

Contents

Statics and dynamics – what does it do for us?	i
Newton's three laws	ii
1 Math tools	1
1.1 Unit systems - SI and USA	1
1.2 Geometry: Ancient Euclid and modern vectors (see Chapters 2, 3, ...).	2
1.3 Circles and their properties	2
1.4 Triangles and ratios of their sides (sine, cosine, tangent)	2
1.4.1 Formulas involving sine and cosine	3
1.4.2 Sine and cosine as functions (Euler, circa 1730)	3
1.5 Types of scalars: Variable, specified, constant	3
1.6 Differentiation	3
1.6.1 Definition of an ordinary derivative of a scalar function	3
1.6.2 Definition of a partial derivative of a scalar function	4
1.6.3 Short table of derivatives frequently encountered in engineering	4
1.6.4 Good product rule for differentiation (for scalars, \vec{v} ectors, matrices, ...)	4
1.6.5 Quotient rule for derivatives (or remember $\frac{u}{v} = u v^{-1}$ and use exponent and product rules)	4
1.6.6 Chain rule for ordinary derivatives	5
1.6.7 Example: Partial and ordinary differentiation	5
1.6.8 Implicit differentiation: A useful tool for calculating derivatives	5
1.7 Integration and a short table of integrals	5
1.8 Solutions of <i>polynomial</i> equations (roots) quadratic equation	6
1.9 Solutions of linear and nonlinear algebraic equations	6
1.10 Solving ODEs (ordinary differential equations)	6
2 Vectors ($\triangleq \vec{a} + \vec{b}, -\vec{b}, \angle(\vec{a}, \vec{b}), \vec{a} \cdot \vec{b}, \vec{a} \times \vec{b}$)	7
2.1 Examples of scalars vectors and dyadics	7
2.2 Definition of a vector	7
2.3 Zero vector $\vec{0}$, a vector whose magnitude is zero	8
2.4 Unit \hat{v} ectors: Vectors with magnitude 1 and no units (typeset with a \hat{v} hat)	8
2.5 Equal vectors (=) vectors with the same magnitude and direction	8
2.6 Vector addition (+)	9
2.7 Vector multiplied or divided by a scalar (* or /)	9
2.8 Vector negation and subtraction (-)	9
2.9 Vector dot product (\cdot)	10
2.9.1 Properties of the dot-product (\cdot)	10
2.9.2 Uses for the dot-product (\cdot)	10
2.9.3 Dot-products to change vector equations to scalar equations (see Hw 1.29)	10
2.9.4 Special case: Dot-products with orthogonal unit vectors	10
2.9.5 Examples: Vector dot-products (\cdot)	11
2.10 Vector cross product (\times)	11

2.10.1	Uses for the cross-product (\times) in geometry, statics, motion analysis,	11
2.10.2	Determinants and cross-products (with right-handed unit vectors)	12
2.11	Optional: Scalar triple product ($\cdot \times$ or $\times \cdot$)	12
2.11.1	Scalar triple product and the volume of a tetrahedron	12
2.11.2	($\times \cdot$) to change vector equations to scalar equations (see Hw 1.29)	12
3	Position vectors and vector geometry	13
3.1	What is a point, particle, position vector (see examples in Hw 3)	13
3.2	Area of a triangle	13
3.3	Geometry example: Length/distances, angle, surface area, volume	14
4	Vector basis	15
4.1	What is a vector basis?	15
4.2	Expressing a vector in terms of the basis $\vec{a}_1, \vec{a}_2, \vec{a}_3$	15
4.3	Concept: What is the vector vs. how is it expressed	16
5	Rotation matrices (resolving unit vectors into components)	17
5.1	Uses for the rotation matrix ${}^aR^b$ (for geometry, statics, motion, stress . . .)	17
5.2	How to use a rotation matrix (angles, inverse, express, dot, cross)	18
5.3	Simple rotation matrices and the “hug rule”	19
5.4	Forming rotation matrices with matrix multiplication	20
5.5	What is an angle?	21
6	Vector differentiation	23
6.1	Definition: Derivative of a vector in a rigid basis (or reference frame)	23
6.2	What is a constant vector (i.e., a vector fixed in a reference frame)?	23
