

12.1 Matrix rows and columns

Given: $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ Row 1 of M = $\begin{bmatrix} \square & \square \end{bmatrix}$ Row 2 of M = $\begin{bmatrix} \square & \square \end{bmatrix}$
 $M_{2,1} = \square$ Column 1 of M = $\begin{bmatrix} \square \\ \square \end{bmatrix}$ Column 2 of M = $\begin{bmatrix} \square \\ \square \end{bmatrix}$

12.2 Matrix transpose

Transpose $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$ Transpose $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \square$

12.3 Matrix addition and subtraction (+, -)

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$ $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} = \square$

12.4 Scalar-matrix multiplication (*)

$5 * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$ $5 * \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \square$

12.5 Matrix-matrix multiplication (*)

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} \square \\ \square \end{bmatrix}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 & x \\ 5 & y \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$
 $\begin{bmatrix} a \\ b \end{bmatrix} * \begin{bmatrix} x & y \end{bmatrix} = \square$ $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} * \begin{bmatrix} x & 3 \\ y & 5 \\ z & 7 \end{bmatrix} = \square$

12.6 Matrix determinants

$\det [5] \triangleq 5$ $\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \square * \square - \square * \square = -2$ $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \square$
 $\det [a] \triangleq \square$

Calculate the following determinant three ways, namely by expanding along the first row, expanding along the first column, and expanding along the second row.

$\det \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{bmatrix} = +1 \det \begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + -2 \det \begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix} + +3 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$
 $= +1 \det \begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + -4 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + +7 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$
 $= -4 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + +5 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + -6 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$

Calculate the following determinant by expanding along the third column.

$\det \begin{bmatrix} a & b & c \\ d & e & 0 \\ g & h & 0 \end{bmatrix} = \square * \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$

12.7 Optional: Matrix inverse with determinants and adjugate (adjunct) matrices

Calculate the following matrix inverses (the 3×3 matrix inverses are optional).

$$\text{inv} [5] = \frac{\text{adj} [5]}{\det [5]} = \frac{[1]}{5} = [0.2] \quad \text{inv} [a] = \text{[Yellow Box]}$$

$$\text{inv} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \frac{\text{adj} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}{\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} = \frac{\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}}{\text{[Yellow Box]}} = \text{[Yellow Box]}$$

$$\text{inv} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{\text{adj} \begin{bmatrix} a & b \\ c & d \end{bmatrix}}{\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}} = \frac{\text{[Yellow Box]}}{\text{[Yellow Box]}}$$

$$\text{inv} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{bmatrix} = \begin{bmatrix} -0.9375 & 0.375 & 0.0625 \\ -0.125 & 0.25 & -0.125 \\ 0.7291667 & -0.2916667 & 0.0625 \end{bmatrix}$$

$$\text{inv} \begin{bmatrix} a & b & c \\ d & e & 0 \\ g & h & 0 \end{bmatrix} = \begin{bmatrix} 0 & h/(d * h - e * g) & -e/(d * h - e * g) \\ 0 & -g/(d * h - e * g) & d/(d * h - e * g) \\ 1/c & -(a * h - b * g)/(c * (d * h - e * g)) & (a * e - b * d)/(c * (d * h - e * g)) \end{bmatrix}$$

12.8 Matrix form of scalar equations (matrix multiplication in reverse)

Put the following sets of scalar equations into matrix form.

$\begin{aligned} ax + by &= 12 \\ dx + ey &= 15 \end{aligned}$ $\text{[Yellow Box]} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$	$\begin{aligned} ax + by + cz &= 12 \\ dx + ey + fz &= 15 \end{aligned}$ $\text{[Yellow Box]} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$	$\begin{aligned} ax + by - 12 &= 0 \\ dx + ey - 15 &= 0 \end{aligned}$ $\text{[Yellow Box]} \begin{bmatrix} a \\ b \\ d \\ e \end{bmatrix} = \begin{bmatrix} \text{[Yellow Box]} \\ 15 \end{bmatrix}$
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12.9 Optional: Solving sets of linear algebraic equations.

$$ax + by = 1$$

$$dx + ey = 2$$

Solve for x, y :

$$x = \frac{e - 2b}{ae - bd} \quad y = \frac{2a - d}{ae - bd}$$

Solve for x, y, z :

$$x = \frac{-1 - 2b + 2c}{2b - a - c} \quad y = \frac{2 + 2a - 2c}{2b - a - c} \quad z = \frac{-1 - 2a + 2b}{2b - a - c}$$

$$ax + by + cz = 1$$

$$2x + 3y + 4z = 2$$

$$2x + 4y + 6z = 4$$

12.10 Optional: Matrix computation with MotionGenesis and/or MATLAB[®]

Use MotionGenesis and/or MATLAB[®] to do Homework 12.1 - Homework 12.9 (include optional problems but exclude Homework 12.8). Print and submit a MotionGenesis input command file named MatrixAlgebra.txt that uses the following commands.

GetElement	GetRow	GetColumn	GetTranspose
+ - *	GetDeterminant	GetInverse	Solve