} \right)$ (20.4)
ω_z	Rotate	$\hat{\mathbf{b}}_z$	B	Draw	B_{cm}	$\hat{\mathbf{b}}_z \cdot \left(\vec{\mathbf{M}}^{B/B_{cm}} = \frac{{}^N d {}^N \vec{\mathbf{H}}^{B/B_{cm}}}{dt} \right)$ (20.4)

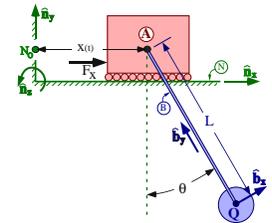


Solution and simulation link at www.MotionGenesis.com \Rightarrow [Textbooks](#) \Rightarrow [Resources](#).

Note: The “about point” is somewhat arbitrary. When B_{cm} is chosen: ${}^N \vec{\mathbf{H}}^{B/B_{cm}} = \vec{\mathbf{I}}^{B/B_{cm}} \cdot {}^N \vec{\boldsymbol{\omega}}^B$.
(15.1)

21.1.5 MG road-map: Bridge crane equations of motion (2D)

A payload (particle) Q is welded to a light rigid cable B which swings in a Newtonian frame N . Cable B is pinned to a massive trolley A that can move horizontally along a frictionless slot fixed in N with a **specified** (known) displacement $x(t)$. A translational actuator with force measure F_x connects trolley A to point N_o of N .



MG road-map for pendulum angle θ , actuator force F_x , and cable tension

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	<i>MG road-map equation</i>
θ	Rotate	$\hat{\mathbf{n}}_z = \hat{\mathbf{b}}_z$	B, Q	Draw	A	$\hat{\mathbf{n}}_z \cdot \left(\vec{\mathbf{M}}^{S/A} = \frac{{}^N d {}^N \vec{\mathbf{H}}^{S/A}}{dt} + \dots \right)$
F_x	Translate	$\hat{\mathbf{n}}_x$	A, B, Q	Draw	Not applicable	$\hat{\mathbf{n}}_x \cdot \left(\vec{\mathbf{F}}^S = m^S {}^N \vec{\mathbf{a}}^{S_{cm}} \right)$
Tension	Translate	$\hat{\mathbf{b}}_y$	Q	Draw	Not applicable	$\hat{\mathbf{b}}_y \cdot \left(\vec{\mathbf{F}}^Q = m^Q {}^N \vec{\mathbf{a}}^Q \right)$

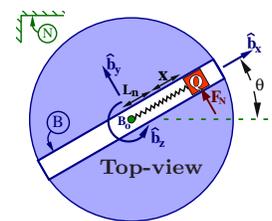
Student/Instructor version at www.MotionGenesis.com \Rightarrow [Textbooks](#) \Rightarrow [Resources](#)

Note: Only the θ road-map equation is needed to predict this system's motion. The others are shown for illustrative purposes.

21.1.6 MG road-map: Particle on spinning slot (2D)

A particle Q slides on a straight slot B . The slot is connected with a revolute joint to a Newtonian frame N at point B_o so that B rotates in a horizontal plane perpendicular to $\hat{\mathbf{b}}_z$ ($\hat{\mathbf{b}}_z$ is vertically-upward and fixed in both B and N).

Note: Homework 14.7 completes the MG road-map calculations for x and θ .



MG road-map for x , θ , and F_N ($\hat{\mathbf{b}}_y$ measure of normal force on Q from B)

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	<i>MG road-map equation</i>
x	Translate	$\hat{\mathbf{b}}_x$	Q	Draw	Not applicable	$\hat{\mathbf{b}}_x \cdot \left(\vec{\mathbf{F}}^Q = m^Q {}^N \vec{\mathbf{a}}^Q \right)$
θ	Rotate	$\hat{\mathbf{b}}_z$	B, Q	Draw	B_o	$\hat{\mathbf{b}}_z \cdot \left(\vec{\mathbf{M}}^{S/B_o} = \frac{{}^N d {}^N \vec{\mathbf{H}}^{S/B_o}}{dt} \right)$
F_N	Translate	$\hat{\mathbf{b}}_y$	Q	Draw	Not applicable	$\hat{\mathbf{b}}_y \cdot \left(\vec{\mathbf{F}}^Q = m^Q {}^N \vec{\mathbf{a}}^Q \right)$

Note: The F_N road-map equation is needed to predict motion if a friction force depends on μF_N .

