Prelab	Participation	Lab
$\sqrt{+}$ $\sqrt{}$ $\sqrt{-}$ 0	$\sqrt{+}$ $\sqrt{}$ $\sqrt{-}$ 0	$\sqrt{+}$ $\sqrt{}$ $\sqrt{-}$ 0

Name: _____

4   Lab: Forcing functions for a 2nd-order, linear, ODE
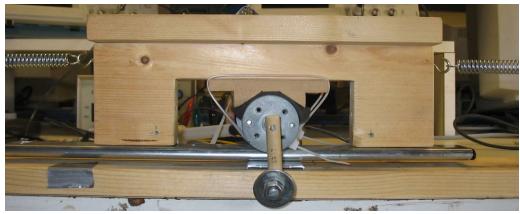
Some **_inputs_** to dynamic systems result in interesting dynamic **_response_**. Inputs that excite dynamic systems include periodic inputs (such as a vibrating motor) or one-time disturbances (such as the step displacement or impulse used in previous labs to determine system parameters). This lab investigates **_harmonic_** (periodic) sinusoidal inputs because

- It frequently occurs in physical/engineering systems.
- A rotary motor with an offset mass creates a sinusoidal forcing function.

The input forcing function is created by a motor with an offset mass. The motor's base is rigidly attached to a cart that moves horizontally along a flat track. The ODE that describes the horizontal displacement x of the cart is similar to the ODEs associated with the Scotch-yoke and air-conditioner homework problems, e.g.,

$$m\,\ddot{x} \;+\; b\,\dot{x} \;+\; k\,x \;=\; A\,\Omega^2\,\sin(\Omega\,t)$$

The input motor's angular speed Ω will be measured by an encoder. The cart's horizontal acceleration $\ddot{x}$ will be measured using an accelerometer. You will see how Ω affects x and $\ddot{x}$.

4.1   PreLab: Working Model and brainstorming

1. Download the following Working Model (.wm2d) simulations.
   Get these files from: www.MotionGenesis.com $\Rightarrow$ Textbooks $\Rightarrow$ Resources

HarmonicForcingPogoStick.wm2d	AirConditionerOnRoofWithDamping.wm2d
ParticleSlidingOnSpinningSlot.wm2d	HarmonicForcingWithSingleParticle.wm2d

2. Run the Working Model simulations.
   Record results on the Working Model PreLab (.pdf on www.MotionGenesis.com).

3. The parameter that usually requires experimental determination is: $m/b/k$ (circle one).
   Hint: This parameter is usually measured when the system is in motion.

4. Loosely speaking, damping ratio ζ determines   **speed/shape**.

5. Loosely speaking, natural frequency ω_n determines   **speed/shape**.

6. The magnitude of the output response is relatively   **small/large**   when $\Omega \ll \omega_n$

7. The magnitude of the output response is relatively   **small/large**   when $\Omega \approx \omega_n$

8. The magnitude of the output response is relatively   **small/large**   when $\Omega \gg \omega_n$

4.2 Experimental

The first step in solving an ***inhomogeneous*** ODE, is to solve the ***homogeneous*** part of the ODE. Similarly, the first step in determining a system's ***forced response*** is to determine the system's ***unforced*** (natural) response. This lab uses hardware similar to Lab 2 (mass/spring cart system) but employs an offset-mass motor that is rigidly attached to the cart.

4.2.1 Determination of physical and electrical parameters

Determine the natural frequency ω_n and damping ratio ζ for horizontal motions x of the cart ***with*** the motor attached to it. Use Part 1 of the Arduino Lab4.ino program to capture necessary data (described in Section 4.2.3). Hint: Remember Lab 2.

$$\omega_n \;=\; \underline{\quad\quad} \; \frac{\text{rad}}{\text{sec}} \qquad\qquad\qquad \zeta \;=\; \underline{\quad\quad} \; \text{NoUnits}$$

Record the A/D steady-state value of the accelerometer (in A/D Ticks) and calculate the associated voltage offset V_{0g} when the cart is not moving. Hint: 5 Volts = 1024 A/D Ticks

Result: (These values will be used later when calculating the magnitude of the accelerometer's signal).

$$A/D_{\text{SteadyState}} \underset{\text{(lab data)}}{} \;=\; 491 \; \text{A/D Ticks} \qquad\qquad V_{0g} \;=\; \underline{\quad\quad} \; \text{Volts}$$

Using a computer, plot the time-history of acceleration $\ddot{x}_{\text{ss}}(t)$. This graph helps calculate numerical estimates for ζ and ω_n.
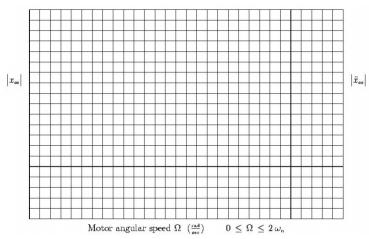
4.2.2 Analytical prediction

