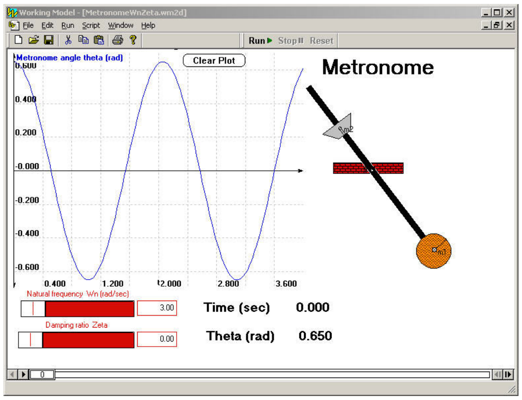
Lab 5 (associated with Hw 5): Dynamic response of rotational systems

The objective of this laboratory is to develop physical intuition for ***2^{nd}-order ODEs*** and associated mathematical quantities, including decay ratio, period of vibration, natural frequency, and damping ratio.

Lab 5.1 Effect of damping ratio (ζ) and natural frequency (ω_n) on dynamic response.

In Lab 3.1, you varied ζ and ω_n and observed their effect on the vertical (translational) motions of a car with a spring-damper suspension system. In this experiment, you will vary ζ and ω_n and observe their effect on the rotational motions of a metronome. To begin this problem, double-click on the file `MetronomeWnZeta.wm2d`.



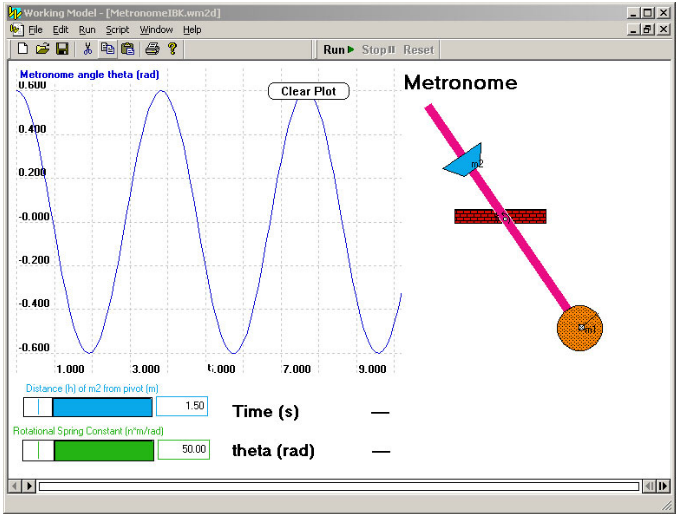
To answer each question with Working Model, click the **Reset** button (if necessary) and click and drag the sliders that control the numerical values for ω_n (measured in rad/sec) and ζ (dimensionless). To start the simulation, click the **Run** button, and to stop it, click the **Stop** button.

Decide which of the following statements are **true** or **false**. If a statement is **false**, provide a counter-example. If a statement involves ζ, assume ω_n is known, and vice-versa.

Statement	**True/False**	Counter-example if **false**
Increasing the damping ratio ζ always makes the metronome move slower	**True/False**	
For an ***oscillating*** metronome, increasing ζ always decreases the frequency of oscillation and makes the period longer	**True/False**	
For an ***oscillating*** metronome, increasing ζ always makes the decay ratio larger	**True/False**	
Increasing ζ always makes the metronome settle to equilibrium quicker	**True/False**	
For an ***oscillating*** metronome, increasing ω_n always increases the frequency of oscillation and makes the period shorter	**True/False**	
For an ***oscillating*** metronome, increasing ω_n always makes the decay ratio larger	**True/False**	
For a damped ***oscillating*** metronome, increasing ω_n decreases the settling time	**True/False**	
When $\zeta > 0$, increasing ω_n always makes the metronome settle to equilibrium quicker	**True/False**	

Lab 5.2 **Effect of moment of inertia and spring stiffness on dynamic response.**

For each simulation that follows, determine the effect of k (the rotational spring constant) and h (the distance between the particle of mass m_2 and the pivot point) on the rotational motions of the metronome. To begin this problem, double-click on the file `MetronomeIBK.wm2d`.



(a) After running as many simulations as necessary, fill in the following table with $--$ (decreases), $-$ (slightly decreases), **0** (no change), $+$ (slightly increases), or $++$ (increases).

	I (moment of inertia)	τ_{period}	ω_n
Effect of increasing k on:			
Effect of Increasing h on:			

(b) Based on your observations, circle the appropriate answer in the following statements.

- A metronome with a higher natural frequency uses a **softer/stiffer** spring
- A metronome with a lower natural frequency uses a **smaller/larger** value of h

(c) In Homework 5.8, a small angle approximation $\sin(\theta) \approx \theta$ is used to convert a ***nonlinear*** ODE to a ***linear*** ODE. To see the difference between the actual value of $\theta(t)$ and the value of $\theta(t)$ predicted with the small angle approximation, double-click on the file `MetronomeNonlinearAngle.wm2d`.

Initial value of θ	Value of $\theta(t{=}10)$ predicted with small angle approximation	Actual value of $\theta(t{=}10)$	Difference
0.3 rad	rad	rad	rad
0.6 rad	rad	rad	rad
0.9 rad	rad	rad	rad
1.2 rad	rad	rad	rad

(d) When is the small angle approximation most accurate?
Result: