Homework 1. Chapters 2.
Basis independent vectors and their properties

Show work – except for ♣ fill-in-blanks (print .pdf from www.MotionGenesis.com ⇒ Textbooks ⇒ Resources).

1.1 ♣ Solving problems – what engineers do.

Understanding dynamics results from **doing** problems. Many problems herein guide you to help you synthesize processes (imitation). Please **do** these problems by yourself or with colleagues/instructors and use the textbook and other resources.

Confucius 500 B.C.
"I hear and I forget.
I see and I remember.
I ______ and I understand."

"By three methods we may learn wisdom:
First, by reflection, which is noblest;
Second, by imitation, which is easiest;
Third by experience, which is the bitterest."

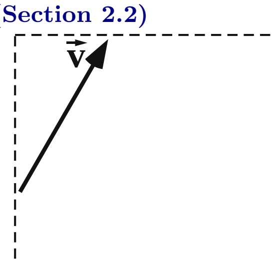
1.2 ♣ What is a vector (defined by Gibbs circa 1897)? (Section 2.2)
Two properties (attributes) of a vector are ______ and ______ (fill in the blanks).

1.3 ♣ What is a zero vector? (Section 2.3)
A zero vector $\vec{0}$ has a magnitude of **0/1/2/∞** (circle an answer).
A zero vector $\vec{0}$ has no direction. **True/False** (circle true or false).

1.4 ♣ Unit vectors. (Section 2.4)
A unit vector has a magnitude of **0/1/2/∞**.
All unit vectors are equal. **True/False**.

1.5 ♣ Draw the following vectors: (Section 2.2)
- Long, horizontally-right vector $\vec{a}$
- Short, vertically-upward vector $\vec{b}$
- Outwardly-directed **unit** vector $\hat{c}$.

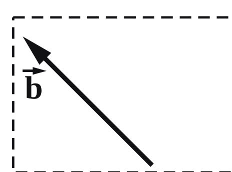
1.6 ♣ Optional: Vector magnitude and direction (orientation and sense). (Section 2.2)
The figure to the right shows a vector $\vec{v}$. **Draw** the following vectors.



$\vec{a}$ Same magnitude and direction as $\vec{v}$ ($\vec{a} = \vec{v}$).
$\vec{b}$ Same magnitude and orientation as $\vec{v}$, but different sense (direction).
$\vec{c}$ Same magnitude as $\vec{v}$, but different orientation (direction).
$\vec{d}$ Same direction as $\vec{v}$, but smaller magnitude.
$\vec{e}$ Different magnitude and different direction (orientation) as $\vec{v}$.

1.7 ♣ Magnitude of a vector. (Section 2.2)
Knowing x is a real positive number and $\hat{i}$ is a horizontally-right unit vector, the ***magnitude*** of the vector $-x\,\hat{i}$ is (circle **one**): positive negative non-negative non-positive.

1.8 ♣ Negating a vector. (Section 2.8)
Complete the figure to the right by **drawing** the vector $-\vec{b}$.

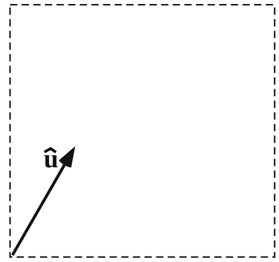


Negating the vector $\vec{b}$ results in a vector with different (circle **all** that apply):
magnitude direction orientation sense

Historical note: Negative numbers (e.g., −3) were not widely accepted until 1800 A.D.

1.9 ♣ Multiplying a vector by a scalar. (Section 2.7)

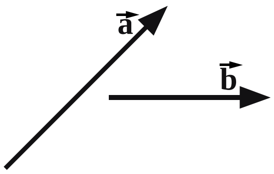
Complete the figure to the right by drawing the vectors $2\,\hat{\mathbf{u}}$ and $-2\,\hat{\mathbf{u}}$.



The following statements involve a unit vector $\hat{\mathbf{u}}$ and a real scalar s $(s \neq 0)$.
If a statement is true, provide a numerical value for s that supports your answer

$s\,\hat{\mathbf{u}}$ can have a different ***magnitude*** than $\hat{\mathbf{u}}$.	**True/False**	$s =$
$s\,\hat{\mathbf{u}}$ can have a different ***direction*** than $\hat{\mathbf{u}}$.	**True/False**	$s =$
$s\,\hat{\mathbf{u}}$ can have a different ***sense*** than $\hat{\mathbf{u}}$.	**True/False**	$s =$
$s\,\hat{\mathbf{u}}$ can have a different ***orientation*** than $\hat{\mathbf{u}}$.	**True/False**	$s =$

1.10 ♣ Graphical vector addition/subtraction - draw. (Sections 2.6,2.8)



Draw $\vec{\mathbf{a}} + \vec{\mathbf{b}}$

Draw $\vec{\mathbf{a}} - \vec{\mathbf{b}}$

1.11 ♣ Visual representation of a vector dot-product. (Section 2.9)

Write the ***definition*** of the dot-product of a vector $\vec{\mathbf{a}}$ with a vector $\vec{\mathbf{b}}$.
Include a ***sketch*** with ***each symbol*** in your definition clearly labeled.

Result: $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \triangleq$ Sketch should include $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, $|\vec{\mathbf{a}}|$, $|\vec{\mathbf{b}}|$, …

1.12 ♣ Visual representation of a vector cross-product. (Section 2.10)

Write the ***definition*** of the cross-product of a vector $\vec{\mathbf{a}}$ with a vector $\vec{\mathbf{b}}$.
Include a ***sketch*** with ***each symbol*** in your definition clearly labeled.

Result: $\vec{\mathbf{a}} \times \vec{\mathbf{b}} \triangleq$ $(\theta)\,\hat{\mathbf{u}}$ Sketch should include $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, $|\vec{\mathbf{a}}|$, $|\vec{\mathbf{b}}|$, …

where $\hat{\mathbf{u}}$ is

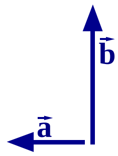
and θ is

1.13 ♣ Properties of vector dot-products and cross-products. (Sections 2.9.1 and 2.10)

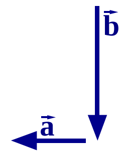
When $\vec{\mathbf{a}}$ is ***parallel*** to $\vec{\mathbf{b}}$,	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$	**True/False**	$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{0}}$	**True/False**
When $\vec{\mathbf{a}}$ is ***perpendicular*** to $\vec{\mathbf{b}}$,	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$	**True/False**	$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{0}}$	**True/False**
For arbitrary vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$,	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{a}}$	**True/False**	$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{a}}$	**True/False**

1.14 ♣ Calculating vector dot-products and cross-products via definitions. (Sections 2.9 and 2.10)

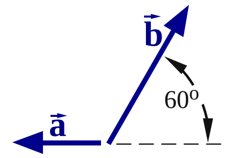
Draw a unit vector $\hat{\mathbf{k}}$ outward-normal to the plane of the paper (perpendicular to vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$).
Knowing $|\vec{\mathbf{a}}| = 2$ and $|\vec{\mathbf{b}}| = 4$, calculate the expressions below (2^+ significant digits) using only the **definitions** of dot-product and cross-product.



