Complex numbers, circuits, Laplace transforms, frequency response, motors and sensors

Show work – except for ♣ fill-in-blanks (print .pdf from www.MotionGenesis.com ⇒ Textbooks ⇒ Resources).

8.1 Euler's formula and trigonometry functions (Chapter 15).

Show every step to express the right-hand side of the following expressions in terms of trigonometric functions of the real scalar variable θ (without the imaginary number i).

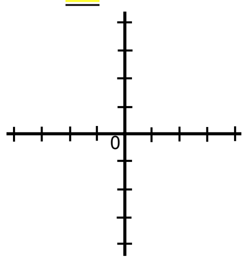
$$\frac{e^{i\,\theta} + e^{-i\,\theta}}{2} = \boxed{} \qquad\qquad \frac{e^{i\,\theta} - e^{-i\,\theta}}{2\,i} = \boxed{}$$

8.2 ♣ Putting real, imaginary, and complex numbers in magnitude-phase form (Section 15.1).

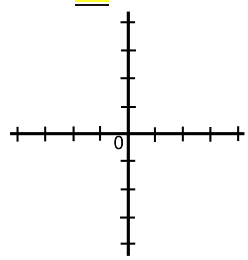
Clearly **mark** each of the following number's location in the complex plane.
Next, express the number in the magnitude-phase form $z = |z|\,e^{(\theta+2\,\pi\,n)\,i}$, where

- $|z|$ is the magnitude of z and n is any integer (e.g., $n=0,1,2,\dots$)
- $-\pi \le \theta \le \pi$ is the angle between the positive real axis and the line connecting 0 to z

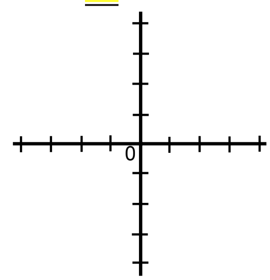
$2 = \underline{\boxed{}}\,e^{(\underline{\boxed{}}+2\,\pi\,n)\,i}$



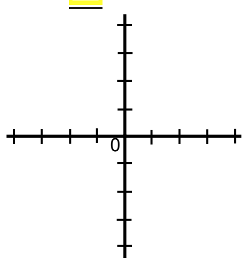
$-2 = \underline{\boxed{}}\,e^{(\underline{\boxed{}}+2\,\pi\,n)\,i}$



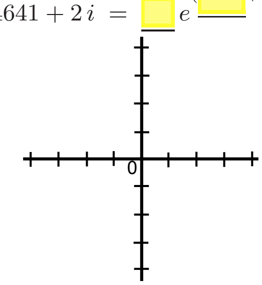
$4\,i = \underline{\boxed{}}\,e^{(\underline{\boxed{}}+2\,\pi\,n)\,i}$



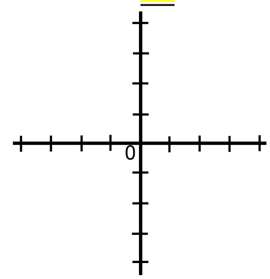
$-4\,i = \underline{\boxed{}}\,e^{(\underline{\boxed{}}+2\,\pi\,n)\,i}$



$3.4641 + 2\,i = \underline{\boxed{}}\,e^{(\underline{\boxed{}}+2\,\pi\,n)\,i}$



$-3.4641 - 2\,i = \underline{\boxed{}}\,e^{(\underline{\boxed{}}+2\,\pi\,n)\,i}$



8.3 ♣ Why does multiplying two negative numbers produce a positive number? (Section 15.5)

Using magnitude-phase form, show $(-2) * (-2) = +4$.

$$(-2)*(-2) = \left[2\,e^{(\boxed{}+\boxed{})\,i}\right] * \left[2\,e^{(\boxed{}+\boxed{})\,i}\right] = 4\,e^{(\boxed{}+\boxed{})\,i} = 4\left[\cos(\boxed{}+\boxed{}) + \boxed{}\right] = 4$$

8.4 ♣ Complex numbers and exponentiation (Section 15.6).

Find ***all*** complex numbers (in Cartesian form) equal to the following.

$\sqrt{4} = \boxed{} = \underline{\boxed{}}$ or $\underline{\boxed{}}$

$\sqrt{i} = \boxed{} \approx \underline{0.707} + \underline{0.707}\,i$ or $\underline{\boxed{}} - \underline{\boxed{}}\,i$

$1^{\frac{1}{2\pi}} = \boxed{} = 1,\ \ 0.54+0.84\,i,\ \ \boxed{}\ \ \dots$

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8.5 ♣ Optional: How does a number change when multiplied by -1 or exponentiation by -1.

