Homework 1. Chapter 2.
Geometry, trigonometry, & calculus review: Triangles, functions, derivatives, integrals.

Show work – except for ♣ fill-in-blanks.

1.1 ♣ Solving problems – what engineers do.

Understanding math and physics results from **doing** problems. Many problems herein guide you to help you synthesize processes (imitation). Please **do** these problems by yourself or with colleagues/instructors and use the textbook and other resources.

Confucius 500 B.C.
"I hear and I forget.
I see and I remember.
I $\boxed{}$ and I understand."

"By three methods we may learn wisdom:
1st by reflection, which is noblest;
2nd by imitation, which is easiest;
3rd by experience, which is the bitterest."

1.2 ♣ PEMDAS (Parentheses, Exponents, Multiplication/Division, Addition/Subtraction).

$36 \,/\; 3*3 \;-\; 12 \;=\; \boxed{}$ $\qquad 2*5^{\;2} \;-\; 25 \;=\; \boxed{}$ $\qquad -3^2 \;=\; \boxed{}$? (ambiguous)

$2^{3^2} \;=\; \boxed{}$? (ambiguous) $\qquad \left[{}^{+}\sqrt{(3^3 \;+\; 23) \;*\; \tfrac{1}{2}} \;*\; 2 \;+\; 2+3 \;\div\; 3\right] * (5+6) \;=\; \boxed{}$

1.3 ♣ Unit conversions between U.S. and SI (Standard International). (Guess and check Section 2.2).

Complete each blank with one of the following numbers: $\boxed{\mathbf{0.45,\ \ 1,\ \ 2.54,\ \ 32.2}}$.

Length 1 inch $\triangleq \boxed{}$ cm	**Mass** 1 lbm $\approx \boxed{}$ kg	1 slug $\approx \boxed{}$ lbm	
Force 1 Newton $\triangleq \boxed{}\ \frac{\text{kg m}}{\text{s}^2}$	1 lbf $\triangleq \boxed{}\ \frac{\text{slug ft}}{\text{s}^2}$	1 lbf $\approx \boxed{}\ \frac{\text{lbm ft}}{\text{s}^2}$	

1.4 ♣ (1900 BC). Sine, cosine, tangent as ratios of sides of a right triangle. (Section 2.6)

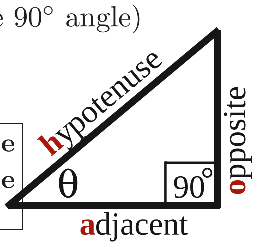
Below is a ***right triangle*** (triangle with a 90° angle) with one angle labeled as θ. Write definitions for sine, cosine, and tangent in terms of:

- **h**ypotenuse – the triangle's longest side (opposite the 90° angle)
- **o**pposite – the side opposite to θ
- **a**djacent – the side adjacent to θ

Memorize: ***Soh Cah Toa***

$$\sin(\theta) \triangleq \frac{\boxed{}}{\boxed{}} \qquad \cos(\theta) \triangleq \frac{\boxed{}}{\boxed{}} \qquad \tan(\theta) \triangleq \frac{\boxed{}}{\boxed{}}$$

I can draw a triangle with a negative-length side	**True/False**
Using the limited definition shown right, sin(θ) (the sine of an angle) can be negative.	**True/False**



1.5 ♣ (1900 BC - 1400 AD) Pythagorean theorem & Law of cosines. (Section 2.6.2).

Draw a right-triangle with a hypotenuse of length c and other sides of length a and b. Relate c to a and b with the ***Pythagorean theorem***.

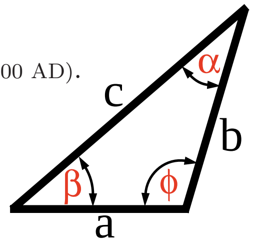
Result: Babylonians 1900 BC to Pythagoreus 525 BC. $\boxed{c^2 \;=\; \boxed{} \;+\; \boxed{}}$ ***memorize***

Shown right is a triangle with angles α, β, ϕ opposite sides a, b, c, respectively. Complete each formula below using the ***law of cosines*** (Euclid 300 BC - Al-Kashi 1400 AD).

Result: $\boxed{c^2 \;=\; \boxed{} \;+\; \boxed{} \;-\; \boxed{}}$ ***memorize***

$a^2 \;=\; \boxed{} \;+\; \boxed{} \;-\; \boxed{}$

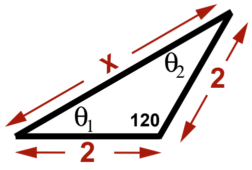
$b^2 \;=\; \boxed{} \;+\; \boxed{} \;-\; \boxed{}$

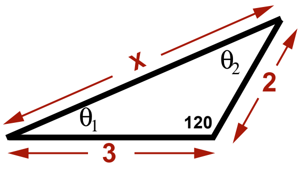


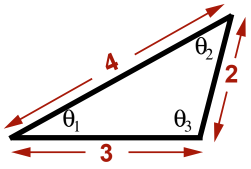
The ***Pythagorean theorem*** is a special case of the ***law of cosines***. **True/False**. (circle one).

1.6 ♣ Law of cosines - examples. (Section 2.6)

For each triangle below, use the ***law of cosines*** to determine values for x, θ_1, and θ_2.







