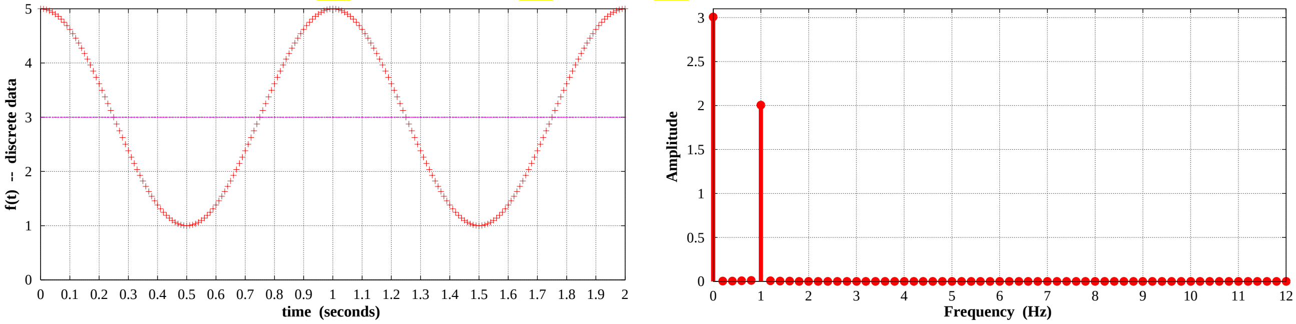
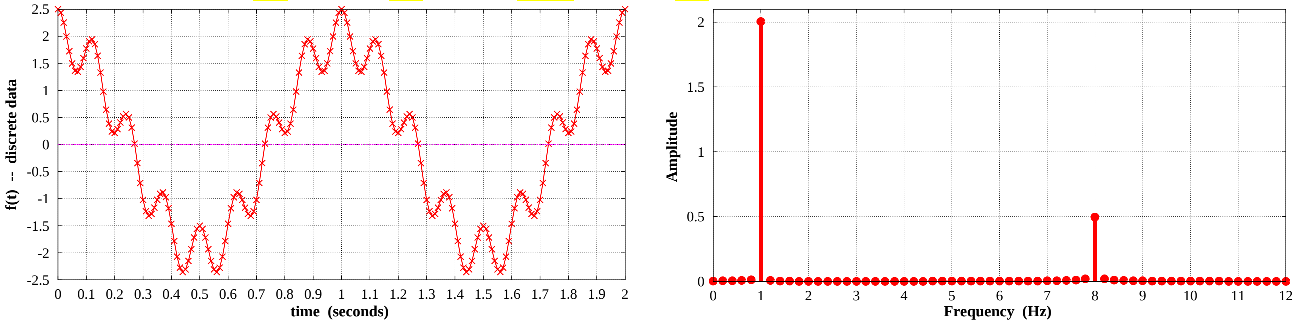
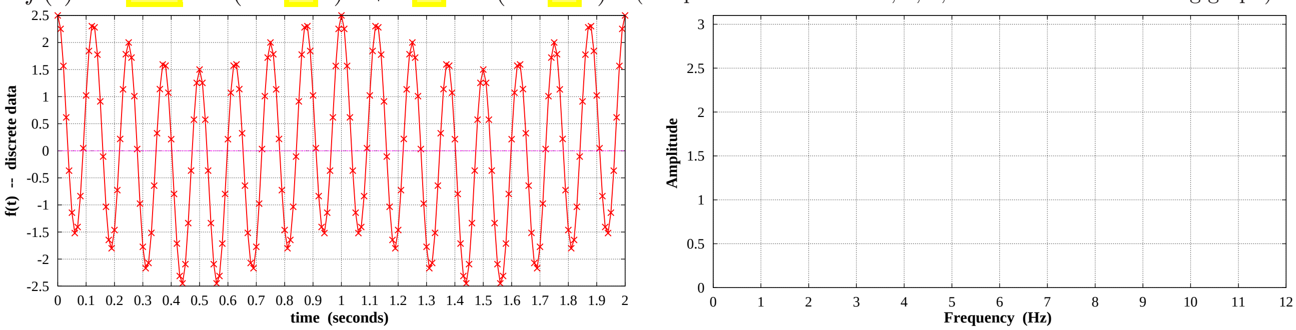
$f(t) \;=\; \square\,\cos(0\,t) \;+\; \square\,\cos(2\,\pi\,\square\,t)$ (complete blanks with 1, 2, or 3).



