Lab 6 (associated with Hw 6): Dynamic response with harmonic forcing

The objective of this laboratory is to develop physical intuition into how a forcing function effects the behavior of a physical system governed by a second-order, linear, ODE.

Lab 6.1 Effect of forcing function frequency (Ω) on the dynamic response of a pogo stick.

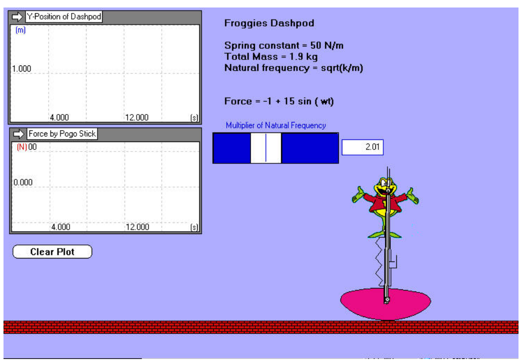
The power/energy-rate principal relates ${}^N\mathrm{P}^S$ (the power of a system S in a Newtonian reference frame N) with the time-rate of change of ${}^N\mathrm{K}^S$ (the kinetic energy of S in N) as

$$ {}^N\mathrm{P}^S \;=\; \frac{d\,{}^N\mathrm{K}^S}{dt} $$

The power associated with $\vec{\mathbf{F}}^P$ (a force applied to a point P) is denoted ${}^N\mathrm{P}^{\vec{\mathbf{F}}^P}$ and is defined in terms of ${}^N\vec{\mathbf{v}}^P$ (the velocity of P in N) as

$$ {}^N\mathrm{P}^{\vec{\mathbf{F}}^P} \;\triangleq\; \vec{\mathbf{F}}^P \cdot {}^N\vec{\mathbf{v}}^P $$

In view of these equations and the definition of the vector dot-product, one may see that $\vec{\mathbf{F}}^P$ adds power to a system (increases kinetic energy) when $\vec{\mathbf{F}}^P$ is applied in the same direction as ${}^N\vec{\mathbf{v}}^P$. Alternately, $\vec{\mathbf{F}}^P$ removes power from a system (decreases kinetic energy) when $\vec{\mathbf{F}}^P$ is applied in the direction opposite ${}^N\vec{\mathbf{v}}^P$.



To begin, double-click on the file `HarmonicForcingPogoStick.wm2d`. To answer each question with Working Model, click the **Reset** button (if necessary) and click and drag the slider that controls Ω/ω_n (the ratio of the forcing frequency to the system's natural frequency). To start the simulation, click the **Run** button, and to stop it, click the **Stop** button.

(a) When the forcing frequency Ω is approximately equal to the system's natural frequency ω_n, i.e., $\frac{\Omega}{\omega_n} \approx 1$, the pogo stick jumps **very high/high/low/none**.

(b) When the forcing frequency Ω is much higher than the system's natural frequency ω_n, i.e., $\frac{\Omega}{\omega_n} > 2$, the pogo stick jumps **very high/high/low/none**.

(c) When the forcing frequency Ω is much lower than the system's natural frequency ω_n, i.e., $\frac{\Omega}{\omega_n} < 0.5$, the pogo stick jumps **very high/high/low/none**.

(d) Explain your observations based on the power/energy-rate principle.

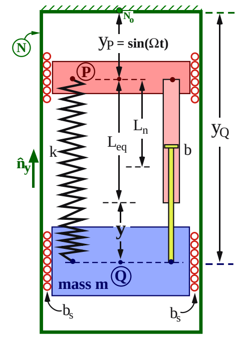
(e) Optional: What happens when the center of mass position is moved?

Lab 6.2 Harmonic forcing of a mass-spring-damper system

Homework 5.4 showed that when the Scotch-yoke mechanism moved P so that $y_P(t) = \bar{A}\,\sin(\Omega\,t)$, the equation of motion governing $y(t)$ (when b_s=0) was

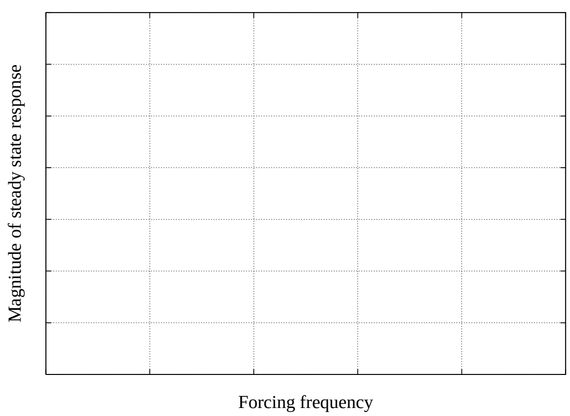
$$\ddot{y} \;+\; \frac{b}{m}\,\dot{y} \;+\; \frac{k}{m}\,y \;=\; \bar{A}\,\Omega^2\,\sin(\Omega\,t)$$

Quantity	Symbol	Type
Mass of Q	m	Constant
Linear spring constant of spring connecting P and Q	k	Constant
Natural length of spring connecting P and Q	L_n	Constant
Static equilibrium length of spring connecting P and Q	L_{eq}	Constant
Earth's gravitational acceleration	g	Constant
Linear damper constant of damper connecting P and Q	b	Constant
Linear damper constant associated with Q's sliding in N	b_s	Constant
Measure of Q's equilibrium position from P	y	Variable
Measure of P's position from a line fixed in N	y_P	**Specified**
Amplitude of harmonic forcing	A	Constant
Frequency of harmonic forcing	Ω	Constant

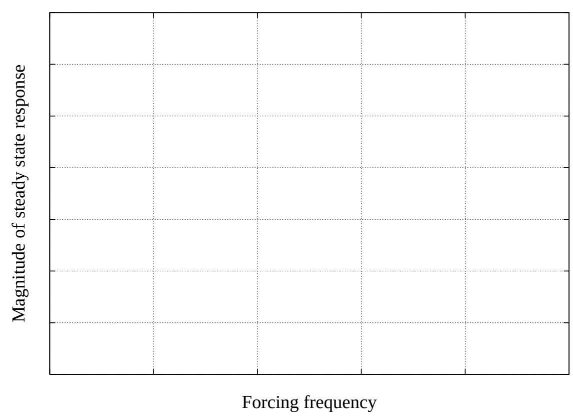


(a) Complete the following sketches. Note: $y(t)$ characterizes how Q moves relative to P and $y_Q(t) \triangleq y_P(t) + y(t)$ characterizes how Q moves relative to N.

$\left|y_{\text{ss}}(t)\right|$ vs. Ω



$\left|y_{Q\,\text{ss}}(t)\right|$ vs. Ω



(b) The particle Q moves very little in N at **low/high** (circle one) frequency whereas at **low/high** frequency, Q appears to be rigidly connected to P. The most energetic motion of Q in N occurs when $\Omega \approx$ ______.