$x = \sqrt{\boxed{}}$

$\theta_1 = \boxed{}^\circ$

$\theta_2 = \boxed{}^\circ$

$x = \sqrt{\boxed{}}$

$\theta_1 = \text{acos}(\frac{4}{\sqrt{\boxed{}}}) \approx 23.4^\circ$

$\theta_2 = \text{acos}(\frac{3.5}{\sqrt{\boxed{}}}) \approx 36.6^\circ$

$\theta_1 = \text{acos}(\frac{\boxed{}}{\boxed{}}) \approx 29.0^\circ$

$\theta_2 = \text{acos}(\frac{\boxed{}}{\boxed{}}) \approx 46.6^\circ$

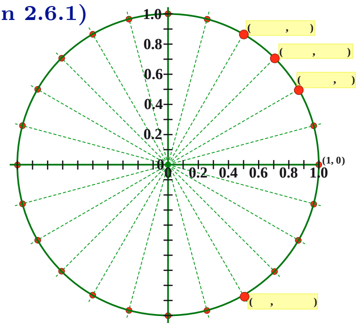
$\theta_3 = \text{acos}(\frac{\boxed{}}{\boxed{}}) \approx \boxed{}^\circ$ use a calculator

1.7 ♣ (140 BC - 1500 AD) Unit circle concept of sine and cosine. (Section 2.6.1)

Angle θ	$\sin(\theta)$	$\cos(\theta)$
0°	☐	☐
30°	0.5	≈ 0.866
45°	≈ ☐	≈ ☐
60°	≈ ☐	☐
90°	☐	☐
120°	≈ ☐	☐
150°	☐	≈ ☐

Label the blanked coordinates on the unit circle to the right.

Note: The unit circle expands the concepts of sine and cosine to **negative** values and its tabulated values provide data for Euler's functions (below). Negative numbers were invented ≈ 650 AD, developed 900 AD – 1200 AD, and widely adopted 1500 AD.



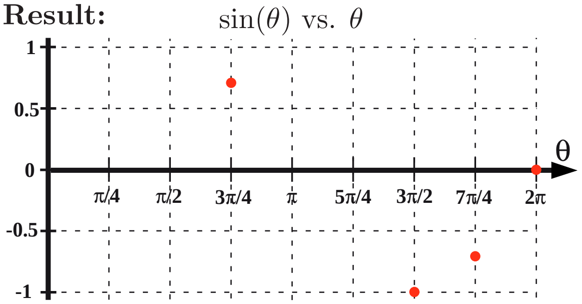
The triangle definition of sine and cosine in Hw 1.4 results in	$0^\circ < \theta < 90^\circ$	$0 < \sin(\theta) < 1$	$0 < \cos(\theta) < 1$
The unit circle extends the range for θ and sine and cosine to	$0^\circ \le \theta \le 360^\circ$	$\boxed{} \le \sin(\theta) \le \boxed{}$	$\boxed{} \le \cos(\theta) \le \boxed{}$

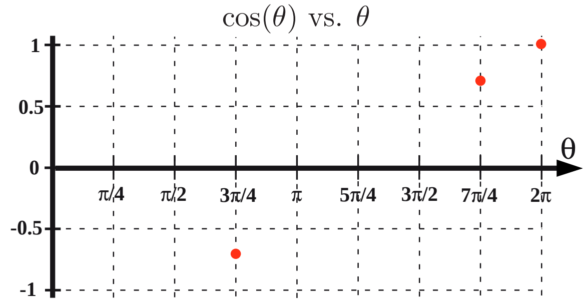
1.8 ♣ (Euler 1730 AD) Sine and cosine as functions. (Section 2.6.3)

Graph sine and cosine as functions of the angle θ over the range $0 \le \theta \le 2\pi$ radians.

Note: Euler invented the sine and cosine **functions** (more than just ratios of sides of a triangle).

Result:





1.9 ♣ Graph sine functions and identify amplitude, frequency, and phase (Section 2.6.3).

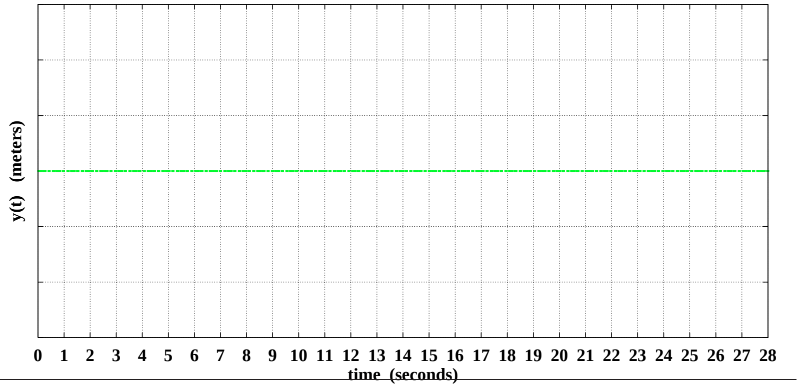
For $0 \le t \le 28$ sec, graph the following functions (label your axes).

$$y_A(t) = 3 * \sin(\frac{\pi}{12}t)$$

$$y_B(t) = 3 * \sin(\frac{\pi}{12}t - \frac{\pi}{4})$$

The ***phase*** in $y_B(t)$ is $\boxed{}$ radians.