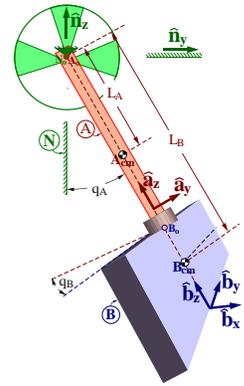
$${}^N \vec{\mathbf{H}}^{S/B_o} = {}^N \vec{\mathbf{H}}^{B/B_o} + {}^N \vec{\mathbf{H}}^{Q/B_o} \quad \text{where} \quad {}^N \vec{\mathbf{H}}^{B/B_o} = \mathbf{I}_{zz} {}^N \vec{\boldsymbol{\omega}}^B \quad \text{and} \quad {}^N \vec{\mathbf{H}}^{Q/B_o} = \mathbf{r}^{Q/B_o} \times m^Q {}^N \vec{\mathbf{v}}^Q. \quad (10.3)$$

21.1.7 MG road-map: Motion of a chaotic double pendulum (3D)

Shown right is a mechanical model of a swinging babyboot (uniform plate B) attached by a shoelace (thin uniform rod A) to a fixed rigid support N . Rod A is attached to N by a revolute joint at point N_o of N . B is attached to A with a second revolute joint at point B_o so B can rotate freely about A 's axis.

Note: The revolute joints' axes are *perpendicular*, not parallel.

- The plate, rod, and support are rigid.
- The revolute joints are ideal (massless, frictionless, no slop/flexibility).
- Earth is a Newtonian reference frame N .
- Forces due to Earth's gravitation are uniform and constant.
- Other distance forces (electromagnetic and gravitational) and air-resistance are negligible.



Right-handed sets of unit vectors $\hat{n}_x, \hat{n}_y, \hat{n}_z$; $\hat{a}_x, \hat{a}_y, \hat{a}_z$; $\hat{b}_x, \hat{b}_y, \hat{b}_z$ are fixed in N, A, B , respectively, with $\hat{n}_x = \hat{a}_x$ parallel to the revolute axis joining A to N , \hat{n}_z vertically-upward, $\hat{a}_z = \hat{b}_z$ parallel to the rod's long axis (and the revolute axis joining B to A), and \hat{b}_z perpendicular to plate B . q_A is the angle from \hat{n}_z to \hat{a}_z with $+\hat{n}_x$ sense. q_B is the angle from \hat{a}_y to \hat{b}_y with $+\hat{a}_z$ sense.

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	<i>MG road-map equation</i>
q_A	Rotate	\hat{a}_x	A, B	Draw	A_o	$\hat{a}_x \cdot (\vec{M}^{S/A_o} = \frac{N d^N \vec{H}^{S/A_o}}{dt})$
q_B	Rotate	\hat{b}_z	B	Draw	B_{cm}	$\hat{b}_z \cdot (\vec{M}^{B/B_{cm}} = \frac{N d^N \vec{H}^{B/B_{cm}}}{dt})$

Solution and simulation link at www.MotionGenesis.com \Rightarrow [Textbooks](#) \Rightarrow [Resources](#).