The steady-state solution (i.e., the long-term behavior) of the ***stable***, constant-coefficient, linear, ***2nd-order ODE*** that governs this sinusoidally-forced laboratory system is

$$x_{\text{ss}}(t) \;=\; B\,\sin(\Omega t + \phi) \qquad \text{where} \qquad B = \frac{m_{offset}\, r \,/\, (m_{offset} + m_{cart})}{\sqrt{\left[\left(\frac{\omega_n}{\Omega}\right)^2 - 1\right]^2 + \left[2\zeta\left(\frac{\omega_n}{\Omega}\right)\right]^2}}$$

Differentiate $\ddot{x}_{\text{ss}}$ to form $\ddot{x}_{\text{ss}}(t)$. Determine its magnitude $\left|\ddot{x}_{\text{ss}}\right|$ in terms of B and Ω.

$$\ddot{x}_{\text{ss}}(t) \;=\; \boxed{} \qquad\qquad \left|\ddot{x}_{\text{ss}}(t)\right| \;=\; \boxed{}$$

Using a computer,[10] plot $\left|x_{\text{ss}}(t)\right|$ for $0 \le \Omega \le 2\omega_n$ (use your experimentally-determined values of ω_n and ζ). Similarly, plot $\left|\ddot{x}_{\text{ss}}(t)\right|$ versus Ω. **Label** the scale on the **left**-axis appropriately for the range of values of $\left|x_{\text{ss}}(t)\right|$ and **label** the scale of the **right**-axis for the range of values of $\left|\ddot{x}_{\text{ss}}(t)\right|$. (Note: The left-scale and right-scale will be different). Submit a printed plot with your post-lab report.

Motor angular speed Ω $\left(\frac{\text{rad}}{\text{sec}}\right)$ $0 \le \Omega \le 2\omega_n$

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[10]Ensure the scales on **both** the left-axis and right-axis are both labeled.

4.2.3   Data acquisition

1. Power the Arduino by plugging-in (in order):
   (a). 12 Volt adaptor (between the board and wall socket)
   (b). USB cable (between the board and the computer)

2. Navigate to the Lab4 folder and open Lab4.ino

3. Click the magnifying glass button (or type Ctrl+Shift+m) to open the serial monitor

4. On the serial monitor screen, a menu should appear. Enter "a" to start reporting data.

5. Have one group member hold the cart

6. Input a desired PWM value (0-255) to select motor voltage for driving the cart.
   Do not select a voltage above 4 **Volts** (85 **in the PWM selection**).

7. Release the cart.   Note: At low voltages, you may have to nudge the offset mass to overcome static friction and get the motor rotating.

8. Allow the cart to reach steady state, and press 'q' to get the max accelerometer reading at steady-state and the average motor speed Ω (in tics/millisecond). Record these numbers in the table below.

9. Press 'r' to stop the motor and reset.

10. Convert max acceleration value from step 8 to volts.
    Record this value under $\left|v_{accelerometer}\right|$. Using v_{0g}, convert $\left|v_{accelerometer}\right|$ to $\left|\ddot{x}_{ss}\right|$.

11. Repeat steps 5 through 10 to record $\left|\ddot{x}_{ss}\right|$ for $0 \leq \frac{\Omega}{\omega_n} \leq 2$ in the table below.

12. Ensure the power to the board is off.

Ω (tics/millisec)	Ω (rad/sec)	Sensor $\left\lvert v_{accelerometer}\right\rvert$ (A/D ticks)	$\left\lvert v_{accelerometer}\right\rvert$ (volts)	$\left\lvert \ddot{x}_{ss}\right\rvert$ (m/s^2)	$\left\lvert x_{ss}\right\rvert = \left\lvert \ddot{x}_{ss}\right\rvert / \Omega^2$ (meters)
Sample table - use lab data in Excel spread sheet					

Note: To convert Ω from $\frac{tics}{millisec}$ to $\frac{rad}{sec}$, see Section 4.3.3 – there are $1000\ \frac{tics}{rev}$.
The Arduino lab data (see Excel spreadsheet) has 1 millisec intervals between samples.
The accelerometer's range of 0 to 5 volts has linear correspondence to 0 to 1024 A/D ticks.

Plot your experimental data for $\left|\ddot{x}_{ss}\right|$ on the same graph as analytical $\left|\ddot{x}_{ss}\right|$ versus Ω (Section 4.2.2).
How well does the analytic model compare to the experimental data? (circle one).

Great $< 1\%$	Good $< 10\%$	Fair $< 50\%$	Poor $< 100\%$

- Experimental errors in the physical plant include:
- Experimental errors in the mechatronics include:

On a new plot, graph both your experimental data and analytical calculation for $\left|x_{ss}\right|$ versus Ω.

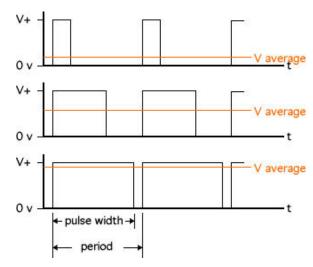## 4.3 Optional Info: Pulse Width Modulation (PWM)

To control a motor, it is often desirable to power the motor using a wide range of voltage values – even though many power supplies have **one** constant voltage. One way to overcome this obstacle is with **P**ulse **W**idth **M**odulation (PWM). In PWM, the micro-controller turns the motor on and off at a specified frequency, varying the ratio between the On and Off periods to control the average voltage provided to the motor. For example, the micro-controller can quickly vary the voltage to the motor from 0 volts (Off) to 12 volts (On) for equal amounts of time so is seems the motor receives 6 Volts.