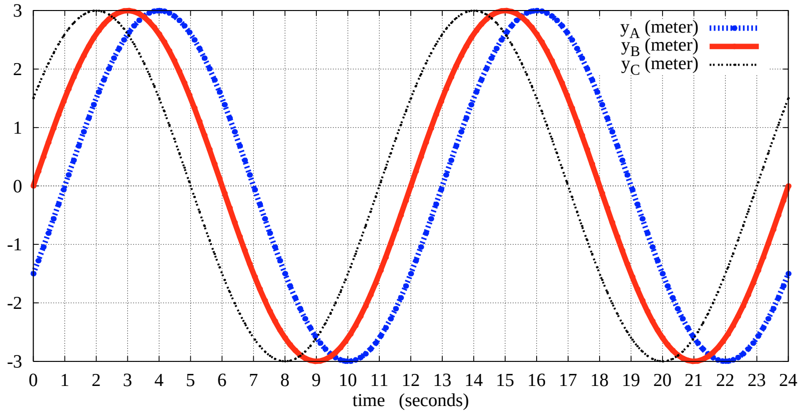
$y_B(t)$ **leads/lags** $y_A(t)$.



In general **negative/positive** phase is **lag** (later), which shifts a curve **left/right**.
In general **negative/positive** phase is **lead** (earlier), which shifts a curve **left/right**.

1.10 ♣ Identifying amplitude, frequency, and phase for sine functions (Section 2.6.3).

Graphed below are the time-dependent functions $y_A(t)$, $y_B(t)$, $y_C(t)$. Determine numerical values and units for their non-negative ***amplitudes*** B, non-negative ***frequencies*** Ω, and ***phase*** ϕ ($-\pi < \phi_i \le \pi$).



$$y_i(t) = B\,\sin(\Omega\,t + \phi_i) \quad (i = A, B, C)$$

	Value	Units
$B =$		
$\Omega =$		
$\phi_A =$		
$\phi_B =$		
$\phi_C =$		

1.11 ♣ Memorize sine and cosine addition formulas (Section 2.6.2).

$$\sin(\alpha+\beta) = \sin(\alpha) * \boxed{\quad} + \boxed{\quad} * \boxed{\quad}$$
Addition formula for sine

$$\cos(\alpha+\beta) = \cos(\alpha) * \boxed{\quad}\ \boxed{\quad}\ \boxed{\quad} * \boxed{\quad}$$
Addition formula for cosine

1.12 ♣ Ranges for arguments and return values for inverse trigonometric functions.

Determine all real return values and argument values for the following **real** trigonometric and inverse-trigonometric functions in computer languages such as Java, C^{++}, MATLAB®, **M**otion**G**enesis, ...

Range of return values for z	Function	Range of argument values for x	Note
$\boxed{-1} \le z \le \boxed{\quad}$	$z = \cos(x)$	$\boxed{\quad} < x < \boxed{\quad}$	
$\boxed{\quad} \le z \le \boxed{\quad}$	$z = \sin(x)$	$\boxed{\quad} < x < \boxed{\quad}$	
$\boxed{-\infty} < z < \boxed{\infty}$	$z = \tan(x)$	$\boxed{-\infty} < x < \boxed{+\infty}$	$x \ne \pm\frac{\pi}{2},\ \pm\frac{3\pi}{2}, \ldots$
$\boxed{\quad} \le z \le \boxed{\quad}$	$z = \text{acos}(x)$	$\boxed{\quad} \le x \le \boxed{\quad}$	
$\boxed{\quad} \le z \le \boxed{\quad}$	$z = \text{asin}(x)$	$\boxed{\quad} \le x \le \boxed{\quad}$	
$\boxed{-\pi/2} < z < \boxed{\pi/2}$	$z = \text{atan}(x)$	$\boxed{-\infty} < x < \boxed{+\infty}$	
$\boxed{\quad} < z \le \boxed{\quad}$	$z =$ `atan2`(y, x)	$\boxed{-\infty} < y < \boxed{+\infty}$ $\boxed{\quad} < x < \boxed{\quad}$	`atan2`(0, 0) is undefined

1.13 ♣ Notations for derivatives (complete the blanks). (Section 2.8.1).

Symbol for	1^{st},	2^{nd},	3^{rd} derivative	**Idea**	Date	Name of mathematician
Compact	$\dot{y}$	$\boxed{\quad}$	$\boxed{\dddot{y}}$	**Geometry/slope**	1675	$\boxed{\quad}$
Explicit	$\frac{dy}{dt}$	$\boxed{\frac{d^2y}{dt^2}}$	$\boxed{\quad}$	**Differentials**	1675	$\boxed{\quad}$ (taught Bernoullis, who tutored **Euler**)
Keyboard	y'	$\boxed{\quad}$	$\boxed{\quad}$	**Functions**	1797	Euler and $\boxed{\quad}$ (trained by **Euler**)
$\lim\limits_{h\to 0}\frac{y(t+h)-y(t)}{h}$		$\boxed{?}$	$\boxed{?}$	**Limits** **delta-epsilon**	1850 1872	Cauchy (trained by **Lagrange**) Weierstrass
	$\frac{\partial y}{\partial x}$	$\frac{\partial^2 y}{\partial x^2}$	$\frac{\partial^3 y}{\partial x^3}$		1786 1841	Legendre (introduced partials, abandoned them) Jacobi (re-introduced partials again)

There was bitter rivalry between Newton and Leibniz about the concepts and notation for a derivative.

1.14 ♣ (1675 AD) Leibniz's shorthand notation for 3^{rd} derivatives. (Section 2.8.1).