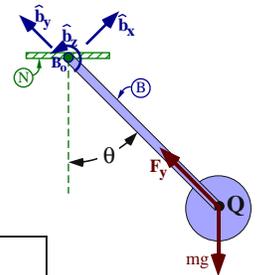
Note: The "about point" for the q_B road-map can be shifted from B_o to B_{cm} since $\hat{b}_z \cdot \vec{M}^{B/B_{cm}} = \hat{b}_z \cdot \vec{M}^{B/B_o}$. (17.4)

$${}^N \vec{H}^{S/A_o} = {}^N \vec{H}^{A/A_o} + {}^N \vec{H}^{B/A_o} \quad \text{where} \quad {}^N \vec{H}^{A/A_o} = \mathbf{I}_{xx}^{A/A_o} * {}^N \vec{\omega}^A \quad \text{and} \quad {}^N \vec{H}^{B/A_o} = \mathbf{I}^{B/B_{cm}} \cdot {}^N \vec{\omega}^B + \mathbf{r}^{B_{cm}/A_o} \times m^B N \vec{v}^{B_{cm}}. \quad (15.3) \quad (15.2, 15.1)$$

21.1.8 MG road-map: Particle pendulum (2D) – angle and tension

A particle Q is welded to the distal end of a light rigid rope B . The rope's other end attaches to a point B_o , fixed in a Newtonian reference frame N . The swinging motion of B and Q is in a vertical plane that is perpendicular to unit vector \hat{b}_z .

MG road-map for pendulum angle θ and tension F_y (\hat{b}_y measure of force on Q from B)



Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	<i>MG road-map equation</i>
θ	Rotate	\hat{b}_z	B, Q	Draw	B_o	$\hat{b}_z \cdot (\vec{M}^{S/B_o} = \frac{N d^N \vec{H}^{S/B_o}}{dt})$ (20.4)
F_y	Translate	\hat{b}_y	Q	Draw	Not applicable	$\hat{b}_y \cdot (\vec{F}^Q = m^Q {}^N \vec{a}^Q)$ (20.1)

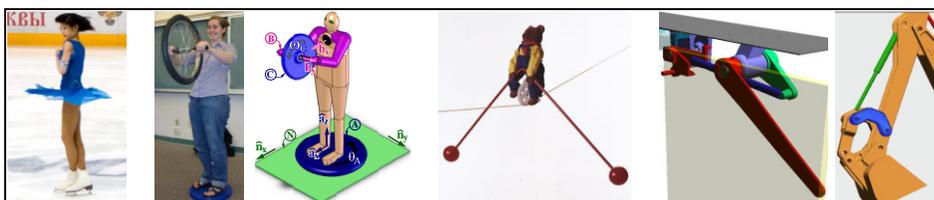
Solution and simulation link at www.MotionGenesis.com \Rightarrow [Textbooks](#) \Rightarrow [Resources](#).

Note: Only the θ road-map equation is needed to predict motion. The other is shown for illustrative purposes.

$$\text{Note: } {}^N \vec{H}^{S/N_o} = {}^N \vec{H}^{Q/N_o} = \mathbf{r}^{Q/N_o} \times m^Q N \vec{v}^Q. \quad (10.3)$$

Section 24.3.2 completes all MG road-map calculations for θ .

Draw FBDs



Many additional MG road-map examples at www.MotionGenesis.com \Rightarrow [Textbooks](#) \Rightarrow [Resources](#).

21.1.9 MG road-map: Dynamicist on a turntable (ice-skater)

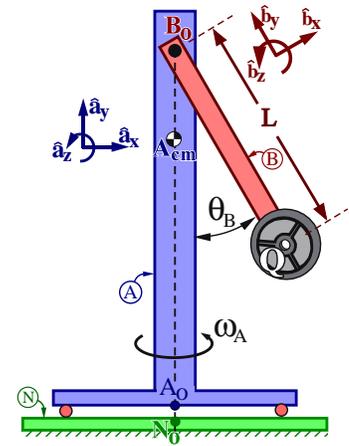
A dynamics instructor stands on a spinning turntable and swings a heavy dumbbell Q inward and outward to change his spin-rate (similar to the ice-skater). Q is modeled as a particle rigidly attached (welded) to the end of the instructor's hands.

The schematic (below-right) shows a rigid body A (modeling the instructor's legs, torso, and head) that rotates (without friction) relative to Earth (a Newtonian reference frame N) about a vertical axis that is fixed in both A and N and which passes through both point N_o of N and point A_{cm} (A 's center of mass).

