A brief history of mathematics for dynamic systems

Advances in mathematics and technology are usually made at a painstakingly slow pace, with small sparks of individual brilliance that are accompanied by good luck or divine inspiration. Over the past millennia, various cultures have produced groups of gifted individuals that have had a significant impact on modern mathematics, engineering, and technology, including: Egypt (3000 BC), Babylon (2000 BC), Greece and China (500 BC), India (500 AD), Africa (800 AD), Europe and Asia (1500 AD), United States and Soviet Union (1900 AD), and Worldwide Web (2000^+ AD).

2.1.1 Real number systems

- **Egyptians (3000 BC):** Egyptians used hieroglyphics (picture writing) for numerals. The system was based on 10, but did not include a zero or the principle of place value. The (approximate) hieroglyphic symbols shown below combine to depict the number 1,326 as `| @@@ ^^ //////`.

1	`/`	Stroke
10	`^`	Arch
100	`@`	Coiled Rope
1,000	`|`	Lotus Flower
10,000	`>`	Finger
100,000	`~`	Tadpole

- **Babylonians (2000 BC):** Developed a number system based on 60. Their number system was more consistent and structured than the Egyptian system and was simpler for mathematical calculations. The number of seconds in a minute (60) and number of minutes in an hour (60) are a consequence of the Babylonian number system.
- **Hittites in Mesopotamia (2000-700 BC):** Developed the precursor to the Roman numeral system e.g., their first four numbers were `I`, `II`, `III`, and `IIII` (note how `I` looks like a finger or a stick).
- **Romans (500 BC):** The early Roman system was based on the Hittite system, e.g., its first four numbers were `I`, `II`, `III`, and `IIII`. The Roman system had special symbols for 5 (`V`), 10 (`X`), 50 (`L`), 100 (`C`), 500 (`D`), and 1000 (`M`). The early Roman system only used **addition** (not subtraction) thus 4 was `IIII` (not `IV`), 6 was `VI`, 9 was `VIIII` (not `IX`). The late Roman numeral system was invented in France in 1500 AD by clock makers to save space on clocks. The Roman system was used throughout the Middle Ages, and by law was the only acceptable system in Europe until 1300 AD.
- **Greeks (500 BC):** Developed a system based on 10. The first nine letters of their alphabet represented the numbers 1-9. The next nine letters stood for tens, from 10 through 90. The last nine letters were for hundreds, 100 through 900. The Greeks combined their symbols like the Egyptians.
- **Hindus (200 BC - 700 AD):** Around 200-300 BC, the Hindus in India used a system based on 10. They had symbols for each number from 1 to 9 and a symbol for each power of 10. Thus a Hindu wrote "1 sata, 3 dasan, 5" to write the number 135. Around 600 AD the Hindus found a way to eliminate place names. They invented the symbol "sunya" (meaning empty), which we call zero. With this symbol they could write "105" instead of "1 sata, 5".

  The use of ***negative numbers*** in solving problems can be traced as far back as the Indian Brahmagupta (7th century AD) who used zero and negative numbers in his algebraic work. He even gave the rules for arithmetic, e.g., "a negative number divided by a negative number is a positive number." This may be the earliest [known] systemization of negative numbers as entities in themselves.

  The ancient Chinese calculated with **red** (positive) and **black** (negative) rods (opposite to today's accounting practices), but, like other cultures, they did not accept a negative number as a solution to a problem. Instead, the problem was stated so its result was a positive quantity.

  Note: A 4-year old child's answer to the problem of 3 apples - 5 apples was a profoundly definite "***you can not do that***". Humans use negative numbers to convey **opposites**, e.g., negative/positive temperatures convey cold and hot, debt is negative whereas assets are positive, electrons and protons have positive and negative charge, etc.

Nineteenth century Europeans were the first to broadly accept **negative** numbers. Before 1800 AD, negative numbers were treated with great suspicion. For example, Pascal regarded $0-4$ as utter nonsense. Maseres and Frend wrote algebra texts renouncing both negative and imaginary numbers on the grounds that mathematicians were unable to explain their use except by analogy.

- **Mayans (700-800 AD):** The Mayan civilization invented a highly sophisticated vertical number system that used zero and positional base notation. Unfortunately, their knowledge did not survive the centuries and has had little impact on modern mathematics.
- **Arabs (700-800 AD):** Muhammad ibn Musa Al-Khwarizmi (780-850 AD) learned the Hindu number system and in 800 AD wrote a book that extended it by using **zero** as a place holder in positional base notation. Many mathematicians consider the Hindu-Arabic number system the world's greatest mathematical invention because it introduced the idea of place value and zero.
- **Europeans (1000-1200 AD):** Encouragament by the mathematician Gerbert (who later was Pope Sylvester II) in 980 AD, the Latin translation of Al-Khwarizmi's book and the notational work of Leonardo of Pisa (Fibonacci) in 1202 AD, resulted in the number system used worldwide today. Political and cultural conflicts delayed widespread adoption of the Arabic number system until 1500 AD.[4]
- **Europeans (1700 AD):** Gottfried Wilhelm Leibniz (1646-1716) developed the ***binary number system*** (used by most computers) in which he interpreted 1 for God and 0 for "the void".
- ***Fractions*** were used by the Egyptians (3000 BC), Babylonians (2000 BC), and Greeks (500 BC). Representing fractions with two stacked integers was essentially due to the Hindus (628 AD). The horizontal fraction bar (called the ***vinculum***) was introduced by the Arabs circa 1200 AD. Soonthereafter, the diagonal fraction bar (called a ***solidus*** or ***virgule***) was used for print because of typographical difficulties with the horizontal fraction bar.
- ***Percent symbols*** (denoting $\frac{1}{100}$) were introduced in an anonymous Italian manuscript in 1425 AD.
- ***Decimal points*** were first written as a blank space by the Persian astronomer Al-Kashi (1426). In 1530, Christoff Rudolff used a vertical bar exactly as we use a decimal point today. Before 1617 when Napier used both a comma and period to separate units and tenths, several other notations were in use, e.g., Simon (1585), Viete (1600), and Kepler (1616). Modern monetary and measurement systems (e.g, the SI system) take advantage of the conveniences afforded by the base-10 number system and decimal point [denoted with either a period . or comma ,].

