Prelab	Participation	Lab
$\sqrt{+}$ $\sqrt{}$ $\sqrt{-}$ 0	$\sqrt{+}$ $\sqrt{}$ $\sqrt{-}$ 0	$\sqrt{+}$ $\sqrt{}$ $\sqrt{-}$ 0

Name: _____

2   Lab: 2nd-order systems and vibrations

Lab 1 investigated motors and 1st-order ODEs, whereas this lab investigates **2nd-order ODEs**.

$$m \frac{d^2y}{dt^2} + b \frac{dy}{dt} + k\,y = 0$$

m	**effective mass**	constant (non-zero)
b	**effective viscous damping**	constant
k	**effective spring constant**	constant

This system's response is better described with an ODE having only two constants:

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2\,y = 0 \qquad \zeta = \frac{b}{2\sqrt{m\,k}} \text{ (damping ratio)} \qquad \omega_n = \frac{k}{m} \text{ (natural frequency)}$$

2.1   PreLab: Motor spin-down (analytical and Working Model)

1. The following Homeworks 3, 4 problems (from the book) facilitate this lab:

Hw 3.13 System response and damping	**Hw 3.14** System response for various of ζ
Hw 4.6 Calculating ζ and ω_n from empirical data for an underdamped 2nd-order ODE	

2. Download the following Working Model (.wm2d) simulations.
   Get these files from: www.MotionGenesis.com $\Rightarrow$ Textbooks $\Rightarrow$ Resources

CarSuspensionWithWnAndZeta.wm2d	**CarSuspensionWithSpringDamper.wm2d**

3. Run the Working Model simulations.
   Record results on the Working Model PreLab (.pdf on www.MotionGenesis.com).
   Show your lab-section TA your completed pre-lab at the start of lab.

2.2   Slinky. (Show data taken and calculations. Use SI units for all answers.)

To model a slinky, we use a relatively light spring and relatively large mass. Ideally, this improves the massless-spring assumption.   Measure the (lumped) mass of the system, and determine the spring constant k. Next, analytically calculate the system's natural frequency ω_n.

Result:
$$k = \boxed{} \; \tfrac{\text{N}}{\text{m}} \qquad\qquad \omega_n = \boxed{} \; \tfrac{\text{rad}}{\text{sec}}$$

Use a stopwatch to determine the system's damped natural frequency ω_d.

Result: (Hint: Taking an average of a few runs will probably improve your accuracy.)
$$\omega_d = \boxed{} \; \tfrac{\text{rad}}{\text{sec}}$$

How well do the predicted and measured value agree? (circle one)

Great ($\leq 0.1\%$ deviation)   Very good ($\leq 1\%$ deviation)   Good ($\leq 10\%$ deviation)

OK ($\leq 50\%$ deviation)   Poor ($\leq 100\%$ deviation)   Very poor ($> 100\%$ deviation)

Use a stopwatch to estimate the 1% settling time for the system. Hint: Just eyeball it.

Knowing $t_{\text{settling}} \approx \dfrac{4.6}{\zeta\,\omega_n}$, calculate the value of ζ.[4]

Result:
$$t_{\text{settling}} = \boxed{} \; \text{sec} \qquad\qquad \zeta = \boxed{} \; \text{no units}$$

† What would the system's mass need to be so t_{settling} is less than 10 seconds?

Result:
$$m_{new} = \boxed{} \; \text{kg}$$

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[4] The value of ζ is calculated with an experimental value of t_{settling} and a calculated value of ω_n.

2.2.1 Estimation with an accelerometer

We use several pieces of equipment to measure and record the cart's horizontal acceleration, namely, we use an accelerometer, a microprocessor, a transceiver, and a computer.[5]

- **Accelerometer: ADXL 311 from Analog Devices**

  The accelerometer is mounted on a cart and measures acceleration in up to three directions (we use data from only one direction). The accelerometer is relatively small and lightweight as compared to the cart - so its effect on the acceleration of the cart is negligible. The accelerometer's output signal is a linear $0.3 \frac{\text{volts}}{g}$ signal over a range of $\pm$ 2 g. The accelerometer is designed to output 2.5 volts when there is **no** acceleration, but there is some variation from one accelerometer to the next.