6.3	Properties of derivatives of vectors	23
6.4	Example: Derivative of a vector	24
6.5	Differentiation concepts: Changes in magnitude and direction	24
7	Angular velocity & angular acceleration	25
7.1	Angular velocity concepts: Moon and Earth celestial systems	25
7.2	What is a reference frame or a Newtonian reference frame?	26
7.3	Defining property of angular velocity (golden rule for vector differentiation)	26
7.3.1	Example: Angular velocity and vector differentiation	26
7.3.2	Example: Vector differentiation and angular momentum – spinning body	27
7.3.3	Simple angular velocity	27
7.3.4	Angular velocity negative property	28
7.3.5	Angular velocity addition theorem (<i>squash rule for adding angular velocities</i>)	28
7.3.6	Angular velocity example: Welding robot	28
7.3.7	Angular velocity example: Chaotic plate pendulum	28
7.3.8	Angular velocity – a property of a rigid object	28
7.4	Angular acceleration	29
7.5	Optional: Angular velocity proofs	29
7.5.1	Proof of angular velocity and orthogonal basis vectors	29
7.5.2	Proof of simple angular velocity	30
7.5.3	Proof of angular velocity addition theorem	30
7.5.4	Proof of angular velocity negative property	30

8 Points: Velocity and acceleration	31
8.1 Definition of a point's velocity and acceleration	31
8.2 Velocity and acceleration of two points <u>fixed</u> on a rigid body	33
8.3 Relationship between position, velocity, and acceleration	34
8.3.1 Constant acceleration along a curve or line	34
8.3.2 Circular motion and centripetal acceleration – a special case	34
8.3.3 Analogy between translational motion and planar (2D) rotational motion	34
8.3.4 Example: Free-fall of a sky-diver with constant downward acceleration	35
8.4 Speed and distance-traveled in a reference frame (see Hw 11.5)	35
8.5 Optional: Acceleration vocabulary	35
8.6 Optional: How g 's affect human health (biomechanics)	36
8.7 Optional: Velocity and acceleration proofs	36
9 Constraints: Rods, rolling, gears, ...	37
9.1 Rods, ropes, and separators	37
9.2 Linear actuator	37
9.3 Ball and socket joint	38
9.4 Revolute joint (also called a hinge or pin joint)	38
9.5 Revolute motor (revolute joint with additional angular constraint)	38
9.6 Prismatic joint (also called a slider joint or square-slot joint)	38
9.7 Contact (an intermittent inequality constraint)	39
9.8 Sliding and rolling (a special type of contact)	39
9.9 Gears and constraint equations	40
10 Particles (points with mass)	43
10.1 $\vec{F} = m\vec{a}$ recipe for translational motion (examples: Sections 10.8, 10.9, Hw 9)	44
10.2 Systems with particles	44
10.3 Translational (linear) momentum for particles	44
10.4 Angular momentum and shift theorem for particles	45
10.5 Shift theorems for moments of momentum, force, inertia,	45
10.6 Kinetic energy for particles	46
10.7 Example: Momentum and energy for a child (particle) on a swing	47
10.8 Example: $\vec{F} = m\vec{a}$ recipe for translational motion (rocket-sled)	48
10.9 Example: $\vec{F} = m\vec{a}$ recipe for translational motion (particle in spinning slot)	49
10.10 Optional: Proofs for translational/angular momentum	50
10.10.1 Proof of translational (linear) momentum of a system of particles	50
10.10.2 Proof of shift theorem for angular momentum	50
11 Mass, center of mass, centroid	51
11.1 Mass	51
11.2 Center of mass and centroid	51
11.3 Optional: Figures and their length, area, and volume	53
11.4 Optional: Integrals for mass, center of mass, and centroid	53
11.5 Optional: Experiments, tables, CAD/CAE, and medical scanning	54