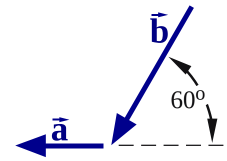
$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} =$ ____
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} =$ ____



$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} =$ ____
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} =$ ____



$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} =$ ____
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} =$ ____



$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} =$ ____
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} =$ ____

1.15 ♣ Vector exponentiation: $\vec{\mathbf{v}}^2$ and $\vec{\mathbf{v}}^3$. Complete the 3-step proofs. (Section 2.9)

Step 1: Complete the **definition** of $\vec{\mathbf{v}}^2$ in terms of $|\vec{\mathbf{v}}|$.
Step 2: Use the **definition** of the dot-product to show how $\vec{\mathbf{v}} \cdot \vec{\mathbf{v}}$ can be expressed in terms of $|\vec{\mathbf{v}}|$.
Step 3: Combine these two definitions to provide an alternate way to calculate $\vec{\mathbf{v}}^2$ with a vector dot-product.

Result: $\vec{\mathbf{v}}^2 \triangleq |\vec{\mathbf{v}}|^{\square}$ $\qquad \vec{\mathbf{v}} \cdot \vec{\mathbf{v}} \underset{(2.2)}{=}$ ____ $\qquad \vec{\mathbf{v}}^2 = \square \cdot \square$

Complete the 3-step proof that relates $\vec{\mathbf{v}}^3$ to $\vec{\mathbf{v}} \cdot \vec{\mathbf{v}}$ raised to a real number.

Result: $\vec{\mathbf{v}}^3 \triangleq |\vec{\mathbf{v}}|^{\square} \underset{(2.4)}{=} (\sqrt{})^{\square} = (\vec{\mathbf{v}} \cdot \vec{\mathbf{v}})^{\square}$

1.16 ♣ $|c\,\hat{\mathbf{a}}_x|$ Calculate vector magnitude with dot products. (Section 2.9 and Hw 1.15)

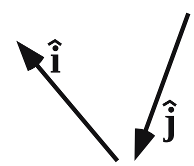
Show how the vector dot-product can be used to show that the magnitude of the vector $c\,\hat{\mathbf{a}}_x$ (c is a positive or **negative** number and $\hat{\mathbf{a}}_x$ is a unit vector) can be written solely in terms of c (without $\hat{\mathbf{a}}_x$).

Result: $|c\,\hat{\mathbf{a}}_x| = {}^+\sqrt{\square \cdot \square} = {}^+\sqrt{c^2 * \square \cdot \square} = {}^+\sqrt{c^2} = \text{abs}(c)$

1.17 † Magnitude of the vector $\vec{\mathbf{v}}$. **Show work.** (Section 2.9)

Knowing the angle between a unit vector $\hat{\mathbf{i}}$ and unit vector $\hat{\mathbf{j}}$ is 120°, calculate a numerical value for the magnitude of $\vec{\mathbf{v}} = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$.

Result: $|\vec{\mathbf{v}}| = \sqrt{13}$ $\qquad$ Note: The answer is *not* $\sqrt{25} = 5$.



1.18 ♣ Property of scalar triple product. (Section 2.11).

For arbitrary non-zero vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$: $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot \vec{\mathbf{c}}$ **Never/Sometimes/Always**
A property of the ***scalar triple product*** is $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \times \vec{\mathbf{a}} = 0$. **True/False**.

1.19 Property of vector triple cross-product. (Sections 2.10 and 2.11)

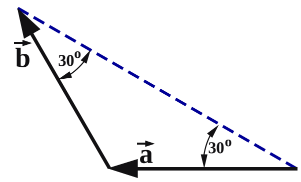
Complete the following equation: $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \vec{\mathbf{b}}() - \vec{\mathbf{c}}()$
For arbitrary vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$: $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}} + \vec{\mathbf{b}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{c}})$ **True/False** (show work).

1.20 ♣ Optional: Proof of magnitude of vector cross product property. (Sections 2.9 and 2.10)

Letting $\hat{\boldsymbol{\lambda}}$ be a ***unit vector*** and $\vec{\mathbf{v}}$ be ***any vector***, prove[1] $|\vec{\mathbf{v}} \times \hat{\boldsymbol{\lambda}}|^2 = \vec{\mathbf{v}} \cdot \vec{\mathbf{v}} - (\vec{\mathbf{v}} \cdot \hat{\boldsymbol{\lambda}})^2$.

[1] One way to prove this is to write $(\vec{\mathbf{v}} \times \hat{\boldsymbol{\lambda}})^2 = (\vec{\mathbf{v}} \times \hat{\boldsymbol{\lambda}}) \cdot (\vec{\mathbf{v}} \times \hat{\boldsymbol{\lambda}}) \underset{(2.11)}{=} \vec{\mathbf{v}} \cdot [\hat{\boldsymbol{\lambda}} \times (\vec{\mathbf{v}} \times \hat{\boldsymbol{\lambda}})]$ and then use the vector triple cross-product property $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \vec{\mathbf{b}}(\vec{\mathbf{a}} \cdot \vec{\mathbf{c}}) - \vec{\mathbf{c}}(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})$ from Section 2.10. Alternatively, it is helpful to write $\vec{\mathbf{v}} = \mathbf{v}_\perp \hat{\boldsymbol{\lambda}}_\perp + \mathbf{v}_{||} \hat{\boldsymbol{\lambda}}$ where $\mathbf{v}_\perp \hat{\boldsymbol{\lambda}}_\perp$ is the component of $\vec{\mathbf{v}}$ that is perpendicular to $\hat{\boldsymbol{\lambda}}$ and $\mathbf{v}_{||} \hat{\boldsymbol{\lambda}}$ is the component of $\vec{\mathbf{v}}$ that is parallel to $\hat{\boldsymbol{\lambda}}$.

1.21 ♣ Angle between vectors. **(Section 2.9)**



For the figure shown right, determine the numerical value for the angle between vector $\vec{a}$ and vector $\vec{b}$.

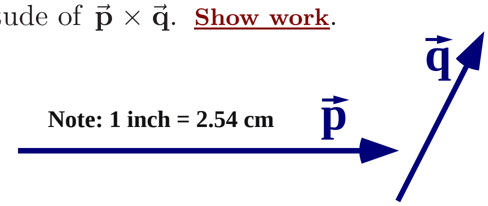
Result: $\angle(\vec{a},\ \vec{b}) = \_\_\_\_^\circ$

1.22 Visual estimation of vector dot/cross-products. **Show work.** **(Sections 2.9 and 2.10)**

Estimate (e.g., using your pinky) the magnitudes of the vectors $\vec{p}$ and $\vec{q}$ shown below.
Estimate the angle between $\vec{p}$ and $\vec{q}$, $\vec{p} \cdot \vec{q}$, and the magnitude of $\vec{p} \times \vec{q}$. **Show work**.

Result: (Provide numerical results with 1 or more significant digits).

$\lvert\vec{p}\rvert \approx$ ____ cm	$\lvert\vec{q}\rvert \approx$ ____ cm	$\angle(\vec{p},\ \vec{q}) \approx$ ____ °
$\vec{p} \cdot \vec{q} \approx$ ____ cm^2		$\lvert\vec{p} \times \vec{q}\rvert \approx$ ____ cm^2



1.23 ♣ Form the *unit* vector $\hat{u}$ having the same direction as $c\,\hat{a}_x$. **(Section 2.4)**

Result: $\hat{u} = \_\_\_\_\ \hat{a}_x$ Note: $\hat{a}_x$ is a unit vector and c is a non-zero real number, e.g., 3 or −3.

1.24 ♣ Coefficient of $\hat{u}$ in cross products – definitions and trig functions. **(Section 2.10)**

The ***cross product*** of vectors $\vec{a}$ and $\vec{b}$ can be written in terms of a real scalar s as $\vec{a} \times \vec{b} = s\,\hat{u}$ where $\hat{u}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ in a direction defined by the **right-hand rule**. The coefficient s of the unit vector $\hat{u}$ is inherently non-negative. **True/False**.

1.25 ♣ Insights into orthogonal vectors via drawing. **(Section 2.10)**

Draw unit vectors $\hat{a}$, $\hat{b}$, $\hat{c}$ such that:

- $\hat{b}$ is perpendicular to $\hat{a}$
- $\hat{c}$ is perpendicular to $\hat{b}$ but $\hat{c}$ is neither parallel or perpendicular to $\hat{a}$.

1.26 Calculating distance between a point and a line via cross-products. **(Section 2.10.1)**

Draw a horizontally-right unit vector $\hat{a}_x$ and vertically-upward unit vector $\hat{a}_y$.
Draw a point P and a line L through P that is parallel to $\hat{u} = \frac{3}{5}\,\hat{a}_x + \frac{4}{5}\,\hat{a}_y$.
Draw a point Q whose position vector from point P is $\vec{r} = 5\,\hat{a}_x$ (also **draw** $\vec{r}$).
Draw the ***distance*** d between Q and L. Calculate d with both formulas in eqn (2.9).

Result: $\text{d} \underset{(2.9)}{=} \_\_\_\_\_\_\_\_ = \boxed{4}$ $\text{d} \underset{(2.9)}{=} \_\_\_\_\_\_\_\_ = \boxed{4}$

1.27 ♣ Ranges of angles from dot-product and cross-product calculations. **(Sections 2.9 and 2.10)**

Given: Unit vectors $\hat{a}$ and $\hat{b}$ and the numerical values of $c \triangleq \hat{a} \cdot \hat{b}$ and $s \triangleq \lvert\hat{a} \times \hat{b}\rvert$.
Complete the following table with an appropriate range of numerical values.

Quantity	Range of values
c	____ $\le c \le$ ____
s	____ $\le s \le$ ____
Angle θ_c between $\hat{a}$ and $\hat{b}$ that can be uniquely determined **solely** from c	____° $\le \theta_c \le$ ____°
Angle θ_s between $\hat{a}$ and $\hat{b}$ that can be uniquely determined **solely** from s	____° $\le \theta_s \le$ ____°
Angle θ between $\hat{a}$ and $\hat{b}$	____° $\le \theta \le$ ____°

Note: The range of θ_s is smaller than the range for θ. Hence, s and θ_s are insufficient to correctly calculate θ.

1.28 ♣ Vector operations and units. **(Chapter 2)**

Circle the vector operations below (scalar multiplication, addition, dot-product, etc.) that are **defined** for a position vector $\vec{\mathbf{a}}$ (with **units** of m) and a velocity vector $\vec{\mathbf{b}}$ (with **units** of $\frac{\text{m}}{\text{s}}$).

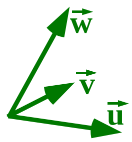
$-\vec{\mathbf{a}}$ $\quad 5\,\vec{\mathbf{a}}$ $\quad \vec{\mathbf{a}}\,/\,5$ $\quad \vec{\mathbf{a}} + \vec{\mathbf{b}}$ $\quad \vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$ $\quad \vec{\mathbf{a}} \times \vec{\mathbf{b}}$

1.29 ♣ Using vector identities to simplify expressions **(refer to Homework 1.13).**

One reason to treat vectors as ***basis-independent*** quantities is to simplify vector expressions ***without*** resolving the vectors into orthogonal "$\vec{\mathbf{x}}, \vec{\mathbf{y}}, \vec{\mathbf{z}}$" or "$\vec{\mathbf{i}}, \vec{\mathbf{j}}, \vec{\mathbf{k}}$" components. Simplify the following vector expressions using mathematical properties of dot-products and cross-products.

Express results in terms of dot-products $\cdot$ and cross-products $\times$ of the arbitrary vectors $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{w}}$.

$\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{w}}$ are not necessarily orthogonal or co-planar.



Vector expression	Simplified vector expression
$(3\,\vec{\mathbf{u}} - 2\,\vec{\mathbf{v}}) \times (\vec{\mathbf{u}} + \vec{\mathbf{v}})$	$\_\_\;\vec{\mathbf{u}} \times \vec{\mathbf{v}}$
$(3\,\vec{\mathbf{u}} - 2\,\vec{\mathbf{v}}) \cdot (\vec{\mathbf{u}} + \vec{\mathbf{v}})$	$\_\_\;\vec{\mathbf{u}}^2 - \_\_\;\vec{\mathbf{v}}^2 + \_\_\;\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$
$(\vec{\mathbf{u}} - \vec{\mathbf{v}}) \cdot (\vec{\mathbf{u}} + \vec{\mathbf{v}})$	$\_\_ - \_\_$
$(3\,\vec{\mathbf{u}} - 2\,\vec{\mathbf{v}}) \times (\vec{\mathbf{u}} + \vec{\mathbf{v}}) \cdot (2\,\vec{\mathbf{u}} - 7\,\vec{\mathbf{v}})$	$\_\_$
$(\vec{\mathbf{u}} + \vec{\mathbf{v}}) \times (\vec{\mathbf{v}} + 2\,\vec{\mathbf{w}}) \cdot (\vec{\mathbf{w}} + 2\,\vec{\mathbf{u}})$	$\_\_\;\vec{\mathbf{u}} \times \vec{\mathbf{v}} \cdot \vec{\mathbf{w}}$

1.30 ♣ Vector concepts: Solving a vector equation. **(Section 2.9.5)**

Shown right is a vector equation and a process that solves for $\dot{\theta}$
($\widehat{\mathbf{a}}_{\text{x}}$ is a unit vector and v_x, $\dot{\theta}$, R are scalars).

This is a valid process to solve for $\dot{\theta}$. **True/False**.

$$v_x\,\widehat{\mathbf{a}}_{\text{x}} = \dot{\theta}\,R\,\widehat{\mathbf{a}}_{\text{x}}$$

$$\dot{\theta} = \frac{v_x\,\widehat{\mathbf{a}}_{\text{x}}}{R\,\widehat{\mathbf{a}}_{\text{x}}} = \frac{v_x}{R}$$

Explain: __________

1.31 Changing a vector equation to scalar equations. **(Section 2.9.5)**

Draw three mutually orthogonal unit vectors $\widehat{\mathbf{p}}$, $\widehat{\mathbf{q}}$, $\widehat{\mathbf{r}}$.

Use a vector operation (e.g., $+$, $*$, $\cdot$, $\times$) to change the **vector** equation $(2x-4)\,\widehat{\mathbf{p}}$ into **one scalar** equation and subsequently solve the scalar equation for x.

Result: $\quad (2x-4)\,\widehat{\mathbf{p}} = \vec{\mathbf{0}} \quad \overset{??}{\Rightarrow} \quad (2x-4) = 0 \quad \Rightarrow \quad x = 2$

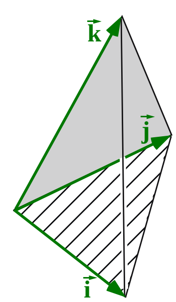
Show ***every*** vector operation (e.g., $+$, $*$, $\cdot$, $\times$) that changes the following **vector** equation into **three scalar** equations and subsequently solve the scalar equations for x, y, z.

$$(2\,x-4)\,\widehat{\mathbf{p}} + (3\,y-9)\,\widehat{\mathbf{q}} + (4\,z-16)\,\widehat{\mathbf{r}} = \vec{\mathbf{0}}$$

Result: $\quad (2\,x-4) = 0 \quad (3\,y-9) = 0 \quad (\_\_\_) = 0$

$\quad x = 2 \quad y = 3 \quad z = 4$

†Optional: The figure to the right shows three **non-orthogonal**, non-coplanar vectors $\vec{\mathbf{i}}, \vec{\mathbf{j}}, \vec{\mathbf{k}}$. Show **every** vector operation that changes the following **vector** equation into **three** uncoupled **scalar** equations and subsequently solve those scalar equations for x, y, z.



$$(2\,x-4)\,\vec{\mathbf{i}} + (3\,y-9)\,\vec{\mathbf{j}} + (4\,z-16)\,\vec{\mathbf{k}} = \vec{\mathbf{0}}$$

Result: $\quad (2\,x-4) = 0 \quad (3\,y-9) = 0 \quad (\_\_\_) = 0$

$\quad x = 2 \quad y = 3 \quad z = 4$

Hint: think $\times$ $\cdot$, not matrix algebra.

1.32 ♣ Number of independent scalar equations from 1 vector equation. (Section 2.9.5)

The **vector** equation shown right is useful for static analyses of a system S. $\vec{\mathbf{F}}^S = \vec{\mathbf{0}}$

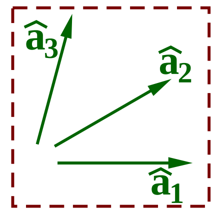
In the table to the right, box **all** integers that could be equal to the number of ***independent scalar*** equations produced by the previous vector equation. Hint: Hw 1.31. Related Hw 13.7.

System type	Integer(s)
1D (line)	0 1 2 3 4^+
2D (planar)	0 1 2 3 4^+
3D (spatial)	0 1 2 3 4^+

Note: 1D/linear means $\vec{\mathbf{F}}^S$ can be expressed in terms of one vector $\widehat{\mathbf{i}}$.
2D/planar means $\vec{\mathbf{F}}^S$ can be expressed in terms of two non-parallel unit vectors $\widehat{\mathbf{i}}$ and $\widehat{\mathbf{j}}$.
3D/spatial means $\vec{\mathbf{F}}^S$ can be expressed in terms of three non-coplanar unit vectors $\widehat{\mathbf{i}}$, $\widehat{\mathbf{j}}$, $\widehat{\mathbf{k}}$.

1.33 ♣ Vector concepts: Solving a vector equation (just circle true or false and fill-in the blank).

Consider the following vector equation written in terms of the scalars x, y, z and three unique non-orthogonal **coplanar** unit vectors $\widehat{\mathbf{a}}_1$, $\widehat{\mathbf{a}}_2$, $\widehat{\mathbf{a}}_3$.



$$(2\,x - 4)\,\widehat{\mathbf{a}}_1 \;+\; (3\,y - 9)\,\widehat{\mathbf{a}}_2 \;+\; (4\,z - 16)\,\widehat{\mathbf{a}}_3 \;=\; \vec{\mathbf{0}}$$

The **unique** solution to this vector equation is $x = 2$, $y = 3$, $z = 4$. **True/False**.

Explain: $\widehat{\mathbf{a}}_2$ can be expressed in terms of $\widehat{\mathbf{a}}_1$ and $\widehat{\mathbf{a}}_3$ (i.e., $\widehat{\mathbf{a}}_2$ is a linear combination of $\widehat{\mathbf{a}}_1$ and $\widehat{\mathbf{a}}_3$).
Hence the vector equation produces ______ linearly independent scalar equations.

1.34 ♣ Concept: A vector revolution for geometry. (Chapter 2)

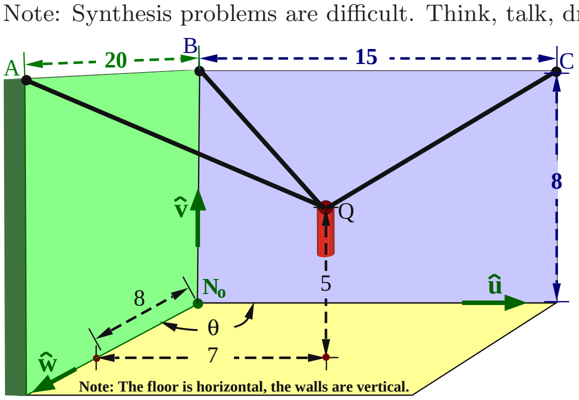
The relatively new invention of vectors (Gibbs ≈ 1900 AD) has revolutionized Euclidean geometry (Euclid ≈ 300 BC). For each geometrical quantity below, circle the vector operation(s) (either the dot-product, cross-product, or both) that is ***most*** useful for their calculation.

Length: $\cdot$ $\times$	Angle: $\cdot$ $\times$
Area: $\cdot$ $\times$	Volume: $\cdot$ $\times$

1.35 † Microphone cable lengths (non-orthogonal walls) "It's just geometry". Show work.

A microphone Q is attached to three pegs A, B, C by three cables. Knowing the peg locations, microphone location, and the angle θ between the vertical walls, express L_A, L_B, L_C solely in terms of numbers and θ. Next, complete the table by calculating L_B when $\theta = 120°$.

Hint: To do this **efficiently**, use only unit vectors $\widehat{\mathbf{u}}$, $\widehat{\mathbf{v}}$, $\widehat{\mathbf{w}}$, and (do **not** introduce an **orthogonal** set of unit vectors).
Hint: Use the distributive property of the vector dot-product as shown in Section 2.9.1 and Homework 2.4.
Note: Synthesis problems are difficult. Think, talk, draw, sleep, walk, get help, ... (if necessary, read Section 3.3).



Distance between A and B	20 m
Distance between B and C	15 m
Distance between N_o and B	8 m
Distance along back wall (see picture)	7 m
Q's height above N_o	5 m
Distance along side wall (see picture)	8 m
L_A: Length of cable joining A and Q	16.9 m
L_B: Length of cable joining B and Q	8.1 m
L_C: Length of cable joining C and Q	14.2 m

$$\vec{\mathbf{r}}^{\,Q/N_o} \;=\; 7\,\widehat{\mathbf{u}} \;+\; 5\,\widehat{\mathbf{v}} \;+\; 8\,\widehat{\mathbf{w}}$$

Result:

$L_A = \sqrt{202 \;-\; 168\,\cos(\theta)}$ $\qquad L_B =$ ______ $\qquad L_C = \sqrt{137 \;-\; \_\_\_\;\cos(\theta)}$

Show work – except for ♣ fill-in-blanks (print .pdf from www.MotionGenesis.com ⇒ Textbooks ⇒ Resources).

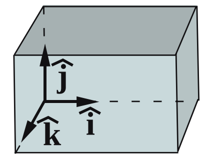
2.1 ♣ Right-handed, orthogonal, unitary, vector basis. (Section 4.1)

Draw a right-handed orthogonal (mutually perpendicular) vector basis consisting of the unit vectors $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$.

2.2 ♣ Adding and subtracting vectors with bases. (Sections 2.6 and 2.8)

Shown right are right-handed orthogonal unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ and vectors $\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}$.
Form the vector sums and differences below.

$$\vec{\mathbf{u}} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}$$
$$\vec{\mathbf{v}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$$
$$\vec{\mathbf{w}} = 5\,\hat{\mathbf{i}} - 6\,\hat{\mathbf{j}} + 7\,\hat{\mathbf{k}}$$



Result:

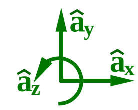
$$\vec{\mathbf{u}} + \vec{\mathbf{v}} = (2+x)\,\hat{\mathbf{i}} + \square\,\hat{\mathbf{j}} + \square\,\hat{\mathbf{k}} \qquad \vec{\mathbf{u}} - \vec{\mathbf{v}} = (2-x)\,\hat{\mathbf{i}} + \square\,\hat{\mathbf{j}} + \square\,\hat{\mathbf{k}}$$

2.3 ♣ Column matrices and vectors (Hint: What is a vector – Hw 1.2). (Section 2.12)

As defined by Gibbs, vectors have magnitude and direction.

