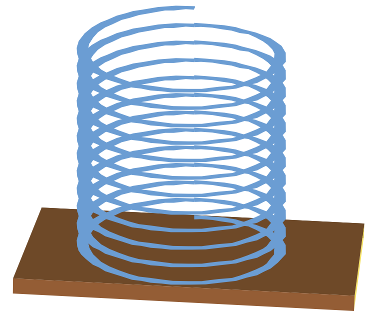
Lab 4 (associated with Hw 4): Period of vibration of a slinky

This laboratory develops physical intuition for 2^{nd}-order, linear, ODEs and associated quantities, including natural length, equilibrium length, period of vibration, natural frequency, decay ratio, and damping ratio.

Lab 4.1 Period of vibration of a massless slinky



To analytically estimate the period of vibration of a mass-spring system ("massless" slinky with attached massive object), gather the following data:

Quantity	Symbol	Value
Mass of object attached to slinky	m_{object}	____ kg
Stretch when object is attached	$stretch$	____ m
Earth's gravitational acceleration	g	$9.8\ \frac{\text{m}}{\text{s}^2}$

Assuming that $\zeta \approx 0$ (little damping) the period of vibration τ_{period} can be estimated as follows:

$$k \;=\; \frac{m_{object}\, g}{stretch} \;=\; \underline{\hspace{2cm}}\ \frac{\text{N}}{\text{m}} \qquad\qquad \omega_n \underset{(5.3)}{=} \sqrt{\frac{k}{m_{object}}} \;=\; \underline{\hspace{2cm}}\ \frac{\text{rad}}{\text{sec}}$$

$$\tau_{\text{period}_{analytical}} \underset{(1)}{=} \frac{2\,\pi}{\omega_d} \underset{(5.7)}{\approx} \frac{2\,\pi}{\omega_n} \;=\; \underline{\hspace{2cm}}\ \text{sec}$$

To experimentally determine τ_{period}, measure several periods of vibration and then average them, i.e.,

$$\tau_{\text{period}_{experimental}} \;=\; \frac{\underline{\hspace{1.5cm}}\ \text{sec}}{\underline{\hspace{1.5cm}}\ \text{cycles}} \;=\; \underline{\hspace{1.5cm}}\ \frac{\text{sec}}{\text{cycle}}$$

The error in the analytical estimate of τ_{period} to the experimental one is

$$\%\text{Error} \;=\; 100 * \frac{\tau_{\text{period}_{analytical}} - \tau_{\text{period}_{experimental}}}{\tau_{\text{period}_{experimental}}} \;=\; 100 * \frac{\underline{\hspace{1.5cm}} - \underline{\hspace{1.5cm}}}{\underline{\hspace{1.5cm}}} \;=\; \underline{\hspace{1.5cm}}\ \%$$

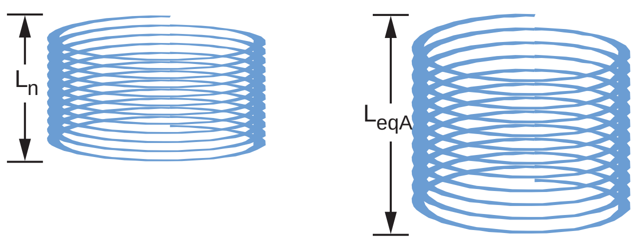
Lab 4.2 A better model: Period of vibration of a massive slinky

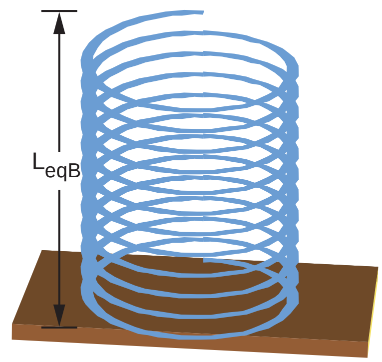
The previous experiment showed how to experimentally determine the period of vibration (τ_{period}) of a massive objected suspended by a "massless" slinky. To account for modeling errors associated with the mass of the slinky, the period of vibration is recalculated with the following data:

Quantity	Symbol	Value
Mass of object attached to slinky	m_{object}	____ kg
Natural length of slinky (***completely*** unstretched)	L_n	____ m
Equilibrium length of vertical slinky ***without*** attached object	L_{eqA}	____ m
Equilibrium length of vertical slinky ***with*** attached object	L_{eqB}	____ m
Earth's gravitational acceleration	g	$9.8 \ \frac{\text{m}}{\text{s}^2}$

By using the equations which govern the static equilibrium lengths of the slinky (with and without the attached object), one can estimate τ_{period}. The process consists of the following steps.