11.6 Optional: Measuring mass	54
11.7 Optional: The language and etymology of “mass”	54
11.8 Optional: What is mass?	54

12 Concepts: Moments/products of inertia	55
12.1 Moment of inertia (also see Section 14.2)	55
12.1.1 Shift theorem for moment of inertia (parallel axis theorem)	56
12.1.2 Example: Moments of inertia of a particle (repeated in Hw 10.2)	56
12.1.3 Example: Moment of inertia concepts (repeated in Hw 10.3)	56
12.1.4 Demo: How moment of inertia affects rolling objects	57
12.1.5 Demo: How moment of inertia affects a spinning book (repeated in Hw 10.4)	57
12.2 Products of inertia (also see Section 14.3)	57
12.2.1 Example: Products of inertia of a particle (repeated in Hw 10.2)	57
12.2.2 Conceptual example of products of inertia (repeated in Hw 10.9)	58
12.2.3 Demo: How product of inertia affects a rotating rattleback (repeated in Hw 10.18) . .	58
12.2.4 Optional: Two sign conventions (\pm) for products of inertia	58
13 Dyadics	59
13.1 Dyadics and 3×3 matrices	60
13.2 Dyadic symmetry	60
14 Inertia dyadics	61
14.1 Inertia dyadic: A “suitcase” for moments and products of inertia	61
14.1.1 Inertia dyadic for a particle or system	61
14.1.2 Shift theorem for an inertia dyadic (for an arbitrary system S)	61
14.1.3 Example: Inertia dyadic of a particle (also see example in Section 14.5)	62
14.2 Moments of inertia and inertia dyadics (also see Section 12.1)	62
14.3 Products of inertia and inertia dyadics (also see Section 12.2)	63
14.4 Inertia matrix (a symmetric matrix)	63
14.5 Example: Inertia properties for a child on a swing	64
14.6 Optional: Motivating concepts for inertia dyadics	65
15 Rigid bodies	67
15.1 What is a rigid body? (see examples in Hw 11, laws of motion in Chapter 20).	67
15.2 $\vec{F} = m\vec{a}$ recipe for translational motion of a rigid body B in a Newtonian frame N	68
15.3 $\vec{M} = I\vec{\alpha}$ recipe for rotational motion of a rigid body B via a point B_p of B	68
15.4 Angular momentum of a rigid body (also see Sections 20.4, 20.9)	68
15.5 Kinetic energy of a rigid body	69
15.6 Example: Momentum of an inverted pendulum on cart	70
15.7 Example: Momentum & kinetic energy of a simplified aircraft	70
15.8 Optional: Proof of angular momentum for a rigid body	71
15.9 Optional: Proof of kinetic energy for a rigid body	72
16 Force and resultant	73
16.1 Force and the law of action/reaction	73
16.2 Statics, dynamics, and resultant forces	74
16.3 Resultant of a set of vectors (e.g., forces on a point, body, or system)	74
16.4 Resultant force on a system (internal force cancellation)	74
16.5 Contact and distance forces	75
16.6 Applied and constraint forces	75
16.7 Optional: The definition and philosophy of force	75

17 Moment and torque	77
17.1 Moment of a vector	77
17.1.1 Moment of a set of vectors	77
17.1.2 Shift theorem for the moment of a set of vectors	77
17.1.3 $\hat{\mathbf{u}} \cdot \vec{\mathbf{M}}^{S/P} = \hat{\mathbf{u}} \cdot \vec{\mathbf{M}}^{S/O}$ if $\hat{\mathbf{u}}$ is parallel to line \overline{OP} (useful for MG road-maps in Section 21.1)	78
17.1.4 Moment arm of a vector about a point (example at end of Section 17.6)	78
17.2 Moment of internal forces (a separate law of mechanics?)	78
17.3 Statics, dynamics, and moments of forces	78
17.4 Definition of static equilibrium	78
17.5 Torque of a set of vectors (moment of a couple)	79
17.6 Neuromuscular biomechanics example: Muscle tension for curling	80
18 Replacement of forces and bound vectors	83
18.1 Replacement of distance forces (not a unique process)	83