- **Breadboard signal processing**

  The accelerometer's signal is filtered by a low-pass filter to remove high-frequency noise in the signal. Op-amps are used as a buffer to supply additional current and avoid unwanted voltage drops.

- **Arduino UNO microprocessor:**

  The microprocessor's A/D port receives an analog voltage signal from the breadboard in a specified range (i.e., continuous voltages from 0 volts to 5 volts). The A/D port samples the analog signal at 1000 Hz (i.e., at 1 ms intervals). The 10 bit A/D converter on the microprocessor changes the 0 to 5 volt analog signal to bits (ones and zeros) that represent $2^{10} = 1024$ integer values (e.g., 5 volts converts to 1024 and 2.5 volts converts to 512). The Arduino transmits this data to the computer via serial communication.

- **Computer:**

  The computer receives bits from its serial port and uses the Arduino software program to translate the bits to integer numbers which are then printed to the screen. The numbers displayed on the computer screen are integers from 0 to 1024.

Data acquisition proceeds in a manner similar to Lab 1.[6]

1. Power the Arduino by plugging-in (in order):
   (a). 12 Volt adaptor (between the board and wall socket)
   (b). USB cable (between the board and the computer)

2. From the desktop, navigate to the Lab2 folder and open Lab2.ino

3. Under Tools $\rightarrow$ ports, select something other than COM1, COM2, or COM3 (the USB port can be enumerated to anything other than these)

4. Click the magnifying glass button (or type Ctrl+Shift+m) to open the serial monitor

5. On the serial monitor screen, a menu should appear. Enter "a" to start reporting data.

6. Have one group member pull the cart to one side and then release it.

7. Enter "a" or "r" to stop recording data after the cart has stopped.

8. Plot the data (e.g., using Excel, MATLAB®, or PlotGenesis).

9. Email the data files and/or graphs to yourself and your group members.

10. Pass in a printed graph of the cart's acceleration $\left(\frac{m}{s^2}\right)$ vs. time (sec) with your lab.

11. Ensure the power to the board is off and the setup is neat for the next lab.

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[5]Most accelerometers do not come assembled with a microprocessor, transceiver, and computer.

[6]Although the output signal can be twice-integrated to generate position data, this is unnecessary for present purposes.

2.2.2 Questions - SHOW YOUR WORK

Use your graph to determine numerical values for period of vibration and decay ratio.[7]
Result:

$$\tau_{\text{period}} = \boxed{} \text{ secs} \qquad\qquad \text{decayRatio} = \boxed{} \text{ no units}$$

Calculate numerical values for the system's natural frequency and damping ratio.
Result:

$$\omega_n = \boxed{} \frac{\text{rad}}{\text{sec}} \qquad\qquad \zeta = \boxed{} \text{ no Units}$$

List three things that could improve the accuracy of these values.

1.
2.
3.

Now add the mass to the platform and repeat the experiment.

1. Pass in a new graph of the cart's acceleration $\left(\frac{m}{s^2}\right)$ vs. time (sec) with your lab.
2. Calculate the new natural frequence and damping ratio
   Result:

$$\omega_n = \boxed{} \frac{\text{rad}}{\text{sec}} \qquad\qquad \zeta = \boxed{} \text{ no units}$$

3. The formula $\omega_n = \boxed{}$ shows adding mass **increases/decreases** natural frequency.

4. The formula $\zeta = \boxed{}$ shows adding mass **increases/decreases** damping ratio.

† What are the physical parameters (m,b,k) for this system without the added mass? Show your work on a separate page. You will use these values again in later labs!
Result:

$$m = \boxed{} \text{ kg} \qquad b = \boxed{} \frac{\text{N sec}}{\text{m}} \qquad k = \boxed{} \frac{\text{N}}{\text{m}}$$

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[7]The textbook chapter on time-specifications for 2nd-**order ODEs** is helpful for determining period of vibration and decay ratio.