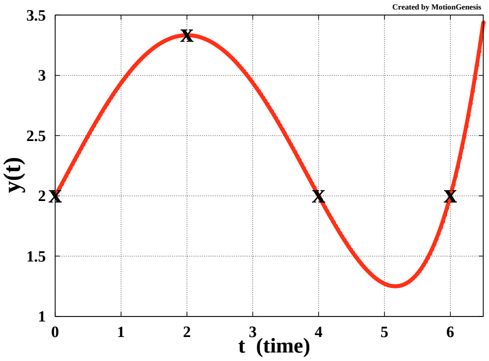
Write the explicit expression for Leibniz's 3^{rd} derivative show right (so it contains three 1^{st} derivatives).

$$\underbrace{\frac{d^3y}{dt^3}}_{\text{shorthand}} \triangleq \underbrace{\frac{d}{dt}\left(\boxed{}\left(\boxed{}\right)\right)}_{\text{explicit}}$$

Write Leibniz's and Newton's shorthand expression for the 9^{th} derivative of y with respect to t.

$$\underbrace{\boxed{}}_{\text{Leibniz}} = \underbrace{\boxed{}}_{\text{Newton}}$$

1.15 ♣ (1675 AD) Newton's idea: Derivative as geometry (slope and curvature). (Section 2.8.1).



Newton related derivatives to geometry (1^{st}-derivative as slope and 2^{nd}-derivative as curvature). Estimate the slope of the function $y(t)$ shown right at $t = 0,\ 2,\ 4,\ 6$.

Result: Pick your answers from: $\boxed{\mathbf{-1,\ 0,\ 1,\ 2}}$.

Slope
($\mathbf{1^{st}}$ **derivative**)

$$\left.\frac{dy}{dt}\right|_{t=0} \approx \boxed{} \qquad \left.\frac{dy}{dt}\right|_{t=2} \approx \boxed{}$$

$$\left.\frac{dy}{dt}\right|_{t=4} \approx \boxed{} \qquad \left.\frac{dy}{dt}\right|_{t=6} \approx \boxed{}$$

Estimate the **sign** of the curvature [2^{nd}-derivative of $y(t)$].

Result: Pick your answers from: $\boxed{\mathbf{<\ ,\ \approx,\ >}}$. Select $\boldsymbol{\approx}$ when the curvature ≈ 0 (i.e., $|\frac{d^2y}{dx^2}| < 0.01$).

Curvature
($\mathbf{2^{nd}}$ **derivative**)

$$\left.\frac{d^2y}{dt^2}\right|_{t=0} \boxed{}\ 0 \qquad \left.\frac{d^2y}{dt^2}\right|_{t=2} \boxed{}\ 0 \qquad \left.\frac{d^2y}{dt^2}\right|_{t=4} \boxed{}\ 0 \qquad \left.\frac{d^2y}{dt^2}\right|_{t=6} \boxed{}\ 0$$

1.16 ♣ (1755 AD) Euler's idea: Derivative of a function is a function. (Section 2.8.4).

Differentiate the following functions that depend on t (time). Express results in terms of x, $\dot{x}$, t so the results are valid when x is constant or depends on time (e.g., when $x = 9$ or $x = t^3$ or $x = t^5$).

Result:

$$\frac{d}{dt}\,t^2 = \boxed{} \qquad \frac{d}{dt}\,t^3 = \boxed{} \qquad \frac{d}{dt}\,t^{-7} = \boxed{}$$

Hint: **Chain rule** $\frac{d\,f[x(t)]}{dt} = \frac{df}{dx}\,\frac{dx}{dt}$

$$\frac{d}{dt}\,\sin(t) = \boxed{} \qquad \frac{d}{dt}\,\cos(t) = \boxed{} \qquad \frac{d}{dt}\,\cos(x) = \boxed{} * \boxed{}$$

$$\frac{d}{dt}\,e^t = \boxed{} \qquad \frac{d}{dt}\,\ln(t) = \frac{\boxed{}}{\boxed{}} \qquad \frac{d}{dt}\,\ln(x) = \frac{\boxed{}}{\boxed{}} * \boxed{}$$

1.17 ♣ Good product rule for differentiation – for scalars, $\vec{\text{v}}$ectors, [matrices], … (Section 2.8.5).

Circle the ***good product rule*** that works when u and v are scalars or $\vec{\mathbf{v}}$ectors, or u is a $\mathbf{2 \times 3}$ matrix and v is a $\mathbf{3 \times 5}$ matrix (if you did not learn the **good product rule**, update your calculus teacher).

$$\frac{d(u*v)}{dt} = \frac{du}{dt} * v + u * \frac{dv}{dt} \qquad \frac{d(u*v)}{dt} = u * \frac{dv}{dt} + v * \frac{du}{dt} \qquad \frac{d(u*v)}{dt} = v * \frac{du}{dt} + u * \frac{dv}{dt}$$

Knowing u, v, w are scalars or matrices that depend on time t, use the ***good product rule for differentiation*** to form the

Good product rule: $$\frac{dy}{dt} = \frac{d(u*v*w)}{dt} = \boxed{}\,\boxed{}\,w + \boxed{}\,\boxed{}\,w + u\,v\,\frac{dw}{dt}$$

1.18 ♣ Example of the "good product rule" for differentiation (if done right, takes ≈ 2 minutes).

Differentiate the function $f(t)$ with the easy-to-use ***good product rule for differentiation***.

$$\begin{array}{llcccccccccc}
\textbf{Function:} & f(t) & = & \sin(t) & * & \cos(t) & * & t^2 & * & e^t & * & \ln(t) \\
\hline
\textbf{Derivative:} & \frac{df}{dt} & = & \cos(t) & * & \cos(t) & * & t^2 & * & e^t & * & \ln(t) \\
 & & + & \sin(t) & * & -\sin(t) & * & t^2 & * & e^t & * & \ln(t) \\
 & & + & \sin(t) & * & \boxed{} & * & \boxed{} & * & \boxed{} & * & \boxed{} \\
 & & + & \sin(t) & * & \boxed{} & * & \boxed{} & * & \boxed{} & * & \boxed{} \\
 & & + & \sin(t) & * & \boxed{} & * & \boxed{} & * & \boxed{} & * & \frac{\boxed{}}{\boxed{}}
\end{array}$$

Hint: The "**good product rule**" is an ***efficient*** way to differentiate expressions with many factors.