A massless rigid arm B (modeling the instructor's arms and hands) attaches to A by a revolute motor (shoulder/muscles) whose revolute axis is horizontal and located at point B_o of B (B_o lies on the vertical axis connecting N_o and A_{cm}).

The motor (muscles) **specifies** B 's angle θ_B relative to A to change in a known (prescribed) manner from 0 to π rad in 4 seconds ($\theta_B = \pi \frac{t}{4}$).

Right-handed orthogonal unit vectors $\hat{a}_x, \hat{a}_y, \hat{a}_z$ and $\hat{b}_x, \hat{b}_y, \hat{b}_z$ are fixed in A and B , respectively, with \hat{a}_y vertically-upward, $\hat{b}_z = \hat{a}_z$ parallel to the revolute motor's axis, and \hat{b}_y directed from Q to B_o .



Quantity	Symbol	Type	Value
Earth's gravitational constant	g	Constant	$9.8 \frac{m}{s^2}$
Distance between Q and B_o	L	Constant	0.7 m
Mass of Q	m	Constant	12 kg
A 's moment of inertia about line $\overline{A_{cm} B_o}$	I_{yy}	Constant	0.6 kg m^2
Angle from \hat{a}_y to \hat{b}_y with $+\hat{a}_z$ sense	θ_B	Specified	$0.25 \pi \text{ t rad}$
\hat{a}_y measure of A 's angular velocity in N	ω_A	Variable	

Complete the **MG road-map** for the turntable's "spin-rate" ω_A (Note: The "about point" is not unique)

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation
ω_A	Rotate	\hat{a}_y	A, B, Q	Draw	B_o	$\hat{a}_y \cdot (\vec{M}^{S/B_o} = \frac{{}^N d {}^N \vec{H}^{S/B_o}}{dt})$

FBD hint: Replace the set \vec{S} of contact forces on A from N by an equivalent set with a force \vec{F}^{A_o} and a torque \vec{T}^A . Note the "about point" can be shifted from A_o to A_{cm} or B_o or N_o since $\hat{a}_y \cdot \vec{M}^{\vec{S}/A_o} \stackrel{(17.4)}{=} \hat{a}_y \cdot \vec{M}^{\vec{S}/B_{cm}} \stackrel{(17.4)}{=} \hat{a}_y \cdot \vec{M}^{\vec{S}/B_o}$.

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21.1.10 MG road-map: Instructor on turntable with spinning wheel (3D)

The pictures to the right shows dynamicist Dr. G standing on a spinning turntable and holding a spinning bicycle wheel.



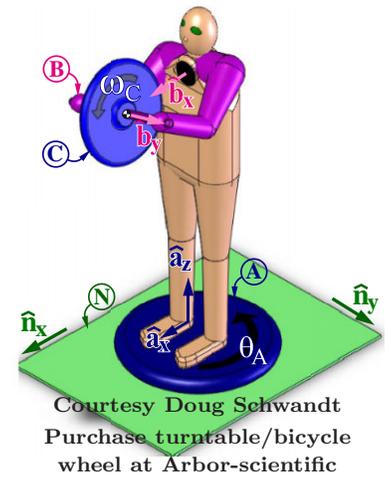
The mechanical model (below right) has a rigid body A (modeling the turntable, legs, torso, and head) that can freely rotate relative to Earth (Newtonian reference frame N) about a vertical axis that is fixed in both A and N and which passes through the center of the turntable (point N_o) and A_{cm} (A 's center of mass).

A light (massless) rigid frame B (modeling the shoulders, arms, hands, and a portion of the bicycle wheel's axle) is attached to A by a revolute motor at point B_o of B (B_o lies on the vertical axis passing through A_{cm}). The motor's revolute axis passes through points B_o and C_{cm} , is horizontal, and is parallel to $\hat{\mathbf{b}}_x = \hat{\mathbf{a}}_x$.

A rigid bicycle wheel C is attached to B by a frictionless revolute joint whose axis passes through C_{cm} (C 's center of mass) and is parallel to $\hat{\mathbf{b}}_y$.