PWM is controlled by a **_duty cycle_**, defined as the ratio of time the signal is On to the total period (total time the voltage is On or Off) – hence it has values between 0 and 1. The average voltage to the motor is calculated as

$$V_{out} = \text{DutyCycle} * V_{supply}$$

For the Arduino micro-controller, duty cycle is selected as an integer between 0 and 255, with 0 corresponding to always Off (0 Volts) and 255 corresponding to always On (12 Volts), hence

$$\text{DutyCycle} = \frac{\text{ArduinoIntegerValue}}{255}$$

Knowing the power supply provides $V_{supply} \approx 12$ Volts, calculate the corresponding Arduino integer value to generate the following desired output voltages.

Result:

$$V_{out} = 2.5 \text{ Volts} \qquad \text{ArduinoIntegerValue} = \boxed{}$$

$$V_{out} = 7 \text{ Volts} \qquad \text{ArduinoIntegerValue} = \boxed{}$$

Knowing V_{out} has 255 discrete values, calculate the smallest voltage increment ΔV_{out}.
Result:

$$\Delta V_{out} = \boxed{} \text{ Volts}$$

4.3.1 Optional Info: Experimental estimation with an accelerometer

We use several pieces of equipment to measure and record the cart's horizontal acceleration, namely, we use an accelerometer, a microprocessor, a transceiver, and a computer.[11]

- **Accelerometer: ADXL 311 from Analog Devices**

  The accelerometer is mounted on a cart and measures acceleration in up to three directions (we use data from only one direction). The accelerometer is relatively small and lightweight as compared to the cart - so its effect on the acceleration of the cart is negligible. The accelerometer's output signal is a linear $0.3 \frac{\text{volts}}{g}$ signal over a range of ± 2 g.[12] The accelerometer is designed to output 2.5 volts when there is **no** acceleration, but there is some variation from one accelerometer to the next. This is why we measured the output of the accelerometer in Section 4.2.1.

$$v_{accelerometer} \approx v_{0g} + \left(\frac{0.3 \text{ volts}}{1 \ g}\right) \ddot{x}$$

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[11]Most accelerometers do not come assembled with a microprocessor, transceiver, and computer.
[12]NIST (National Institute of Standards and Technology) defines 1 g as exactly 9.80665 m/s².

- **Breadboard signal processing**

  The accelerometer's signal is filter by a low-pass filter to remove high-frequency noise in the signal. Op-amps are used as a buffer to supply additional current and avoid unwanted voltage drops.

- **Arduino UNO microprocessor:**

  The microprocessor's A/D port receives an analog voltage signal from the breadboard in a specified range (i.e., continuous voltages from 0 volts to 5 volts). The A/D port samples the analog signal at 1000 Hz (i.e., at 1 ms intervals). The 10 bit A/D converter on the microprocessor changes the 0 to 5 volt analog signal to bits (ones and zeros) that represent $2^{10} = 1024$ integer values (e.g., 5 volts converts to 1024 and 2.5 volts converts to 512). The Arduino transmits this data to the computer via serial communication.

- **Computer:**

  The computer receives bits from its serial port and uses the Arduino software program to translate the bits to integer numbers which are then printed to the screen. The numbers displayed on the computer screen are integers from 0 to 1024.

4.3.2 Optional Info: Determining the motor's $\dfrac{\Omega}{v_{motor}}$ proportionality constant

The maximum (peak) of $\left|x_{\mathrm{ss}}\right|$ versus Ω should occur at $\Omega \approx \omega_n$ (you previously determined ω_n). Determine the linear-proportional constant[13] that relates motor angular speed Ω to motor input voltage v_{motor}.
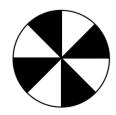
$$\Omega \left(\frac{\mathrm{rad}}{\mathrm{sec}}\right) \;=\; \boxed{} \;*\; v_{motor}\;(\mathrm{rad/s})$$

To complete the previous equation, use an oscilloscope to measure the value of v_{motor} that corresponds to the **maximum** (peak) value of $\left|x_{\mathrm{ss}}\right|$ before breaking out of ***frequency locking***.

What is ***frequency locking***? (Hint: Ask a TA).

4.3.3 Optional Info: Encoder information and Ω conversion factor

The optical quadrature **encoder** measures motor rotational speed by detecting alternating light and dark patterns on a disk. For example, the encoder on the right shows 8 transitions (from light to dark or vice-versa). A quadrature encoder can detect both angular speed **and** direction. Our encoder has 1000 transitions (500 black sections and 500 white sections) and counts $1000\ \frac{\mathrm{tics}}{\mathrm{rev}}$.

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[13]This ***assumes*** a directly proportional relationship between Ω and v_{motor}.