1.19 ♣ Optional: Alternative to quotient rule: Combine product/exponent rules. (Section 2.8.6).

Although the ***quotient rule*** can be used to differentiate the ratio of functions $f(t)$ and $g(t)$, it can be easier to remember $\frac{f(t)}{g(t)} = f(t) * g(t)^{-1}$ and then use the ***product rule*** as shown below.

Given example: $\frac{\sin(t)}{t} = \sin(t) * t^{-1}$ $\quad \frac{d}{dt}\left[\sin(t) * t^{-1}\right] = \cos(t)\, t^{-1} - \sin(t)\, t^{-2}$

Complete this: $\frac{\sin(t)}{t^2} = \sin(t) * t^{\boxed{}}$ $\quad \frac{d}{dt}\left[\sin(t) * t^{\boxed{}}\right] = \boxed{} - \boxed{}$

1.20 ♣ Chain rule for differentiation. $\frac{d\,f[x(t)]}{dt} = \frac{df}{dx}\frac{dx}{dt}$ $\quad \frac{d\,f[x,y]}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$ (Section 2.8.7).

Differentiate the function $f(t)$ with respect to t [$x(t)$ and $y(t)$ depend on the independent variable t (time)].

$$\textbf{Function:}\quad f(t) = \sin(x) + y^2 + (\dot{x})^2 + e^x + \ln(y) + \frac{1}{x} + \cos(x+y)$$

$$\textbf{Derivative:}\quad \frac{df}{dt} = \cos(x)\,\dot{x} + 2\,\boxed{}\,\boxed{} + 2\,\boxed{}\,\boxed{} + \boxed{} + \boxed{} - \boxed{} - \boxed{}\left(\boxed{}\right)$$

1.21 ♣ Ordinary derivative of the function $f(t) = \sin(t) * \cos(x\,y\,z)$. (Sections 2.8.5 and 2.8.7).

Differentiate the function $f(t)$ with respect to t [$x(t)$, $y(t)$, $z(t)$ depend on the independent variable t (time)].

$$\textbf{Result:}\quad \frac{d\,[\sin(t)\,\cos(x\,y\,z)]}{dt} = \boxed{} - \boxed{}$$

1.22 ♣ The amazing function e^x. Related: The hyperbolic cosine and sine functions.

The ***hyperbolic cosine*** and ***hyperbolic sine*** functions are defined below and plotted to the right.

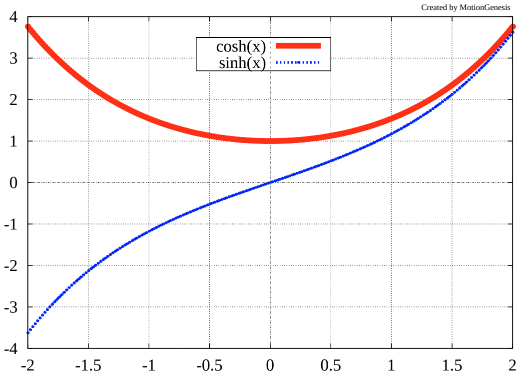
$$\sinh(x) \triangleq \frac{e^x - e^{-x}}{2} \qquad \cosh(x) \triangleq \frac{e^x + e^{-x}}{2}$$

Differentiate each definition with respect to x and express each result in terms of a hyperbolic function.

Result:

$$\frac{d\,[\sinh(x)]}{dx} = \frac{e^x\,\boxed{}\,e^{-x}}{2} = \cosh(x)$$

$$\frac{d\,[\cosh(x)]}{dx} = \frac{\boxed{}}{\boxed{}} = \boxed{}$$



1.23 ♣ Differentiation concepts. (Section 2.8.9 – implicit differentiation).

$$y^4 - 8\,y = 0.3\,t^2 + 9\,\sin(t)$$

Roots: $y = \boxed{}$, $\quad y = \boxed{}$,
Also: $\quad y \approx -1 \pm 1.732\,i$

The equation to the right relates the dependent variable $y(t)$ to the independent variable t. Find two real roots to this equation when $t = 0$.

Form a general expression for $\frac{dy}{dt}$ in terms of y and t and calculate $\frac{dy}{dt}$ when $t = 0$ and $y = 2$.

Result:

In terms of t and y: $\frac{dy}{dt} = \frac{\boxed{}}{\boxed{}}$ $\qquad$ Numerical value: $\left.\frac{dy}{dt}\right|_{\substack{t=0\\ y=2}} = \frac{9}{\boxed{}}$

1.24 ♣ Calculate the following derivatives. (Section 2.8.9 – implicit differentiation).

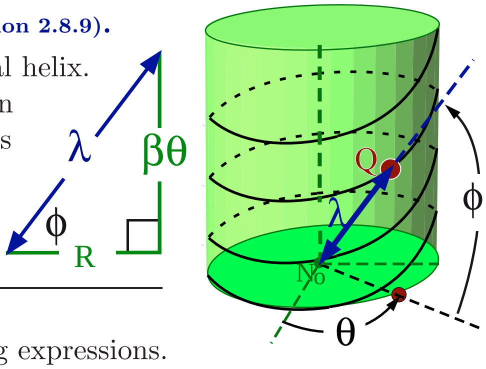
Result: $y = 5^t \;\Rightarrow\; \frac{dy}{dt} = \boxed{}\; \underbrace{5^t}_{y}$ $\qquad z = t^t \;\Rightarrow\; \frac{dz}{dt} = [\boxed{} + \boxed{}]\, z$

1.25 ♣ Review of explicit and implicit differentiation. (Section 2.8.9).

The figure to the right shows a point Q sliding on a cylindrical helix. Two geometrically significant variables are a distance λ and an angle ϕ that are related to **constants** R, β and a variable θ as

$$\lambda^2 = R^2 + (\beta\,\theta)^2 \qquad\qquad \tan(\phi) = \frac{\beta\,\theta}{R}$$

Form $\dot{\lambda}$ and $\dot{\phi}$ using the two methods described below.



Explicit differentiation **1.** Solve explicitly for λ and ϕ.
2. Then differentiate the resulting expressions.

Result:
In terms of R, β, θ, $\dot{\theta}$.

$$\lambda = +\sqrt{R^2 + (\beta\,\theta)^2} \qquad \phi = \text{atan}(\,\tfrac{\beta\,\theta}{R}\,)$$

Physically: $0 \le \phi \le \frac{\pi}{2}$

$$\dot{\lambda} = \frac{\boxed{}}{\boxed{}}\,\dot{\theta} \qquad \dot{\phi} = \frac{\boxed{}}{\boxed{}}\,\dot{\theta}$$

Hint: $\dfrac{\partial\,\text{atan}(x)}{\partial x} = \dfrac{1}{1+x^2}$

Implicit differentiation **1.** Differentiate the equations for λ^2 and $\tan(\phi)$.
2. Then solve for $\dot{\lambda}$ and $\dot{\phi}$.

Result:
In terms of R, β, θ, $\dot{\theta}$, λ.

$$\dot{\lambda} = \frac{\boxed{}}{\boxed{}}\,\dot{\theta} \qquad \dot{\phi} = \boxed{}\,\dot{\theta} \quad \text{or} \quad \dot{\phi} = \frac{\beta\,R}{\lambda^2}\,\dot{\theta}$$

since the triangle shows $\cos(\phi) = \frac{R}{\lambda}$

Forming $\dot{\lambda}$ is easier and computationally more efficient with **explicit/implicit** differentiation.

1.26 ♣ Optional: Partial and ordinary differentiation. (Section 2.8.2).

The kinetic energy K of a bridge-crane (shown right) can be written in terms of constants M, m, L and variables x, $\dot{x}$, θ, $\dot{\theta}$, as

$$\text{K} = \tfrac{1}{2}\,M\,\dot{x}^2 \;+\; \tfrac{1}{2}\,m\,L\,[L\,\dot{\theta}^2 + 2\,\cos(\theta)\,\dot{x}\,\dot{\theta}]$$

- First, regard x, $\dot{x}$, θ, $\dot{\theta}$ as independent variables [so K depends on each separately, i.e., K(x, $\dot{x}$, θ, $\dot{\theta}$)], form the **partial derivatives** below (left).
- Next, regard x, $\dot{x}$, θ, $\dot{\theta}$ as time-dependent variables and form the **ordinary derivatives** below (right).

This type of mathematics is used in ***Lagrange's equations of motion.***

$\frac{\partial \text{K}}{\partial \theta} = \boxed{}$	$\frac{\partial \text{K}}{\partial \dot{\theta}} = \boxed{}$	$\frac{d}{dt}\left(\frac{\partial \text{K}}{\partial \dot{\theta}}\right) = \boxed{}$
$\frac{\partial \text{K}}{\partial x} = \boxed{}$	$\frac{\partial \text{K}}{\partial \dot{x}} = \boxed{}$	$\frac{d}{dt}\left(\frac{\partial \text{K}}{\partial \dot{x}}\right) = \boxed{}$

1.27 ♣ Differentiation concepts – what is wrong? (Section 2.8.3 and previous problem).

The scalar v measures a baseball's upward-velocity. Knowing $v = 0$ only when the ball reaches maximum height, explain what is wrong with the following statement about v's time derivative.

$$\frac{dv}{dt} = \frac{d\,(0)}{dt} = 0 \;\text{ is } \textbf{wrong}.$$

We know the correct answer is: $\frac{dv}{dt} = \text{-}g \approx \text{-}9.8\,\frac{\text{m}}{\text{s}^2}$.

Explain what is wrong: It is incorrect to time-differentiate as shown above because:

1.28 ♣ Leibniz's idea and differentiation concepts: What is dt? (Section 2.8.3).

A continuous function $z(t)$ depends on $x(t)$, $y(t)$, and time t as:	$z \;=\; x \;+\; y^2 \sin(t)$
At a certain instant of time, $y = 1$ and z simplifies to:	$z \;=\; x \;+\; \sin(t)$

Determine the time-derivative of $z(t)$ at the instant when $y = 1$.

Result: $$\frac{dz}{dt}\Big|_{y\,=\,1} = \boxed{}$$

1.29 ♣ Leibniz's idea and differentiation concepts: What is dt? (Section 2.8.3).

A baseball's upward speed $v(t)$ depends on time t and constants v_i (initial upward speed) and g (Earth's gravitational constant) as:	$v \;=\; v_i \;-\; g\,t$
At time $t = 3$ seconds and knowing $g = 9.8\ \frac{\text{m}}{\text{s}^2}$, v simplifies to:	$v \;=\; v_i \;-\; 29.4$

Determine the time-derivative of $v(t)$ at $t = 3$ seconds.