Right-handed orthogonal unit vectors $\hat{\mathbf{a}}_x, \hat{\mathbf{a}}_y, \hat{\mathbf{a}}_z$ and $\hat{\mathbf{n}}_x, \hat{\mathbf{n}}_y, \hat{\mathbf{n}}_z$ are fixed in A and N , respectively. Initially $\hat{\mathbf{a}}_i = \hat{\mathbf{n}}_i$ ($i = x, y, z$), and then rigid body A is subjected to a right-handed rotation characterized by $\theta_A \hat{\mathbf{a}}_z$ where $\hat{\mathbf{a}}_z = \hat{\mathbf{n}}_z$ is directed vertically-upward and $\hat{\mathbf{a}}_x$ points from Dr. G's back to front (parallel to the axis of the revolute motor connecting A and B).

Unit vectors $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$ are fixed in B . Initially $\hat{\mathbf{b}}_i = \hat{\mathbf{a}}_i$ ($i = x, y, z$), then B is subjected to a θ_B ($\hat{\mathbf{a}}_x = \hat{\mathbf{b}}_x$) right-handed rotation in A where $\hat{\mathbf{b}}_y$ is directed along the wheel's axle from Dr. G's right-to-left hand. Dr. G changes θ_B in a **specified** sinusoid manner with amplitude 30° and period 4 seconds.

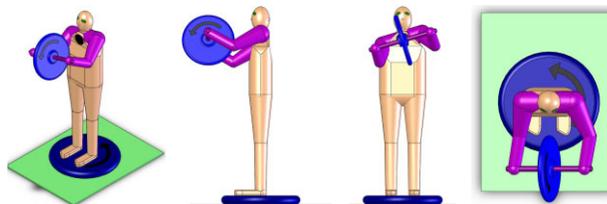


Quantity	Symbol and type	Value
Mass of C	m^C Constant	2 kg
Distance between B_o and C_{cm}	L_x Constant	0.5 m
A 's moment of inertia about B_o for $\hat{\mathbf{a}}_z$	I_{zz}^A Constant	0.64 kg m ²
C 's moment of inertia about C_{cm} for $\hat{\mathbf{b}}_x$	I^C Constant	0.12 kg m ²
C 's moment of inertia about C_{cm} for $\hat{\mathbf{b}}_y$	J^C Constant	0.24 kg m ²
Angle from $\hat{\mathbf{n}}_x$ to $\hat{\mathbf{a}}_x$ with $+\hat{\mathbf{n}}_z$ sense	θ_A Variable	
Angle from $\hat{\mathbf{a}}_y$ to $\hat{\mathbf{b}}_y$ with $+\hat{\mathbf{a}}_x$ sense	θ_B Specified	$\frac{\pi}{6} \sin(\frac{\pi}{2} t)$
$\hat{\mathbf{b}}_y$ measure of C 's angular velocity in B	ω_C Variable	

Complete the **MG road-map** for θ_A and ω_C (the "about points" are not unique).

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation
θ_A	Rotate	$\hat{\mathbf{a}}_z$	A, B, C	Draw	B_o	$\hat{\mathbf{a}}_z \cdot (\vec{M}^{S/B_o} = \frac{N d^N \vec{H}^{S/B_o}}{dt})$
ω_C	Rotate	$\hat{\mathbf{b}}_y$	C	Draw	C_{cm}	$\hat{\mathbf{b}}_y \cdot (\vec{M}^{C/C_{cm}} = \frac{N d^N \vec{H}^{C/C_{cm}}}{dt})$

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21.1.11 MG road-map: Bear riding a unicycle on a high-wire (3D)