18.2 Replacement of contact forces	84
18.3 Applicability of equivalence/replacement?	84
18.4 Replacement of forces associated with constraints	85
19 Encyclopedia of applied force and torque	87
19.1 Weight, mass, and gravity	87
19.2 Uniform gravity on a particle or body (see appendix for g for planets)	87
19.3 Universal gravitational attraction between two particles	88
19.4 Electrostatic forces and Coulomb's law for charged particles	89
19.5 DC (direct current) permanent magnet motors	89
19.6 Kinetic friction and the Continuous Friction Law	90
19.7 Translational spring between two points	91
19.8 Rotational (torsional) spring	91
19.9 Translational damper between two points	92
19.10 Rotational (torsional) damper	92
19.11 Light linear springs and dampers in <i>series</i>	93
19.12 Light linear springs and dampers in <i>parallel</i>	93
19.13 Viscous damper between two surfaces	94
19.14 Fluid forces (lift and drag)	94
19.14.1 Demo: Experimental determination of drag forces on a coffee filter	95
19.14.2 Closed-form solutions with aerodynamic drag	95
19.15 Baseball aerodynamic lift/drag/torque - a case study in philosophy	96
19.16 Optional: Proofs for spring and gravity forces	99
19.16.1 Optional: Proof of light linear springs and dampers in <i>series</i>	99
19.16.2 Optional: Proof of light linear springs and dampers in <i>parallel</i>	99
19.16.3 Optional: Proof for uniform gravity on a body or set of particles	99
20 Equations of motion	101
20.1 Newton's law of motion for a particle $\vec{F} = m\vec{a}$ (also see Chapter 10)	101
20.2 Newton's equation for a system	101
20.3 Translational (linear) momentum principle	102
20.4 Angular momentum principle	102
20.5 Euler's equation for a rigid body (2D) $\vec{M}_z = I_{zz}\vec{\alpha} + \dots$	102
20.6 Euler's equations for a rigid body (3D)	103
20.7 Optional: Lagrange's equations of the second kind	103
20.8 Impulse/momenta	103
20.9 Conservation of translational/angular momentum	104

20.10	Optional: Proof: Translational momentum principle	104
20.11	Optional: Proof: Angular momentum principle	105
21	D'Alembert's method with MG road-maps	107
21.1	MG road-maps for efficient statics and dynamics	108
21.1.1	MG road-map: Projectile motion (2D)	109
21.1.2	MG road-map: Rigid body pendulum (2D)	109
21.1.3	MG road-map: Inverted pendulum on cart (x and θ) (2D)	109
21.1.4	MG road-map: Rotating rigid body (3D)	110
21.1.5	MG road-map: Bridge crane equations of motion (2D)	110
21.1.6	MG road-map: Particle on spinning slot (2D)	110
21.1.7	MG road-map: Motion of a chaotic double pendulum (3D)	111
21.1.8	MG road-map: Particle pendulum (2D) – angle and tension	111
22	Power and work	113
22.1	Power/kinetic energy-rate principle	113
22.2	Forces that do not contribute to power (workless forces)	113
22.3	Why the power/kinetic energy-rate principle is such a useful tool.	114
22.4	Power of a force, set of forces, or torque on a rigid body	115
22.5	Example: Power/kinetic energy-rate to size a torque-motor	115
22.6	Power/kinetic energy-rate for a commercial spring scale	116
22.7	Definition of work with an integral or differential equation	117
22.8	Work/kinetic energy principle	117
22.9	Optional: Proofs with power	118
22.9.1	Optional: Proof of the power/kinetic energy-rate principle	118
22.9.2	Optional: Proof for power of a set of forces (<u>or torque</u>) on a rigid body B	118
23	Potential energy and energy conservation	119
23.1	Work/kinetic energy principle (summarized from Section 22.8)	119
23.2	Conservation of mechanical energy (special case of work/kinetic energy)	119