Result: $$\frac{dv}{dt}\Big|_{t\,=\,3} = \boxed{}$$

1.30 ♣ Euler's idea: Integral of a function is a function. (Section 2.9).

Calculate the following indefinite integrals in terms of an indefinite constant C (regard t as positive).

Result:

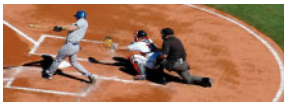
$$\int t^2\,dt = \frac{\boxed{}}{\boxed{}} + C \qquad \int t^3\,dt = \frac{\boxed{}}{\boxed{}} + C \qquad \int t^8\,dt = \frac{\boxed{}}{\boxed{}} + \boxed{}$$

$$\int t^{\text{-}3}\,dt = \frac{\boxed{}}{\boxed{}} + C \qquad \int t^{\text{-}2}\,dt = \frac{\boxed{}}{\text{-}1} + C = \frac{1}{t} + C \qquad \int t^{\text{-}1}\,dt = \boxed{}$$

$$\int \sin(t)\,dt = \boxed{} \qquad \int \cos(t)\,dt = \boxed{} \qquad \int e^t\,dt = \boxed{}$$

$$\int 5\,dt = \boxed{} \qquad \int 5/t\,dt = \boxed{} \qquad \int (5 + \tfrac{1}{t})\,dt = \boxed{} + \boxed{} + C$$

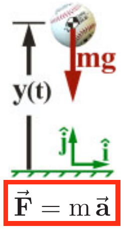
1.31 Solve a 1^{st}-order ODE: Separate variables, integrate, initial value. (Sections 2.9, 4.1).



Solve $\dot{v} \;=\; \text{-}9.8\ \frac{\text{m}}{\text{s}^2}$ with the initial value $v(t\!=\!0) \;=\; 33\ \frac{\text{m}}{\text{s}}$.

Result: $\frac{dv}{dt} = \text{-}9.8 \quad \Rightarrow \quad v(t) = \boxed{} + \boxed{}$ **Show work**

1.32 Solve a 2^{nd}-order ODE: Separate variables, integrate, initial value (2x). (Sections 2.9, 4.1).



Solve $\ddot{y} \;=\; \text{-}9.8\ \frac{\text{m}}{\text{s}^2}$ with initial values $\dot{y}(t\!=\!0) = 33\ \frac{\text{m}}{\text{s}}$, $y(t\!=\!0) = 5$ m. **Show work**

Result: $y(t) = \boxed{} + \boxed{} + \boxed{}$ Hint: $\ddot{y} = \frac{d^2y}{dt^2} \triangleq \frac{d}{dt}(\frac{dy}{dt})$. Separate variables and integrate twice. Use both initial values.

Optional: Show $\ddot{y} \;=\; \text{-}9.8\ \frac{\text{m}}{\text{s}^2}$ results from using $\vec{\mathbf{F}} = \text{m}\,\vec{\mathbf{a}}$ for the baseball and simplifying.

Result: $$\underbrace{\boxed{}}_{\vec{\mathbf{F}}} = \underbrace{\boxed{}}_{m\,\vec{\mathbf{a}}} \quad \Rightarrow \quad \text{-}m\,\text{g} = \text{m}\,\ddot{y} \quad \Rightarrow \quad \boxed{}.$$

1.33 ♣ Solve a 3^{rd}-order ODE with mixed initial/boundary values. (Sections 2.9, 4.1).

Solve $\frac{d^3y}{dt^3} = 6$ with initial/boundary values $y(t\!=\!0) = 5$, $\dot{y}(t\!=\!0) = 0$, $y(t\!=\!3) = 50$.

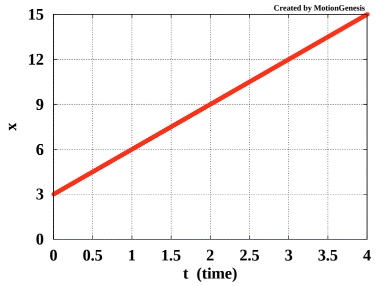
Result: $y(t) = \boxed{}$ Hint: $\frac{d^3y}{dt^3} \triangleq \frac{d}{dt}\,(\,\frac{d}{dt}\,(\frac{dy}{dt}\,)\,)$. Then integrate three times.

1.34 ♣ Geometric interpretations of integrals and derivatives. (Section 2.9).

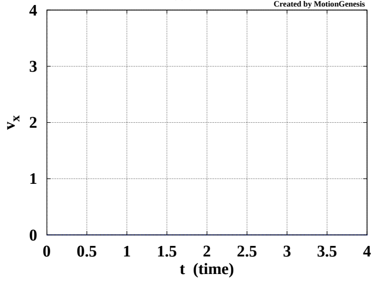
- Complete the blanks and graph the missing functions. Blanks should not have undetermined constants.

Hint: Synthesize information from each vertical column below. Constants of integration can be deduced from graphs. For example, for the 2^{nd} column, start at the bottom with $\frac{d^2x}{dt^2} = 0$ and work upward to determine $\frac{dx}{dt}$ and then $x(t)$.