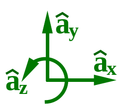
True/False: The vector $\hat{\mathbf{a}}_x + 2\,\hat{\mathbf{a}}_y + 3\,\hat{\mathbf{a}}_z$ is equal to the column matrix $\begin{bmatrix}1\\2\\3\end{bmatrix}$.

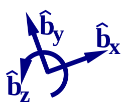
Note: $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ are the orthogonal unit vectors shown right.



True/False: Adding the following vectors and column matrices produce equivalent results.

Note: $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ and $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$ are the sets of orthogonal unit vectors shown below.



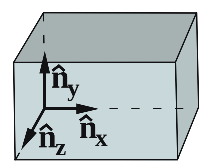


$$\hat{\mathbf{a}}_x + 2\,\hat{\mathbf{a}}_y + 3\,\hat{\mathbf{a}}_z \;+\; 4\,\hat{\mathbf{b}}_x + 5\,\hat{\mathbf{b}}_y + 6\,\hat{\mathbf{b}}_z \;=\; \begin{bmatrix}1\\2\\3\end{bmatrix} + \begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}5\\7\\9\end{bmatrix}$$

2.4 ♣ Calculating vector dot products with bases. (Sections 2.9 and 2.9.3)

Given: Right-handed orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ and:

$$\vec{\mathbf{u}} = 2\,\hat{\mathbf{n}}_x + 3\,\hat{\mathbf{n}}_y + 4\,\hat{\mathbf{n}}_z$$
$$\vec{\mathbf{v}} = x\,\hat{\mathbf{n}}_x + y\,\hat{\mathbf{n}}_y + z\,\hat{\mathbf{n}}_z$$
$$\vec{\mathbf{w}} = 5\,\hat{\mathbf{n}}_x - 6\,\hat{\mathbf{n}}_y + 7\,\hat{\mathbf{n}}_z$$



(a) Use the distributive law for dot products to write $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$ in terms of $\hat{\mathbf{n}}_x \cdot \hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_x \cdot \hat{\mathbf{n}}_y$, etc.
Result:

$$\begin{aligned}\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = \;& 2x\;\hat{\mathbf{n}}_x \cdot \hat{\mathbf{n}}_x + 2y\;\hat{\mathbf{n}}_x \cdot \hat{\mathbf{n}}_y + 2z\;\hat{\mathbf{n}}_x \cdot \hat{\mathbf{n}}_z\\ &+ 3x\;\hat{\mathbf{n}}_y \cdot \hat{\mathbf{n}}_x + 3y\;\square \cdot \square + \square\,\square \cdot \square\\ &+ \square\,\square \cdot \square + \square\,\square \cdot \square + \square\,\square \cdot \square\end{aligned}$$

(b) Use the definition of the dot product in equation (2.2) to calculate $\hat{\mathbf{n}}_x \cdot \hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_x \cdot \hat{\mathbf{n}}_y$, etc.
Result:

$\hat{\mathbf{n}}_x \cdot \hat{\mathbf{n}}_x = \square$	$\hat{\mathbf{n}}_x \cdot \hat{\mathbf{n}}_y = \square$	$\hat{\mathbf{n}}_x \cdot \hat{\mathbf{n}}_z = \square$
$\hat{\mathbf{n}}_y \cdot \hat{\mathbf{n}}_x = \square$	$\hat{\mathbf{n}}_y \cdot \hat{\mathbf{n}}_y = \square$	$\hat{\mathbf{n}}_y \cdot \hat{\mathbf{n}}_z = 0$
$\hat{\mathbf{n}}_z \cdot \hat{\mathbf{n}}_x = \square$	$\hat{\mathbf{n}}_z \cdot \hat{\mathbf{n}}_y = \square$	$\hat{\mathbf{n}}_z \cdot \hat{\mathbf{n}}_z = 1$

(c) In view of your previous two results, calculate $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$.
Result: $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = \square$

(d) Shown right (from Section 2.9.3) is a special formula for the dot product of arbitrary vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ when $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ are ***orthogonal unit*** vectors. Use this special formula to calculate:

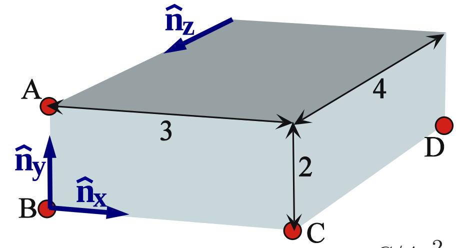
$$\vec{\mathbf{a}} = a_x\,\hat{\mathbf{n}}_x + a_y\,\hat{\mathbf{n}}_y + a_z\,\hat{\mathbf{n}}_z$$
$$\vec{\mathbf{b}} = b_x\,\hat{\mathbf{n}}_x + b_y\,\hat{\mathbf{n}}_y + b_z\,\hat{\mathbf{n}}_z$$
$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_x\,b_x + a_y\,b_y + a_z\,b_z$$

$$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = 2x + 3y + \square\,z \qquad \vec{\mathbf{u}} \cdot \vec{\mathbf{w}} = \square \qquad \vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = \square$$

2.5 ♣ Perpendicular vectors. **(Note: $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ are orthogonal unit vectors).** **(Section 2.9)**

When $\vec{\mathbf{v}} = x\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}} + 3\,\hat{\mathbf{k}}$ is perpendicular to $\vec{\mathbf{w}} = 4\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}} + 6\,\hat{\mathbf{k}}$, $x = \boxed{}$.

2.6 Dot product for calculating angles. **(Sections 2.9 and 3.3)**



The figure to the right shows a rectangular parallelepiped (block) of sides 2, 3, and 4 with points A, B, C located at corners. $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ are right-handed orthogonal unit vectors with $\hat{\mathbf{n}}_x$ directed from B to C and $\hat{\mathbf{n}}_y$ from B to A.

(a) Express $\vec{\mathbf{r}}^{\,C/A}$ (C's position from A) in terms of $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ and find a numerical value for $|\vec{\mathbf{r}}^{\,C/A}|^2$. Next, use equation (2.4) to calculate the magnitude of $\vec{\mathbf{r}}^{\,C/A}$ (the distance between A to C).

Result: $\vec{\mathbf{r}}^{\,C/A} = \boxed{}\,\hat{\mathbf{n}}_x - \boxed{}\,\hat{\mathbf{n}}_y$ $\quad |\vec{\mathbf{r}}^{\,C/A}|^2 = \vec{\mathbf{r}}^{\,C/A} \cdot \vec{\mathbf{r}}^{\,C/A} = \boxed{}$ $\quad |\vec{\mathbf{r}}^{\,C/A}| = \boxed{}$

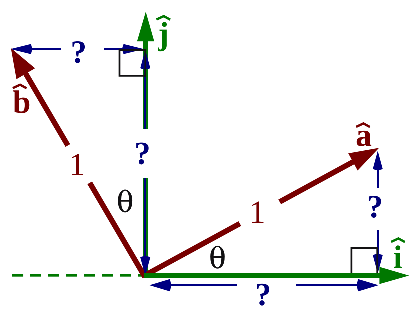
(b) Using equation (2.1), calculate the unit vector $\hat{\mathbf{u}}$ directed from A to C in terms of $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$. Next, find the unit vector $\hat{\mathbf{v}}$ directed from A to D in terms of $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$.

Result: $\hat{\mathbf{u}} = \dfrac{3\,\hat{\mathbf{n}}_x - 2\,\hat{\mathbf{n}}_y}{\sqrt{13}}$ $\qquad \hat{\mathbf{v}} = \dfrac{\boxed{}}{\boxed{}}$

(c) Calculate $\angle BAC$, the angle between line $\overline{AB}$ and line $\overline{AC}$. Next, calculate $\angle CAD$, the angle between line $\overline{AC}$ and line $\overline{AD}$.

Result: $\angle BAC = \boxed{}^\circ$ $\qquad \angle CAD = \boxed{47.97^\circ}$

2.7 ♣ Vector components: Sine and cosine. **(Section 1.4)**



Trigonometry plays a central role in ***rotation matrices***.
Replace each **?** in the figure to the right with $\sin(\theta)$ or $\cos(\theta)$.
Express unit vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ in terms of $\sin(\theta)$, $\cos(\theta)$, $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$.

Result:

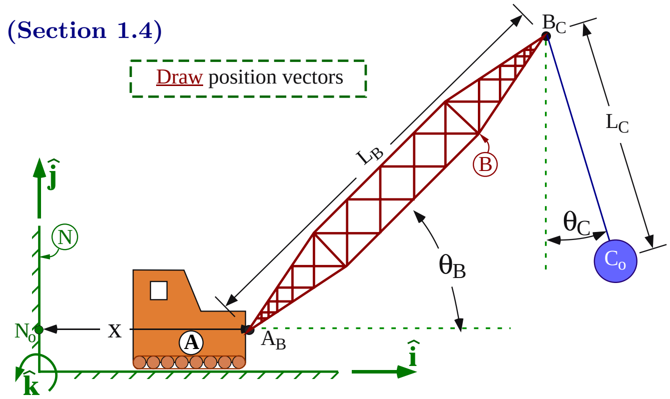
SohCahToa

$$\hat{\mathbf{a}} = \boxed{}\,\hat{\mathbf{i}} + \boxed{}\,\hat{\mathbf{j}}$$

$$\hat{\mathbf{b}} = \boxed{}\,\hat{\mathbf{i}} + \cos(\theta)\,\hat{\mathbf{j}}$$

2.8 ♣ Vector components for a crane-boom. (Section 1.4)

Shown right is a crane whose cab A supports a boom B that swings a wrecking ball C_o.



$\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are right-handed orthogonal unit vectors with $\hat{\mathbf{i}}$ horizontally-right, $\hat{\mathbf{j}}$ vertically-upward, and $\hat{\mathbf{k}}$ perpendicular to the plane containing points N_o, A_B, B_C, C_o.

Draw each position vector listed below and then use your knowledge of sine/cosine to resolve these vectors into $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ components.

A_B's position from N_o $\quad \vec{\mathbf{r}}^{A_B/N_o} = \square\,\hat{\mathbf{i}} + \square\,\hat{\mathbf{j}}$

B_C's position from A_B $\quad \vec{\mathbf{r}}^{B_C/A_B} = \square\,\hat{\mathbf{i}} + \square\,\hat{\mathbf{j}}$

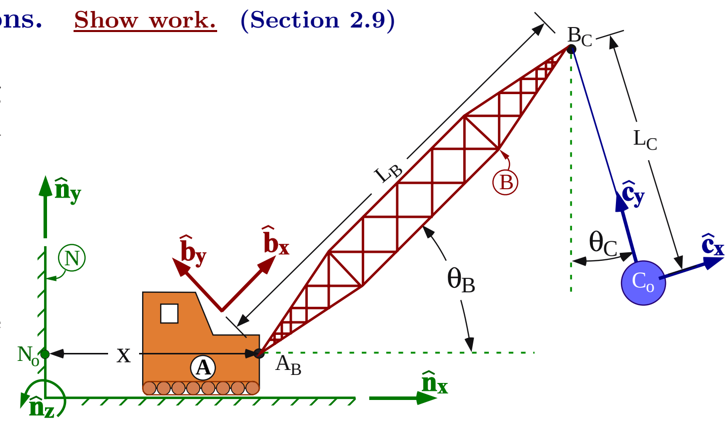
C_o's position from B_C $\quad \vec{\mathbf{r}}^{C_o/B_C} = \square\,\hat{\mathbf{i}} + \square\,\hat{\mathbf{j}}$

B_C's position from N_o $\quad \vec{\mathbf{r}}^{B_C/N_o} = \square\,\hat{\mathbf{i}} + \square\,\hat{\mathbf{j}}$

N_o's position from C_o $\quad \vec{\mathbf{r}}^{N_o/C_o} = \square\,\hat{\mathbf{i}} + [-L_B \sin(\theta_B) + L_C \cos(\theta_C)]\,\hat{\mathbf{j}}$

2.9 Dot products and distance calculations. **Show work.** (Section 2.9)

Shown right is a crane whose cab A supports a boom B that swings a wrecking ball C_o. To prevent the wrecking ball from hitting a car at point N_o, the distance between N_o and the tip of the boom (point B_C) must be controlled.



To start this problem, express B_C's position from N_o in terms of x, L_B, and the unit vectors $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{b}}_x$.

Result: $\vec{\mathbf{r}}^{B_C/N_o} = \square\,\hat{\mathbf{n}}_x + \square\,\hat{\mathbf{b}}_x$

(a) ***Without*** resolving $\vec{\mathbf{r}}^{B_C/N_o}$ into $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{n}}_y$ components (done in the next step), use equation (2.4) and the distributive property to calculate the distance between N_o and B_C in terms of x, L_B, θ_B. Then calculate its numerical value when $x = 20$ m, $L_B = 10$ m, $\theta_B = 30°$.

Result: (If necessary, complete the footnote hint below).[1]

Distance between N_o and B_C: $\quad |\vec{\mathbf{r}}^{B_C/N_o}| = \sqrt{\square + \square + \square} = 29.1$ m

(b) Two colleagues are confused by your use of ***mixed-bases*** vectors (i.e., $x\,\hat{\mathbf{n}}_x + L_B\,\hat{\mathbf{b}}_x$), and ask you to verify B's position from N_o can be expressed in the ***uniform-basis*** as shown below. Use this uniform-basis expression to verify your previous result for $|\vec{\mathbf{r}}^{B_C/N_o}|$.

Note: This inefficient uniform-basis approach requires the simplifying trigonometric identity $\sin^2(\theta_B) + \cos^2(\theta_B) = 1$.

Result: $\vec{\mathbf{r}}^{B_C/N_o} = [x + L_B \cos(\theta_B)]\,\hat{\mathbf{n}}_x + L_B \sin(\theta_B)\,\hat{\mathbf{n}}_y \qquad |\vec{\mathbf{r}}^{B_C/N_o}|$ simplifies to previous result.

(c) **Optional:** Calculate the distance between N_o and C_o in terms of x, L_B, L_C, θ_B, and θ_C.