2.1.2 Imaginary numbers, complex numbers, and quaternions (see Chapter 15)

- Stereometria of Heron of Alexandria (circa 50 AD) first noticed imaginary numbers when he saw that $\sqrt{81-144}$ could not be computed, so he switched it to $\sqrt{144-81}$.
- Arithmetica of Diophantus (c. 275 AD) noticed imaginary numbers when he attempted to compute the sides of a right triangle with a perimeter of 12 and an area of 7. He found it necessary to solve the equation $24\,x^2 - 172\,x + 336 = 0$. He did not understand that the equation had complex roots (there are no real triangles with a perimeter of 12 and an area of 7).
- Mahavira (c. 850 AD) stated that a negative number is not a square and does not have a square root.
- Pacioli (1494 AD) stated in his Summa that the quadratic equation $x^2 + b\,x + c = 0$ could be solved unless b^2 is greater than or equal to $4\,c$. He recognized the impossibility of finding $\sqrt{-1}$.
- Cardan (1545 AD) was the first to use the square root of a negative number in a computation. The problem was to divide 10 into two parts whose product was 40. He determined the number to be $5 + \sqrt{-15}$ and $5 - \sqrt{-15}$ and proved by multiplication that his results were correct.

[4]Until ***Pope Sylvester's*** Latin translation of the Arabic number system, the Europeans resisted the number 0 and using this great "unholy infidel" number system. The Europeans raised philosophical and religious objections to 0 (e.g., 0 cannot exist because there is always God). Adopting the Arabic number system was complicated because of deep animosity between Muslim Arabs and Christian Europeans fueled by the holy wars.

- Wallis (1673 AD) was the first to give a graphical representation of ***imaginary numbers*** in a plane (similar to positive and negative numbers on a line). He stated that the square root of a negative number was thought to imply the impossible, but the same might be said of a negative number.
- Newton's work (1685 AD) with imaginary numbers was confined to the number of roots of an equation.
- Jean Bernoulli (1702 AD) related the atan function and the logarithm of an imaginary number.
- Cotes (1710 AD) stated that $\log[\cos(x) + i\,\sin(x)] = i\,x$.
- Euler (1727 AD) invented the symbol e for the base of natural logs and the corollary of Cotes' formula $e^{i\,x} = \cos(x) + i\,\sin(x)$. Euler also invented the notation $i = \sqrt{-1}$.
- Casper Wessil **(1797 AD)**, a Norwegian surveyor, presented the modern geometric theory of ***complex numbers*** and the ***complex plane*** before the Royal Academy of Denmark.
- Sir William Hamilton (1830 AD) developed a system of hyper-complex numbers called ***quaternions*** which have a "basis" of four elements $(1, i, j, k)$. Quaternions proved adaptable for operations in three dimensional space but have mostly been superseded by vectors. A quaternion is usually written in the form $\mathbf{q} = a_0 + a_1{*}i + a_2{*}j + a_3{*}k$. The coefficients a_0, a_1, a_2, a_3 are called ***Euler parameters*** and are still used for 3D descriptions of orientation of rigid bodies. Note: A quaternion is similar to a complex number in that a complex number may be written as $a_0 + a_1{*}i$ where a_0 and a_1 are real numbers.

2.1.3 Vectors and dyadics: A modern tool for geometry

J. Williard Gibbs (1900 AD) developed ***vectors*** (quantities having magnitude and direction) and ***dyads*** (quantities with magnitude and two directions). Vectors and dyadics are extensively used in forces and motion.

2.1.4 Matrices: A tool to organize linear equations (see Chapter 20)

The Babylonians (300 BC) studied problems that lead to simultaneous linear equations and some of these are preserved in clay tablets. The practical theories of ***matrices*** and ***determinants*** did not develop until nearly **1800 AD**. The idea of a determinant first appeared in Japan (Seki 1683) and Europe (Leibniz 1683) within a few months of each other. In 1750, Cramer gave a general rule for finding a determinant of a $n \times n$ system of equations. In 1772, Laplace discovered a new formula for the expansion of a determinant. In 1773, Lagrange discovered that a 3×3 determinant could be interpreted as the volume of a tetrahedron. The great mathematician Gauss (1801) first used the term "determinant" calculated inverses, and did "Gaussian elimination". In 1812-1826, Cauchy did a detailed study of determinants, minors, eigenvalues, and diagonalization of a matrix. D'Alembert, Jacques Sturm, Jacobi, and Eisenstein made contributions to determinants, and Cayley (1841) introduced the notation of two vertical lines that denotes determinants in modern work. In 1850, Sylvester termed the word "matrix" and shared his work with Cayley who later (1858) defined the word matrix more precisely. The Jordan canonical form appeared in 1870. Frobenius defined the rank of a matrix and orthogonal matrices in 1878.