

Show work – except for ♣ fill-in-blanks (print .pdf from www.MotionGenesis.com ⇒ [Textbooks](#) ⇒ [Resources](#)).

12.1 ♣ Matrix rows and columns

Given: $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Row 1 of M = $\begin{bmatrix} \square & \square \end{bmatrix}$

Row 2 of M = $\begin{bmatrix} \square & \square \end{bmatrix}$

$M_{2,1} = \square$

Column 1 of M = $\begin{bmatrix} \square \\ \square \end{bmatrix}$

Column 2 of M = $\begin{bmatrix} \square \\ \square \end{bmatrix}$

12.2 ♣ Matrix transpose

Transpose $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

Transpose $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \square$

12.3 ♣ Matrix addition and subtraction (+, -)

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} = \square$

12.4 ♣ Scalar-matrix multiplication (*)

$5 * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

$5 * \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \square$

12.5 ♣ Matrix-matrix multiplication (*)

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} \square \\ \square \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 & x \\ 5 & y \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

$\begin{bmatrix} a \\ b \end{bmatrix} * \begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} * \begin{bmatrix} x & 3 \\ y & 5 \\ z & 7 \end{bmatrix} = \square$

12.6 ♣ Matrix determinants

$\det [5] \triangleq 5$

$\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \square * \square - \square * \square = -2$

$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \square$

$\det [a] \triangleq \square$

Calculate the following determinant three ways: expand along the 1st row, 1st column, 2nd row.

$\det \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{bmatrix} = +1 \det \begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + -2 \det \begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix} + +3 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$

$= +1 \det \begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + -4 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + +7 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$

$= -4 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + +5 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + -6 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$

Calculate the determinant by expanding along the 3rd column.

$\det \begin{bmatrix} a & b & c \\ d & e & 0 \\ g & h & 0 \end{bmatrix} = \square * \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$

12.7 ♣ Matrix form of scalar equations (matrix multiplication in reverse)

Put the following sets of scalar equations into matrix form.

$\begin{aligned} ax + by &= 12 \\ dx + ey &= 15 \end{aligned}$ $\boxed{} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$	$\begin{aligned} ax + by + cz &= 12 \\ dx + ey + fz &= 15 \end{aligned}$ $\boxed{} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$	$\begin{aligned} ax + by - 12 &= 0 \\ dx + ey - 15 &= 0 \end{aligned}$ $\boxed{} \begin{bmatrix} a \\ b \\ d \\ e \end{bmatrix} = \begin{bmatrix} \boxed{} \\ 15 \end{bmatrix}$
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12.8 Optional: Solving sets of linear algebraic equations.

$$ax + by = 1$$

$$dx + ey = 2$$

Solve for x, y .

$$x = \frac{e - 2b}{ae - bd} \quad y = \frac{2a - d}{ae - bd}$$

Solve for x, y, z .

$$x = \frac{-1 - 2b + 2c}{2b - a - c} \quad y = \frac{2 + 2a - 2c}{2b - a - c} \quad z = \frac{-1 - 2a + 2b}{2b - a - c}$$

$$ax + by + cz = 1$$

$$2x + 3y + 4z = 2$$

$$2x + 4y + 6z = 4$$

12.9 Concepts: Eigenvalues and eigenvectors

Do all the questions in Section 22.2.

12.10 ♣ Eigenvalues, determinants, and matrix algebra

- One test that the inverse of the $n \times n$ matrix A does **not** exist is $\det(\boxed{}) = 0$
- The eigenvalues of the matrix A can be determined by setting $\det(\boxed{}) = 0$
- If an eigenvalue of the matrix A is zero, A^{-1} does **not** exist. True/False.

12.11 ♣ Concepts: Eigenvalues and eigenvectors

Consider the following set of algebraic equations governing the unknowns u_1, u_2 , and λ .

$$\begin{aligned} \lambda u_1 - u_2 &= 0 \\ 25u_1 + (\lambda - 6)u_2 &= 0 \end{aligned} \quad \text{or equivalently} \quad \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Find “special values” of λ (called **eigenvalues**) that allow for $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Result: $\lambda_1 = \boxed{}$ $\lambda_2 = \boxed{}$

For each special value of λ determine a corresponding “special ratio” of u_2 to u_1 .

Result: (These “special ratios” are called **eigenvectors** and c_1 and c_2 are arbitrary constants.)

$$\text{For } \lambda_1: U_1 \triangleq \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ \boxed{} \end{bmatrix} \quad \text{For } \lambda_2: U_2 \triangleq \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = c_2 \begin{bmatrix} 1 \\ \boxed{} \end{bmatrix}$$

12.12 ♣ Optional: Matrix inverse with determinants and adjugate (adjunct) matrices

Calculate the following matrix inverses (the 3×3 matrix inverses are optional).

$$\text{inv}[5] = \frac{\text{adj}[5]}{\det[5]} = \frac{[1]}{5} = [0.2] \quad \text{inv}[a] = \boxed{}$$

$$\text{inv} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \frac{\text{adj} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}{\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} = \frac{\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}}{\boxed{}} = \boxed{}$$

$$\text{inv} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{\text{adj} \begin{bmatrix} a & b \\ c & d \end{bmatrix}}{\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}} = \frac{\boxed{}}{\boxed{}}$$

$$\text{inv} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{bmatrix} = \begin{bmatrix} -0.9375 & 0.375 & 0.0625 \\ -0.125 & 0.25 & -0.125 \\ 0.7291667 & -0.2916667 & 0.0625 \end{bmatrix}$$

$$\text{inv} \begin{bmatrix} a & b & c \\ d & e & 0 \\ g & h & 0 \end{bmatrix} = \begin{bmatrix} 0 & h/(d * h - e * g) & -e/(d * h - e * g) \\ 0 & -g/(d * h - e * g) & d/(d * h - e * g) \\ 1/c & -(a * h - b * g)/(c * (d * h - e * g)) & (a * e - b * d)/(c * (d * h - e * g)) \end{bmatrix}$$

12.13 Optional: Matrix computation with MotionGenesis and/or MATLAB®

Use **MotionGenesis** or **MATLAB®** to do Hw 12.1 - 12.8 (skip Hw 12.7). For example, print and submit a **MotionGenesis** command file named **MatrixAlgebra.txt** that uses the following commands.

GetElement	GetRow	GetColumn	GetTranspose
+ - *	GetDeterminant	GetInverse	Solve