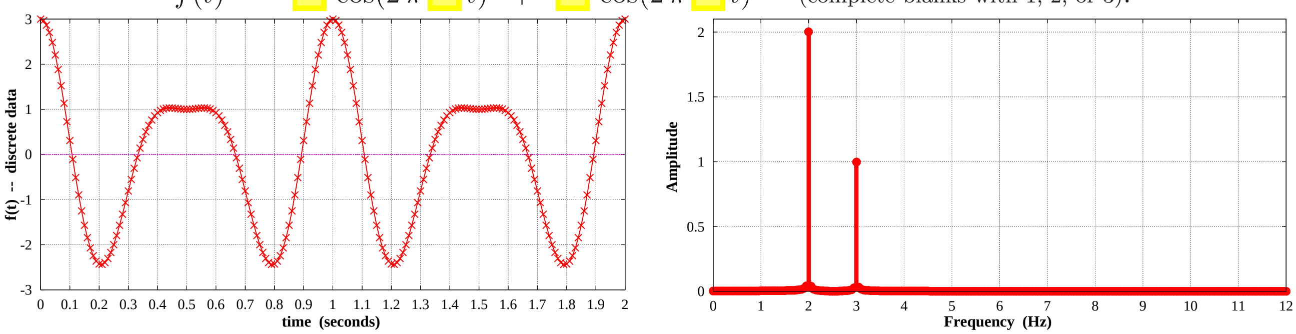
$f(t) \;=\; \square\,\cos(2\,\pi\,\square\,t) \;+\; \square\,\cos(2\,\pi\,\square\,t)$ (complete blanks with 0.5, 1, 2, or 8).



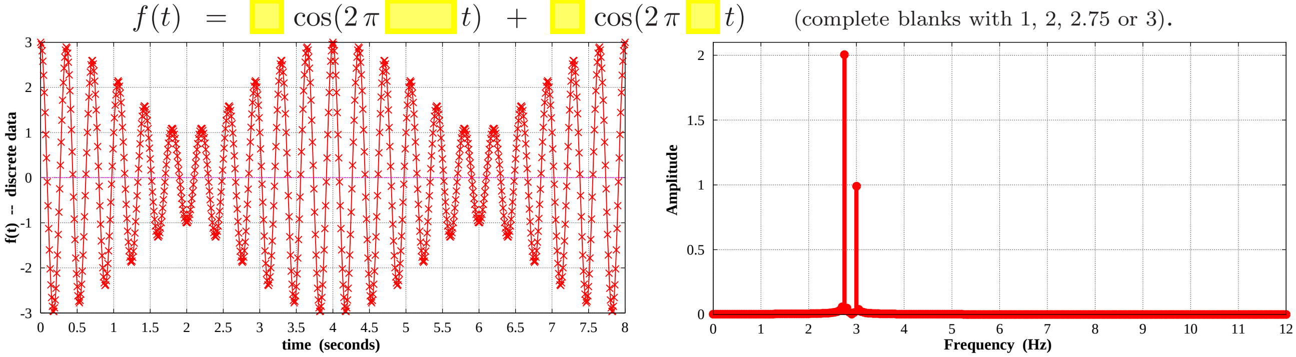
$f(t) \;=\; \square\,\cos(2\,\pi\,\square\,t) \;+\; \square\,\cos(2\,\pi\,\square\,t)$ (complete blanks with 0.5, 1, 2, or 8. **Draw** the missing graph).



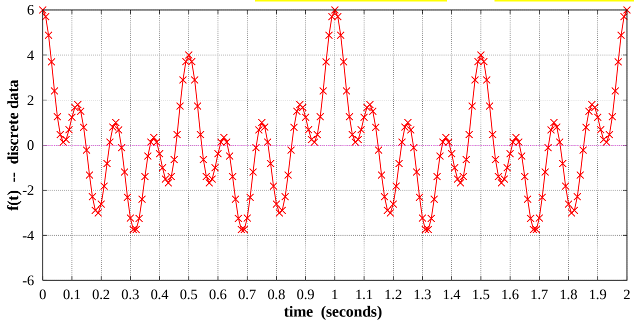
$f(t) \;=\; \square\,\cos(2\,\pi\,\square\,t) \;+\; \square\,\cos(2\,\pi\,\square\,t)$ (complete blanks with 1, 2, or 3).