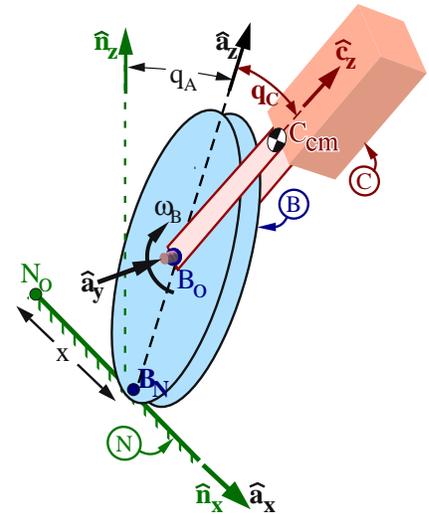
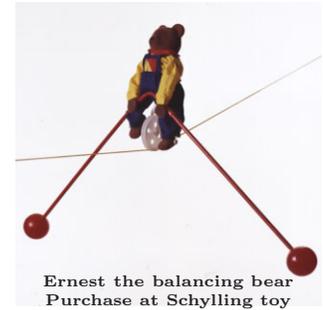
The figures to the right show a (massless) grooved pulley-wheel B that **rolls** along a taut (rigid) cable N (fixed on Earth, a Newtonian frame). Rigid body C (seat, rider, and balancing poles) attach to B with an ideal revolute motor at B_o (B 's centroid). The motor axis is aligned with B 's symmetry axis.

Right-handed orthogonal unit vectors $\hat{n}_x, \hat{n}_y, \hat{n}_z$ are fixed in N with \hat{n}_z vertically-upward and \hat{n}_x directed horizontally along the cable from a point N_o (fixed in N) to B_N (B 's rolling point of contact with N).

Right-handed orthogonal unit vectors $\hat{a}_x, \hat{a}_y, \hat{a}_z$ are directed with $\hat{a}_x = \hat{n}_x$, \hat{a}_y parallel to the motor axis, and \hat{a}_z from B_N to B_o .

Right-handed unit vectors $\hat{c}_x, \hat{c}_y, \hat{c}_z$ are parallel to C 's principal inertia axes about C_{cm} (C 's center of mass), with $\hat{c}_y = \hat{a}_y$ and \hat{c}_z from B_o to C_{cm} (with balancing poles, C_{cm} is below B_o and L_C is negative).

Quantity	Symbol	Type	Value
Earth's gravitational constant	g	Constant	9.8 m/s ²
Radius of B	r_B	Constant	30 cm
\hat{c}_z measure of C_{cm} 's position vector from B_o	L_C	Constant	-35 cm
Mass of C	m^C	Constant	2 kg
C 's moment of inertia about C_{cm} for \hat{c}_x	I	Constant	3.4 kg m ²
C 's moment of inertia about C_{cm} for \hat{c}_y	J	Constant	3.2 kg m ²
C 's moment of inertia about C_{cm} for \hat{c}_z	K	Constant	2.8 kg m ²
\hat{a}_y measure of motor torque on B from C	T_y	Specified	below
Angle from \hat{n}_z to \hat{a}_z with $-\hat{n}_x$ sense	q_A	Variable	
\hat{a}_y measure of ${}^A\vec{\omega}^B$ (${}^A\vec{\omega}^B = \omega_B \hat{a}_y$)	ω_B	Variable	
Angle from \hat{a}_z to \hat{c}_z with $+\hat{a}_y$ sense	q_C	Variable	
\hat{n}_x measure of \vec{r}^{B_N/N_o}	x	Variable	



Form a complete set of **MG road-maps** for this systems's equations of motion (solution is not unique).

If necessary, add more **MG road-maps** so there are the same number of equations as unknowns.

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation	Additional Unknowns
q_A	Rotate	\hat{a}_x	A, B, C	Draw	B_N	$\hat{a}_x \cdot \left(\vec{M}^{S/B_N} = \frac{N d^N \vec{H}^{S/B_N}}{dt} + \dots \right)$	
ω_B	Rotate	\hat{a}_y	B, C	Draw	B_N	$\hat{a}_y \cdot \left(\vec{M}^{S/B_N} = \frac{N d^N \vec{H}^{S/B_N}}{dt} + \dots \right)$	
q_C	Rotate	\hat{a}_y	C	Draw	B_o	$\hat{a}_y \cdot \left(\vec{M}^{C/B_o} = \frac{N d^N \vec{H}^{C/B_o}}{dt} + \dots \right)$	
x	Translate	\hat{a}_x	A, B, C	Draw	Not applicable	$\hat{a}_x \cdot \left(\vec{F}^S = m^S * {}^N \vec{a}^{S_{cm}} \right)$	* F_x

* Additional scalar constraint equation(s): $\dot{x} - r\omega_B = 0$

MG road-map for ω_B is not unique. Alternate (allows for sliding): "System" B , "About point" B_o , "Additional unknown" F_x .
Instructor notes: One way to eliminate F_x from this analysis is to solve $\dot{x} = r\omega_B$ and eliminate the MG road-map for x .