- Measure L_n, the slinky's ***natural length*** (the slinky's length when it is ***completely*** unstretched).
- This experiment models the slinky's mass as a particle of mass m_{spring} attached to the end of the slinky. This is the mass required to stretch the slinky to L_{eqA}. Carefully measure L_{eqA}.
- Attach an object of known mass to the slinky. Measure L_{eqB} (the new ***equilibrium length***).
- ***Show*** how to use $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ to calculate k (the slinky's linear spring constant) and m_{spring}.





$$k \;=\; \frac{m_{object}\; g}{L_{eqB} \;-\; L_{eqA}} \;=\; \text{\_\_\_\_} \ \frac{\text{N}}{\text{m}}$$

$$m_{spring} \;=\; \frac{k}{g}\,(L_{eqA} \;-\; L_n) \;=\; \text{\_\_\_\_} \ \text{kg}$$

- Calculate the motion's ***natural frequency*** ω_n and $\tau_{\text{period}_{analytical}}$ (the mathematically determined ***period of vibration***).

$$\omega_n \;=\; \sqrt{\frac{k}{m_{spring} \;+\; m_{object}}} \;=\; \text{\_\_\_\_} \ \frac{\text{rad}}{\text{sec}}$$

$$\omega_d \;\underset{(5.7)}{\triangleq}\; \omega_n\sqrt{1-\zeta^2} \;\approx\; \omega_n \qquad \text{(when } \zeta \text{ is small)}$$

$$\tau_{\text{period}_{analytical}} \;\underset{(6.1)}{=}\; \frac{2\,\pi}{\omega_d} \;\approx\; \text{\_\_\_\_} \ \text{sec}$$

- Measure $\tau_{\text{period}_{experimental}}$ (the experimentally determined ***period of vibration***) by timing several periods of vibration and then averaging them.

$$\tau_{\text{period}_{experimental}} \;=\; \frac{\text{\_\_\_\_ sec}}{\text{\_\_\_\_ cycles}} \;=\; \text{\_\_\_\_} \ \frac{\text{sec}}{\text{cycle}}$$

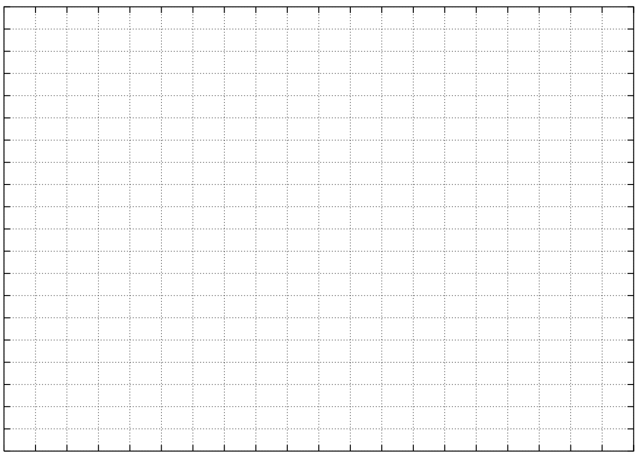
- Calculate the percentage error in the analytical determination of τ_{period}.

$$\text{Error} \;=\; 100\,\frac{\tau_{\text{period}_{experimental}} - \tau_{\text{period}_{analytical}}}{\tau_{\text{period}_{experimental}}} \;=\; \underline{\hspace{2em}}\ \%$$

- Using the same data, calculate the percentage error in the analytical determination of τ_{period} when m_{spring} is assumed to be zero.

$$\text{Error} \;=\; 100\,\frac{\tau_{\text{period}_{experimental}} - \tau_{\text{period}_{analytical}}}{\tau_{\text{period}_{experimental}}} \;=\; \underline{\hspace{2em}}\ \%$$

- The error in calculating τ_{period} is **smaller/larger** (circle one) when one assumes $m_{spring} = 0$.
- Make a rough sketch that is useful for experimentally measuring the ***decay ratio***.



$$\text{decayRatio} \;\approx\; \underline{\hspace{2em}}$$

- Make a rough estimate of the ***damping ratio*** ζ. (See Homework 4.6 for details).

$$\zeta \;\approx\; \underline{\hspace{2em}}$$