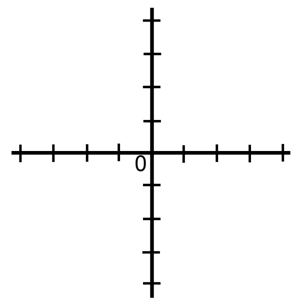
For each value of z below, compare the magnitudes of z and $z_a = -z$ by completing the blank in the 2^{nd} column with $<$ or $=$ or $>$. Complete the 3^{rd} and 4^{th} columns with the phase of z and phase of z_a (in degrees). Similarly for $z_b = z^{-1}$.

	$z_a = -z$			$z_b = z^{-1}$		
z	Compare $\lvert z\rvert$ to $\lvert z_a\rvert$	$\angle z$	$\angle z_a$	Compare $\lvert z_b\rvert$ to $\lvert z\rvert$	$\angle z$	$\angle z_b$
1	$\lvert z\rvert$ ____ $\lvert z_a\rvert$	____°	____°	$\lvert z\rvert$ ____ $\lvert z_b\rvert$	____°	____°
4	$\lvert z\rvert$ ____ $\lvert z_a\rvert$	____°	____°	$\lvert z\rvert$ ____ $\lvert z_b\rvert$	____°	____°
-4	$\lvert z\rvert$ ____ $\lvert z_a\rvert$	____°	____°	$\lvert z\rvert$ ____ $\lvert z_b\rvert$	____°	____°
$4\,i$	$\lvert z\rvert$ ____ $\lvert z_a\rvert$	____°	____°	$\lvert z\rvert$ ____ $\lvert z_b\rvert$	____°	____°
$-4\,i$	$\lvert z\rvert$ ____ $\lvert z_a\rvert$	____°	____°	$\lvert z\rvert$ ____ $\lvert z_b\rvert$	____°	____°
$0.4\,i$	$\lvert z\rvert$ ____ $\lvert z_a\rvert$	____°	____°	$\lvert z\rvert$ ____ $\lvert z_b\rvert$	____°	____°
$-0.4\,i$	$\lvert z\rvert$ ____ $\lvert z_a\rvert$	____°	____°	$\lvert z\rvert$ ____ $\lvert z_b\rvert$	____°	____°
$1+i$	$\lvert z\rvert$ ____ $\lvert z_a\rvert$	____°	____°	$\lvert z\rvert$ ____ $\lvert z_b\rvert$	____°	____°

8.6 Complex numbers and exponentiation (Section 15.6).

Find ***all*** complex numbers (in Cartesian form) equal to the following:

$$\sqrt{-3.4641 - 2\,i} \approx \underline{\qquad} - 1.932\,i \quad \text{or} \quad \underline{\qquad} + \underline{\qquad}\,i$$



Mark the location of the complex number with a circle and mark its square roots with an **X** on the complex plane to the right.

8.7 The complex-plane locations of $8^{\frac{1}{3}}$, the cube root of 8 (Section 15.6).

Consider the complex number $z = a + b\,i = \lvert z\rvert\, e^{\theta\, i}$.

Write z^3 in Cartesian form in terms of a and b and in magnitude-phase form in terms of $\lvert z\rvert$ and θ.

Result:

$$z^3 = (\underline{\qquad} - \underline{\qquad}) + (\underline{\qquad} - \underline{\qquad})\,i \qquad\qquad z^3 = \underline{\qquad}\, e^{\underline{\qquad}\, i}$$

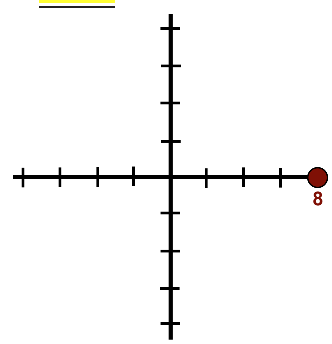
Determine a number z_1 that satisfies the equation $z_1 * z_1 * z_1 = 8$.

Repeat to find two more unique numbers z_2 and z_3.

Report your results in magnitude-phase form.

Result:

$$z_1 = \underline{\quad}\, e^{\underline{\quad}\, i} \qquad z_2 = \underline{\quad}\, e^{\underline{\quad}\, i} \qquad z_3 = \underline{\quad}\, e^{\underline{\quad}\, i}$$



Mark the locations of z_1, z_2, and z_3 with an **X** on the complex plane.

8.8 †Optional: Calculate the imaginary number i to the power i (provide result in Cartesian form).

Result: $i^{\,i} = \underline{\qquad} + \underline{\qquad}\,i$ ____________ (provide all answers)

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