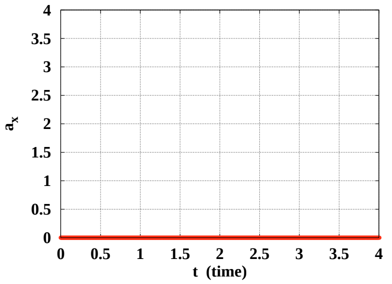
$$x = \boxed{} + \boxed{}$$



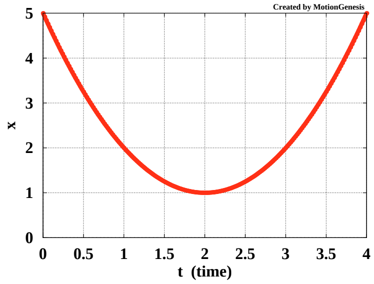
$$v_x \triangleq \frac{dx}{dt} = \boxed{}$$



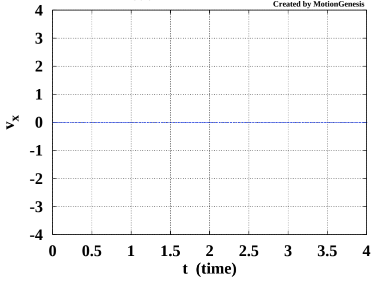
$$a_x \triangleq \frac{d^2x}{dt^2} = \boxed{}$$



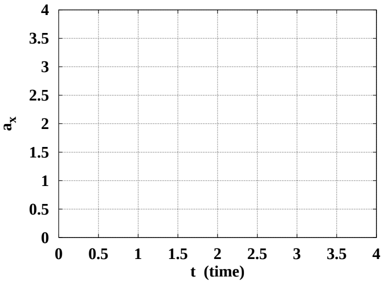
$$x = \boxed{} - \boxed{} + \boxed{}$$



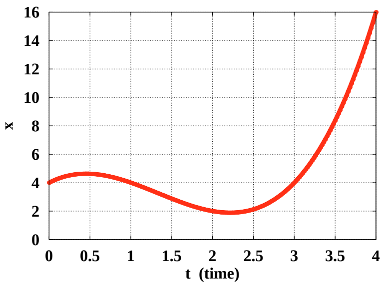
$$v_x \triangleq \frac{dx}{dt} = \boxed{} - \boxed{}$$



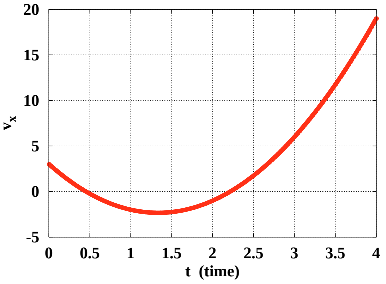
$$a_x \triangleq \frac{d^2x}{dt^2} = 2$$



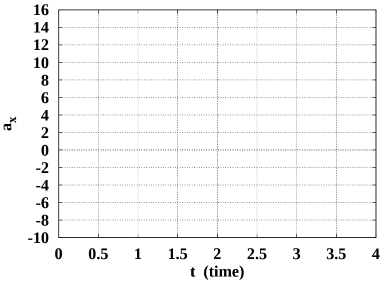
$$x = \boxed{} - \boxed{} + \boxed{} + \boxed{}$$



$$v_x \triangleq \frac{dx}{dt} = 3\,t^2 - 8\,t + 3$$



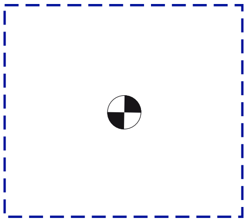
$$a_x \triangleq \frac{d^2x}{dt^2} = \boxed{} - \boxed{}$$



$$\boxed{\vec{\mathbf{F}} = \mathrm{m}\,\vec{\mathbf{a}}}$$

- A rocket-sled/rider is modeled as a particle of mass m whose motion is affected by thrust, normal, and gravity forces. **Draw** its ***free-body diagram*** and write the net force $\vec{\mathbf{F}}_{\text{Net}}$ in terms of scalars F_{T}, F_{n}, $m\,g$ (associated with thrust, normal force, gravity force) and the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

Result: $\vec{\mathbf{F}}_{\text{Net}} = \boxed{}\,\hat{\mathbf{i}} + (\boxed{})\,\hat{\mathbf{j}}$



Thrust $\vec{\mathbf{F}}_{\text{T}} = \boxed{}\,\hat{\mathbf{i}}$

Normal $\vec{\mathbf{F}}_{\text{n}} = \boxed{F_{\text{n}}}\,\boxed{}$

Gravity $\vec{\mathbf{F}}_g = \boxed{}\,\hat{\mathbf{j}}$

$\vec{\mathbf{F}}_{\text{Net}} = \vec{\mathbf{F}}_{\text{T}} + \vec{\mathbf{F}}_{\text{n}} + \vec{\mathbf{F}}_g$

- Set $\vec{\mathbf{F}}_{\text{Net}} = \mathrm{m}\,\vec{\mathbf{a}}$, form scalar equations, solve for $\ddot{x}$, F_{n}.

Result:

$$\underbrace{\boxed{}\,\hat{\mathbf{i}} + (\boxed{})\,\hat{\mathbf{j}}}_{\vec{\mathbf{F}}_{\text{Net}}} = \underbrace{\mathrm{m}\,\ddot{x}\,\hat{\mathbf{i}}}_{\mathrm{m}\,\vec{\mathbf{a}}} \quad \Rightarrow \quad \ddot{x} = \frac{F_{\text{T}}}{\boxed{}} \qquad F_{\text{n}} = \mathrm{m}\,\boxed{}$$

- Given $m = 100$ kg, $F_{\text{T}} = 800$ Newton, $x(t{=}0) = 7$ m, $\dot{x}(t{=}0) = 0\ \frac{\text{m}}{\text{s}}$, show $\boxed{x(t) = 4\,t^2 + 7}$.