### 12.1 Matrix rows and columns

Given: $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Row 1 of M $= \begin{bmatrix} 1 & 2 \end{bmatrix}$  Row 2 of M $= \begin{bmatrix} 3 & 4 \end{bmatrix}$

$M_{2,1} = 3$  Column 1 of M $= \begin{bmatrix} 1 \\ 3 \end{bmatrix}$  Column 2 of M $= \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

### 12.2 Matrix transpose

$$\text{Transpose}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \qquad \text{Transpose}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

### 12.3 Matrix addition and subtraction (+, -)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix} \qquad \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} = \begin{bmatrix} a-2 & b-4 & c-6 \\ d-3 & e-5 & f-7 \end{bmatrix}$$

### 12.4 Scalar-matrix multiplication (∗)

$$5 * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix} \qquad 5 * \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 5a & 5b & 5c \\ 5d & 5e & 5f \end{bmatrix}$$

### 12.5 Matrix-matrix multiplication (∗)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3a + 5b \\ 3c + 5d \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 & x \\ 5 & y \end{bmatrix} = \begin{bmatrix} 3a + 5b & ax + by \\ 3c + 5d & cx + dy \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} * \begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} ax & ay \\ bx & by \end{bmatrix} \qquad \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} * \begin{bmatrix} x & 3 \\ y & 5 \\ z & 7 \end{bmatrix} = \begin{bmatrix} ax+by+cz & 3a+5b+7c \\ dx+ey+fz & 3d+5e+7f \end{bmatrix}$$

### 12.6 Matrix determinants

$$\det\begin{bmatrix} 5 \end{bmatrix} \triangleq 5 \qquad \det\begin{bmatrix} a \end{bmatrix} \triangleq a$$

$$\det\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = 1 * 4 - 2 * 3 = -2 \qquad \det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ab - cd$$

Calculate the following determinant three ways, namely by expanding along the first row, expanding along the first column, and expanding along the second row.

$$\det\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{bmatrix} = +1 \det\begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + -2 \det\begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix} + +3 \det\begin{bmatrix} 4 & 5 \\ 7 & 0 \end{bmatrix} = -48$$

$$= +1 \det\begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + -4 \det\begin{bmatrix} 2 & 3 \\ 0 & 9 \end{bmatrix} + +7 \det\begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} = -48$$

$$= -4 \det\begin{bmatrix} 2 & 3 \\ 0 & 9 \end{bmatrix} + +5 \det\begin{bmatrix} 1 & 3 \\ 7 & 9 \end{bmatrix} + -6 \det\begin{bmatrix} 1 & 2 \\ 7 & 0 \end{bmatrix} = -48$$

Calculate the following determinant by expanding along the third column.

$$\det\begin{bmatrix} a & b & c \\ d & e & 0 \\ g & h & 0 \end{bmatrix} = +c * \det\begin{bmatrix} d & e \\ g & h \end{bmatrix} = c\,(dh - eg)$$

## 12.7 Optional: Matrix inverse with determinants and adjugate (adjunct) matrices

Calculate the following matrix inverses (the 3×3 matrix inverses are optional).

$$\text{inv}\begin{bmatrix} 5 \end{bmatrix} = \frac{\text{adj}\begin{bmatrix} 5 \end{bmatrix}}{\det\begin{bmatrix} 5 \end{bmatrix}} = \frac{\begin{bmatrix} 1 \end{bmatrix}}{5} = \begin{bmatrix} 0.2 \end{bmatrix} \qquad \text{inv}\begin{bmatrix} a \end{bmatrix} = \begin{bmatrix} \frac{1}{a} \end{bmatrix}$$

$$\text{inv}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \frac{\text{adj}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}{\det\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} = \frac{\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}}{-2} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

$$\text{inv}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{\text{adj}\begin{bmatrix} a & b \\ c & d \end{bmatrix}}{\det\begin{bmatrix} a & b \\ c & d \end{bmatrix}} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{a\,d - b\,c}$$

$$\text{inv}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{bmatrix} = \begin{bmatrix} -0.9375 & 0.375 & 0.0625 \\ -0.125 & 0.25 & -0.125 \\ 0.7291667 & -0.2916667 & 0.0625 \end{bmatrix}$$

$$\text{inv}\begin{bmatrix} a & b & c \\ d & e & 0 \\ g & h & 0 \end{bmatrix} = \begin{bmatrix} 0 & h/(d*h-e*g) & -e/(d*h-e*g) \\ 0 & -g/(d*h-e*g) & d/(d*h-e*g) \\ 1/c & -(a*h-b*g)/(c*(d*h-e*g)) & (a*e-b*d)/(c*(d*h-e*g)) \end{bmatrix}$$

## 12.8 Matrix form of scalar equations (matrix multiplication in reverse)

Put the following sets of scalar equations into matrix form.

| | | |
|---|---|---|
| $a\,x + b\,y = 12$<br>$d\,x + e\,y = 15$<br>$\begin{bmatrix} a & b \\ d & e \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$ | $a\,x + b\,y + c\,z = 12$<br>$d\,x + e\,y + f\,z = 15$<br>$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$ | $a\,x + b\,y - 12 = 0$<br>$d\,x + e\,y - 15 = 0$<br>$\begin{bmatrix} x & y & 0 & 0 \\ 0 & 0 & x & y \end{bmatrix}\begin{bmatrix} a \\ b \\ d \\ e \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$ |

## 12.9 Optional: Solving sets of linear algebraic equations.

$$a\,x + b\,y = 1$$
$$d\,x + e\,y = 2$$

$$a\,x + b\,y + c\,z = 1$$
$$2\,x + 3\,y + 4\,z = 2$$
$$2\,x + 4\,y + 6\,z = 4$$

**Solve for $x$, $y$:**

$$x = \frac{e - 2\,b}{a\,e - b\,d} \qquad y = \frac{2\,a - d}{a\,e - b\,d}$$

**Solve for $x$, $y$, $z$:**

$$x = \frac{-1 - 2\,b + 2\,c}{2\,b - a - c} \qquad y = \frac{2 + 2\,a - 2\,c}{2\,b - a - c} \qquad z = \frac{-1 - 2\,a + 2\,b}{2\,b - a - c}$$

## 12.10 Optional: Matrix computation with MotionGenesis and/or MATLAB®

Use **M**otion**G**enesis and/or MATLAB® to do Homework 12.1 - Homework 12.9 (include optional problems but exclude Homework 12.8). Print and submit a **M**otion**G**enesis input command file named `MatrixAlgebra.txt` that uses the following commands.

| | | | |
|---|---|---|---|
| GetElement | GetRow | GetColumn | GetTranspose |
| + − * | GetDeterminant | GetInverse | Solve |