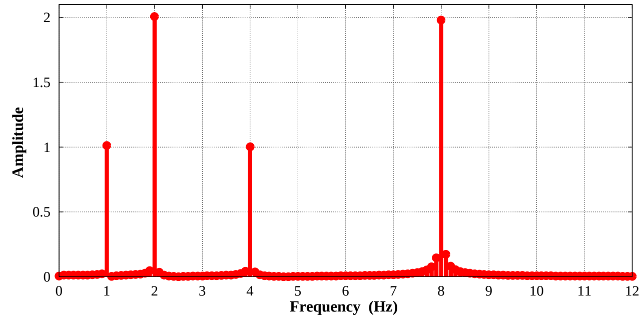


$f(t) \;=\; \square\,\cos(2\,\pi\,\square\,t) \;+\; \square\,\cos(2\,\pi\,\square\,t)$ (complete blanks with 1, 2, 2.75 or 3).



$f(t) = \boxed{} + \boxed{} + \boxed{} + \boxed{}$





Answers to these problems at www.MotionGenesis.com ⇒ Textbooks ⇒ Resources.

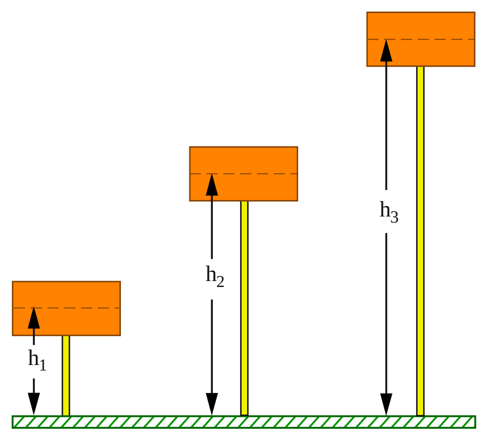
Questions to consider with discrete data: How quickly and how long to sample?

Name	Symbol	Associated question
Highest frequency	B	Highest expected frequency? (Hz)
Frequency resolution	$\Delta\omega_{\text{H}}$	How fine a resolution in the frequency domain?
Sampling rate	$\Delta t < \dfrac{1}{2\,B}$	How often do you sample the function? (***Nyquist criteria***)
Total sampling time	$t_{\text{final}} > \dfrac{1}{\Delta\omega_{\text{H}}}$	How long do you sample?

Note: If you want fine resolution in the frequency domain (e.g., resolve differences between 0.012 Hz and 0.013 Hz) you must sample for a long time. However, you do not necessarily have to take many samples.

Ideal: Sample quickly (small Δt) to "hear" highest frequency. Sample for a long time (large t_{final}) to get fine frequency resolution (small $\Delta\omega_{\text{H}}$). The cost is a large **number of samples** $N = \frac{t_{\text{final}}}{\Delta t}$ (collect data frequently and for a long time).

Earthquakes and building shaking



Listed below are geometry and material for three "buildings":

Description	Symbol	Value
Building heights	h_1	10 cm
	h_2	20 cm
	h_3	30 cm
Mass of block	m	50 g
Radius of wire	r	0.75 mm
Wire elastic modulus	E	200×10^9 N/m^2

The solid cylindrical wire's bending area moment of inertia is

$$I = \frac{\pi\, r^4}{4} = 2.65 \times 10^{-13} \text{ m}^4$$

Each building's bending stiffness k_i is approximated using E, I, and h_i $(i = 1, 2, 3)$ as follows.
Next, each building's natural vibration frequency is approximated as (in $\text{Hz} = \frac{\text{cycles}}{\text{sec}}$)

$$k_1 = \frac{3\,E\,I}{h_1^3} = 159\ \frac{\text{N}}{\text{m}} \qquad k_2 = \frac{3\,E\,I}{h_2^3} = 20\ \frac{\text{N}}{\text{m}} \qquad k_3 = \frac{3\,E\,I}{h_3^3} = 6\ \frac{\text{N}}{\text{m}}$$

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} = 9.0 \text{ Hz} \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} = 3.2 \text{ Hz} \qquad f_3 = \frac{1}{2\pi}\sqrt{\frac{k_3}{m}} = 1.7 \text{ Hz}$$

You are tasked with collecting 50 seconds of ground acceleration data. Approximately what sampling rate would you choose to ensure you see a range of frequencies relevant for concern about building shaking?

[1]Since ***bandwidth*** $B \triangleq \omega_{\text{high}} - \omega_{\text{low}}$ and usually for Fourier transforms $\omega_{\text{low}} = 0$, hence $B = \omega_{\text{high}}$.

Result:

- Concerned about frequencies: ☐ – ☐ Hz
- Sample at a minimum of: ☐ Hz

Consider the time-data (top-right) and its Discrete Fourier Transform (bottom-right).
Based on this information the building that probably shakes most is ☐ cm high.

Note: This data was sampled at 100 Hz (much higher than needed).