23.3	Potential energy - a special type of work for “conservative” forces	120
23.3.1	Potential energy for a constant force or a uniform gravitational field	120
23.3.2	Potential energy for central forces (e.g., springs and inverse-square-law gravity)	121
23.3.3	Potential energy for a simple rotational spring	121
23.4	Optional: Proofs of potential energy	122
23.4.1	Optional: Proof of potential energy for a constant force in reference frame N	122
23.4.2	Optional: Proof of potential energy for a central force	122
24	MIPSI: Classic particle pendulum	123
24.1	Modeling the classic particle pendulum	123
24.2	Identifiers for the classic particle pendulum	124
24.3	Physics: Equations of motion of the classic particle pendulum	124
24.3.1	$\vec{F} = m\vec{a}$ for the particle pendulum	125
24.3.2	Angular momentum principle for the particle pendulum	125
24.3.3	Euler's rigid body equation for the particle pendulum	126
24.3.4	Kinetic energy for the particle pendulum	126
24.3.5	Power/kinetic energy-rate principle for the particle pendulum	126
24.3.6	Conservation of mechanical energy for the particle pendulum	127
24.3.7	Lagrange's method for the particle pendulum	127
24.4	Solution of the classic particle pendulum ODE	127
24.4.1	Numerical solution of pendulum ODE via MotionGenesis and/or MATLAB®	127
24.4.2	Optional: Exact (closed-form) solution of the classic particle pendulum ODE	128

24.4.3 Simplification and analytical solution of the classic particle pendulum ODE	128
24.5 Interpretation of results for the classic particle pendulum	128
25 Example: Inverted pendulum on cart	129
25.1 Kinematics (space and time)	129
25.2 Rotation matrix, angular velocity, angular acceleration	129
25.3 Position vectors, velocity, acceleration	130
25.4 Forces, moments, and 2D free-body diagrams (FBD)	131
25.5 Mass, center of mass, inertia (required by dynamics)	131
25.6 Newton/Euler laws of motion for A and B separately (inefficient)	131
25.7 Dynamics of a rigid body with simple angular velocity (special 2D case)	132
25.8 Optional: Angular momentum principle (2D alternative to Section 25.7)	132
25.9 Equations of motion via MG road-maps/D'Alembert (efficient)	132
25.10 Matrix form of equations of motion (for solution, controls, ...)	132
HOMEWORK	133
Homework 1: Vectors (basis independent)	135
Homework 2: Vector addition and dot/cross-products (with basis)	141
Homework 3: Optional/Advanced: Position vectors and geometry	147
Homework 4: Vector bases and rotation matrices	151
Homework 5: Vector differentiation	163
Homework 6: Angular velocity and angular acceleration	171
Homework 7: Velocity and acceleration	181
Homework 8: Constraints	197
Homework 9: Particle mass, translational/angular momentum, kinetic energy, etc.	203
Homework 10: Moments and products of inertia, dynamic Celt (rattleback)	221
Homework 11: Rigid body: Momentum, energy, and equations of motion.	231
Homework 12: Optional: Forces, force models, and statics	249
Homework 13: Optional: Moments, torques, and static equilibrium	257
Homework 14: Advanced: $\vec{F} = m\vec{a}$ for translational motion (MG road-maps)	269
Homework 15: Advanced: $\vec{M} = \frac{d\vec{H}}{dt} + \dots$ for rotational motion (MG road-maps)	275
Homework 16: Optional: Power, work, potential energy, conservation of energy	289
Homework 17: Optional: MIPS1 simulation project	299
Appendices and index	301
Appendix: Mass and geometry properties of common objects	301
Appendix of constants	306
Summary of equations	307
Bibliography	311