To move the unicycle to $x_{Desired} = 10$ m, use a "PD control law" with $T_y = -0.3(x - x_{Desired}) - 0.6\dot{x}$.

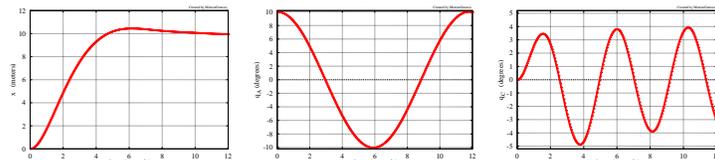
Optional simulation:

Plot x, q_A, q_C for $0 \leq t \leq 12$ sec.

Use initial values:

$$x = 0 \text{ m} \quad q_A = 10^\circ \quad q_C = 0^\circ$$

$$\dot{x} = 0 \quad \dot{q}_A = 0 \quad \dot{q}_C = 0$$

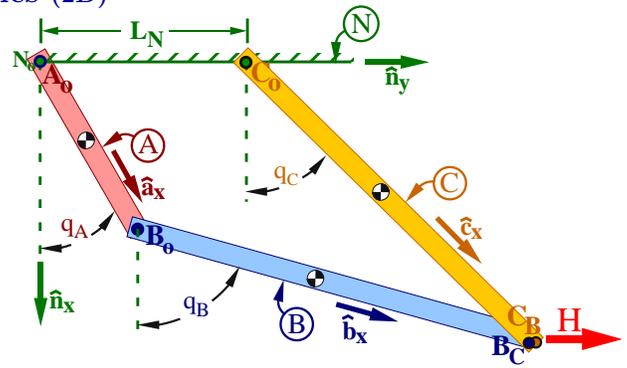


Solution at www.MotionGenesis.com \Rightarrow [Get Started](#) \Rightarrow [Bear on rolling unicycle.](#)

21.1.12 MG road-map: Four-bar linkage statics (2D)

The figure to the right shows a planar four-bar linkage consisting of frictionless-pin-connected uniform rigid links A , B , C and ground N .

- Link A connects to N and B at points A_o and A_B
- Link B connects to A and C at points B_o and B_C
- Link C connects to N and B at points C_o and C_B
- Point N_o of N is coincident with A_o
- Point N_C of N is coincident with C_o



Right-handed orthogonal unit vectors $\hat{\mathbf{a}}_i$, $\hat{\mathbf{b}}_i$, $\hat{\mathbf{c}}_i$, $\hat{\mathbf{n}}_i$ ($i = x, y, z$) are fixed in A , B , C , N , with:

- $\hat{\mathbf{a}}_x$ directed from A_o to A_B
- $\hat{\mathbf{b}}_x$ directed from B_o to B_C
- $\hat{\mathbf{c}}_x$ directed from C_o to C_B
- $\hat{\mathbf{n}}_x$ vertically-downward
- $\hat{\mathbf{n}}_y$ directed from N_o to N_C
- $\hat{\mathbf{a}}_z = \hat{\mathbf{b}}_z = \hat{\mathbf{c}}_z = \hat{\mathbf{n}}_z$ parallel to pin axes

As in Hw 8.7, create the following “loop equation” and dot-product with $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{n}}_y$.

$$L_A \hat{\mathbf{a}}_x + L_B \hat{\mathbf{b}}_x - L_C \hat{\mathbf{c}}_x - L_N \hat{\mathbf{n}}_y = \vec{\mathbf{0}}$$