Result: $|\vec{\mathbf{r}}^{C_o/N_o}| = \sqrt{x^2 + L_B^2 + L_C^2 + 2\,x\,L_B \cos(\theta_B) + 2\,x\,L_C \sin(\theta_C) - 2\,L_B\,L_C \sin(\theta_B - \theta_C)}$

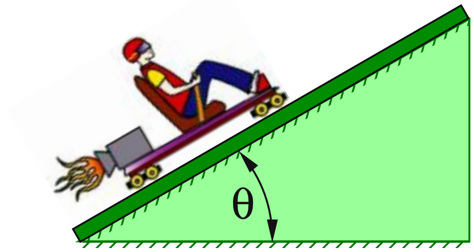
---

[1] Hint: Use the distributive property to express $\vec{\mathbf{r}}^{B_C/N_o} \cdot \vec{\mathbf{r}}^{B_C/N_o}$ in terms of x, L_B, and $\hat{\mathbf{n}}_x \cdot \hat{\mathbf{b}}_x$.
Next, use the ***dot-product definition*** in equation (2.2) to calculate $\hat{\mathbf{n}}_x \cdot \hat{\mathbf{b}}_x = \square$, and then rewrite
$\vec{\mathbf{r}}^{B_C/N_o} \cdot \vec{\mathbf{r}}^{B_C/N_o} = \square^2 + \square^2 + 2\,x\,L_B\,(\square \cdot \square) = \square^2 + \square^2 + 2\,x\,L_B \cos(\square)$.
Note: The distributive property for vector dot-multiplication is $(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{c}} + \vec{\mathbf{d}}) = \vec{\mathbf{a}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{a}} \cdot \vec{\mathbf{d}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{d}}$.

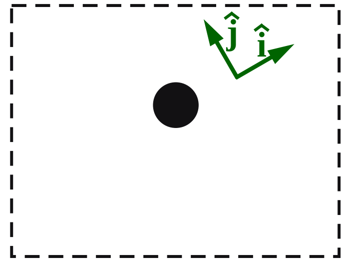
2.10 Vector components, free-body diagram (FBD), and motion graphs for a rocket-sled.

The following figure shows a rocket-sled moving along smooth (**frictionless**) inclined rails.

Description	Symbol	Type
Mass of rocket-sled and rider	m	Constant
Earth's gravitational acceleration	g	Constant
Angle between horizontal and inclined-rails	θ	Constant
$\hat{\mathbf{i}}$ measure of thrust force on sled	F_T	Specified
$\hat{\mathbf{j}}$ measure of normal force on sled	F_N	Variable
$\hat{\mathbf{i}}$ measure of rocket-sled position	x	Variable



Draw a unit vector $\hat{\mathbf{i}}$ upward-right and parallel to the rails.
Draw a unit vector $\hat{\mathbf{j}}$ outward-normal to the rails (perpendicular to $\hat{\mathbf{i}}$) and in the plane of the paper.



Free-body diagram
Draw a particle representing the rocket-sled.
Draw the thrust, normal, and gravity forces on the rocket-sled.
Express the net force on the rocket-sled in terms of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

Result: $\vec{\mathbf{F}}_{\text{Net}} = \vec{\mathbf{F}}_{\text{Thrust}} + \vec{\mathbf{F}}_{\text{Normal}} + \vec{\mathbf{F}}_{\text{gravity}}$
$= \_\_\_\_ \; \hat{\mathbf{i}} + \_\_\_\_ \; \hat{\mathbf{j}}$

$\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ The rocket-sled's acceleration can be calculated as $\vec{\mathbf{a}} = \frac{d^2x}{dt^2}\,\hat{\mathbf{i}} = \ddot{x}\,\hat{\mathbf{i}}$.
Substitute the right-hand sides of $\vec{\mathbf{F}}_{\text{Net}}$ and $\vec{\mathbf{a}}$ into $\vec{\mathbf{F}}_{\text{Net}} = m\,\vec{\mathbf{a}}$ and solve for $\ddot{x}$ and F_N.

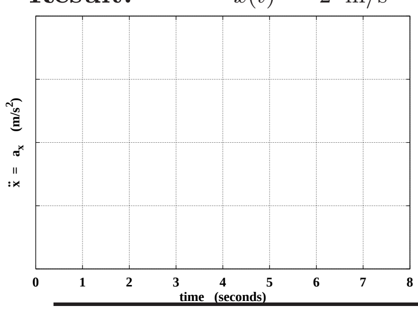
Result: $\_\_\_\_ \; \hat{\mathbf{i}} + \_\_\_\_ \; \hat{\mathbf{j}} = m\,\ddot{x}\,\hat{\mathbf{i}}$

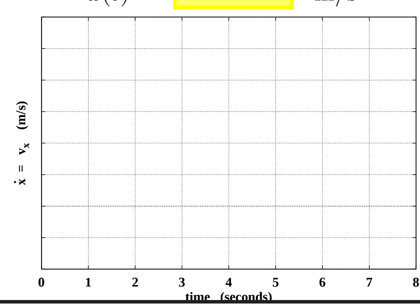
$\hat{\mathbf{i}}: \quad \ddot{x} = \dfrac{\_\_\_\_ - \_\_\_\_}{m} \qquad\qquad \hat{\mathbf{j}}: \quad F_N = \_\_\_\_$

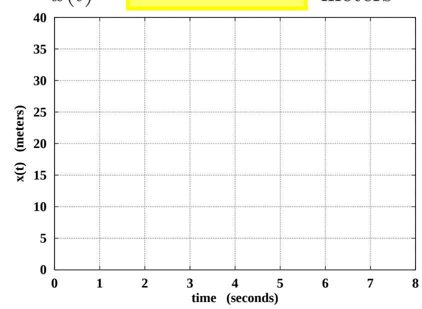
Solve for acceleration: Knowing $m = 100$ kg, $g = 10\ \frac{\text{m}}{\text{s}^2}$, $\theta = 30°$, $F_T = 700$ N, **graph** $\ddot{x}$.

Solve for velocity and position: Knowing the rocket-sled starts at $x = 8$ m, and is initially moving **downward-left** along the rail at $4\ \frac{\text{m}}{\text{s}}$, solve and **sketch** $\dot{x}(t)$ and $x(t)$ for $0 \le t \le 8$.

Result: $\ddot{x}(t) = 2\ \text{m/s}^2$ $\qquad \dot{x}(t) = \_\_\_\_$ m/s $\qquad x(t) = \_\_\_\_$ meters



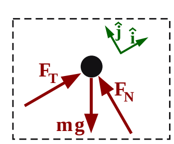




Include friction between the rails and rocket-sled, modeled via a coefficient of kinetic friction μ_k. Express $\ddot{x}$ and F_N in terms of some/all of μ_k and symbols in the table. Knowing $\mu_k \approx 0.115$, $x(0) = 8$ m, and the rocket-sled initially moves **upward-right** at $4\ \frac{\text{m}}{\text{s}}$, find $\dot{x}(t)$ and $x(t)$.

Result: $\ddot{x}(t) = \dfrac{\_\_\_\_}{m} \approx 1\ \frac{\text{m}}{\text{s}^2} \qquad F_N = \_\_\_\_ \qquad \dot{x}(t) \approx \_\_\_\_ \qquad x(t) \approx \_\_\_\_$

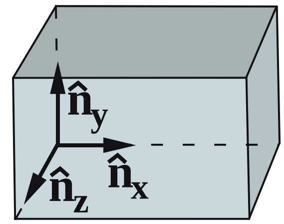
	Calculate the minimum thrust (redraw FBDs) to:	Result
a.	Keep the rocket-sled moving uphill at constant speed	$F_T \approx$ ____ N
b.	Keep the rocket-sled moving downhill at constant speed	$F_T \approx$ ____ N



For part **b**, assume the rocket is initially moving **downward-left** at 4 m/s.
Solution at www.MotionGenesis.com ⇒ Get Started ⇒ Rocket sled.