Quantity	Symbol	Value
Length of link A	L_A	1 m
Length of link B	L_B	2 m
Length of link C	L_C	2 m
Distance between N_o and N_C	L_N	1 m
Mass of A	m^A	10 kg
Mass of B	m^B	20 kg
Mass of C	m^C	20 kg
Earth’s gravitational acceleration	g	$9.81 \frac{\text{m}}{\text{s}^2}$
$\hat{\mathbf{n}}_y$ measure of force applied to C_B	H	200 N
Angle from $\hat{\mathbf{n}}_x$ to $\hat{\mathbf{a}}_x$ with $+\hat{\mathbf{n}}_z$ sense	q_A	Variable
Angle from $\hat{\mathbf{n}}_x$ to $\hat{\mathbf{b}}_x$ with $+\hat{\mathbf{n}}_z$ sense	q_B	Variable
Angle from $\hat{\mathbf{n}}_x$ to $\hat{\mathbf{c}}_x$ with $+\hat{\mathbf{n}}_z$ sense	q_C	Variable

Complete the following **MG road-map** to determine this systems’s **static configuration**.

Make a “cut” between points B_C and C_B and introduce a constraint force $\vec{\mathbf{F}}^{C_B}$ on C_B from B_C .

Variable	Translate/Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation	Additional Unknowns
q_A	Rotate	$\hat{\mathbf{n}}_z$	A, B	Draw	A_o	$\hat{\mathbf{a}}_x \cdot \vec{\mathbf{M}}^{S/A_o} = 0$	$F_x^{C_B}, F_y^{C_B}$
q_B	Rotate	$\hat{\mathbf{n}}_z$	B	Draw	B_o	$\hat{\mathbf{a}}_y \cdot \vec{\mathbf{M}}^{B/B_o} = 0$	$F_x^{C_B}, F_y^{C_B}$
q_C	Rotate	$\hat{\mathbf{n}}_z$	C	Draw	C_o	$\hat{\mathbf{a}}_y \cdot \vec{\mathbf{M}}^{C/C_o} = 0$	$F_x^{C_B}, F_y^{C_B}$
* Additional scalar constraint equation:				$-L_A \sin(q_A) \dot{q}_A - L_B \sin(q_B) \dot{q}_B + L_C \sin(q_C) \dot{q}_C = 0$			
* Additional scalar constraint equation:				$L_A \cos(q_A) \dot{q}_A + L_B \cos(q_B) \dot{q}_B - L_C \cos(q_C) \dot{q}_C = 0$			

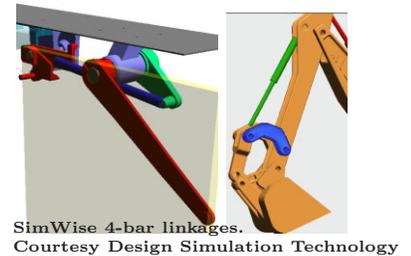
Hint: If you use efficient replacement of gravity forces, MG road-maps results can be identical to those in Hw 22.6.

Using the **MG road-map**, determine the **static equilibrium** values of q_A, q_B, q_C . Using your intuition (guess), circle the **stable** solution.

Solution 1	$q_A \approx 20.0^\circ$	$q_B \approx 71.7^\circ$	$q_C \approx 38.3^\circ$
Solution 2	$q_A \approx 249.3^\circ$	$q_B \approx 140.2^\circ$	$q_C \approx 199.1^\circ$
Solution 3	$q_A \approx 30.7^\circ$	$q_B \approx 226.1^\circ$	$q_C \approx 254.7^\circ$

Determine at least one solution. Solutions are for $H = 200 \text{ N}$.

Solution at www.MotionGenesis.com ⇒ [Get Started](#) ⇒ [Four-bar linkage](#)



Efficient replacement of gravity forces: **Real** gravity forces are complicated as there is a gravity force on each of the $\approx 6.02 \times 10^{23}$ particles of the rod. Although it is common to replace the **real** gravitational forces on each rod with “ $m g$ ” at each link’s center of mass, an efficient alternative (similar to Hw 21.7) replaces gravity forces on each rod with half the gravity force at each end as shown right.