2.11 ♣ **Construct a unit vector $\hat{\mathbf{u}}$ in the direction of each vector given below.** (Section 2.9.2)

Vector	Unit vector $\hat{\mathbf{u}}$
$3\,\hat{\mathbf{n}}_x$	$\hat{\mathbf{n}}_x$
$-3\,\hat{\mathbf{n}}_x$	
$3\,\hat{\mathbf{n}}_x - 4\,\hat{\mathbf{n}}_y$	
$3\,\hat{\mathbf{n}}_x - 4\,\hat{\mathbf{n}}_y + 12\,\hat{\mathbf{n}}_z$	
$c\,\hat{\mathbf{n}}_x$ c is a real non-zero number	or



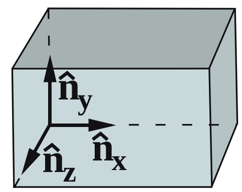
Note: $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ are orthogonal unit vectors.

Note: Ensure your last answer agrees with your first two answers, e.g., if $c = 3$ or $c = -3$.

2.12 ♣ **Calculating vector cross products with bases.** (Section 2.10)

Given: Right-handed orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ and:

$$\vec{\mathbf{u}} = 2\,\hat{\mathbf{n}}_x + 3\,\hat{\mathbf{n}}_y + 4\,\hat{\mathbf{n}}_z$$
$$\vec{\mathbf{v}} = x\,\hat{\mathbf{n}}_x + y\,\hat{\mathbf{n}}_y + z\,\hat{\mathbf{n}}_z$$
$$\vec{\mathbf{w}} = 5\,\hat{\mathbf{n}}_x - 6\,\hat{\mathbf{n}}_y + 7\,\hat{\mathbf{n}}_z$$



(a) Use the distributive law for cross products to write $\vec{\mathbf{u}} \times \vec{\mathbf{v}}$ in terms of $\hat{\mathbf{n}}_x \times \hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_x \times \hat{\mathbf{n}}_y$, etc.

Result: $\vec{\mathbf{u}} \times \vec{\mathbf{v}} = 2\,x\,\hat{\mathbf{n}}_x \times \hat{\mathbf{n}}_x + 2\,y\,\hat{\mathbf{n}}_x \times \hat{\mathbf{n}}_y + 2\,z\,\hat{\mathbf{n}}_x \times \hat{\mathbf{n}}_z$
$+ \quad\quad + \quad\quad +$
$+ \quad\quad + \quad\quad +$

(b) Use the definition of the cross product to calculate $\hat{\mathbf{n}}_x \times \hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_x \times \hat{\mathbf{n}}_y$, etc.

Result:

$\hat{\mathbf{n}}_x \times \hat{\mathbf{n}}_x = \vec{\mathbf{0}}$ $\quad$ $\hat{\mathbf{n}}_x \times \hat{\mathbf{n}}_y = \hat{\mathbf{n}}_z$ $\quad$ $\hat{\mathbf{n}}_x \times \hat{\mathbf{n}}_z = -\hat{\mathbf{n}}_y$

$\hat{\mathbf{n}}_y \times \hat{\mathbf{n}}_x =$ $\quad$ $\hat{\mathbf{n}}_y \times \hat{\mathbf{n}}_y =$ $\quad$ $\hat{\mathbf{n}}_y \times \hat{\mathbf{n}}_z =$

$\hat{\mathbf{n}}_z \times \hat{\mathbf{n}}_x =$ $\quad$ $\hat{\mathbf{n}}_z \times \hat{\mathbf{n}}_y =$ $\quad$ $\hat{\mathbf{n}}_z \times \hat{\mathbf{n}}_z =$

(c) In view of your previous two results, calculate $\vec{\mathbf{u}} \times \vec{\mathbf{v}}$.

Result: $\vec{\mathbf{u}} \times \vec{\mathbf{v}} = \quad\quad \hat{\mathbf{n}}_x + \quad\quad \hat{\mathbf{n}}_y + \quad\quad \hat{\mathbf{n}}_z$

2.13 **Cross products and determinants.** (Section 2.10.2)

Given arbitrary vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ expressed in terms of **right-handed orthogonal unit** vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ as shown right, show that calculating $\vec{\mathbf{a}} \times \vec{\mathbf{b}}$ with the distributive property of the cross product happens to be equal to the determinant of the matrix shown to the right.

$$\vec{\mathbf{a}} = a_x\,\hat{\mathbf{n}}_x + a_y\,\hat{\mathbf{n}}_y + a_z\,\hat{\mathbf{n}}_z$$
$$\vec{\mathbf{b}} = b_x\,\hat{\mathbf{n}}_x + b_y\,\hat{\mathbf{n}}_y + b_z\,\hat{\mathbf{n}}_z$$
$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \det \begin{bmatrix} \hat{\mathbf{n}}_x & \hat{\mathbf{n}}_y & \hat{\mathbf{n}}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix}$$

Next use this determinant method to calculate the following cross products (refer to Hw 2.12).

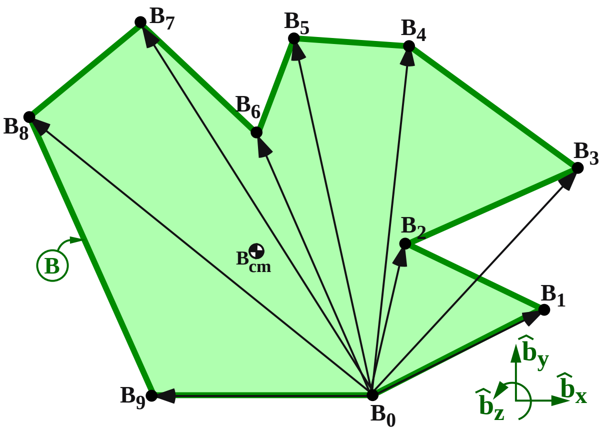
Result: $\vec{\mathbf{u}} \times \vec{\mathbf{v}} = (3\,z - 4\,y)\,\hat{\mathbf{n}}_x + (4\,x - 2\,z)\,\hat{\mathbf{n}}_y + (\quad\quad)\,\hat{\mathbf{n}}_z$

Optional: $\vec{\mathbf{u}} \times \vec{\mathbf{w}} = 45\,\hat{\mathbf{n}}_x + 6\,\hat{\mathbf{n}}_y - 27\,\hat{\mathbf{n}}_z$

Optional: $\vec{\mathbf{v}} \times \vec{\mathbf{w}} = (\quad\quad)\,\hat{\mathbf{n}}_x + (\quad\quad)\,\hat{\mathbf{n}}_y - (\quad\quad)\,\hat{\mathbf{n}}_z$

2.14 ♣ Cross products: Commercial algorithm for area calculations (surveying). (Section 2.10.1)

One reason triangles are important is that complex **planar objects** such as the polygon B below can be decomposed into triangles. Planar measurements are important in various professions, including farming acreage, building costs, and mass and area properties of 2D objects.



$$
\begin{aligned}
\vec{\mathbf{r}}^{\,B_1/B_0} &= 2.0\,\widehat{\mathbf{b}}_{\text{x}} + 2.0\,\widehat{\mathbf{b}}_{\text{y}}\\
\vec{\mathbf{r}}^{\,B_2/B_0} &= 0.5\,\widehat{\mathbf{b}}_{\text{x}} + 2.5\,\widehat{\mathbf{b}}_{\text{y}}\\
\vec{\mathbf{r}}^{\,B_3/B_0} &= 3.0\,\widehat{\mathbf{b}}_{\text{x}} + 4.0\,\widehat{\mathbf{b}}_{\text{y}}\\
\vec{\mathbf{r}}^{\,B_4/B_0} &= 0.2\,\widehat{\mathbf{b}}_{\text{x}} + 6.0\,\widehat{\mathbf{b}}_{\text{y}}\\
\vec{\mathbf{r}}^{\,B_5/B_0} &= -0.5\,\widehat{\mathbf{b}}_{\text{x}} + 7.0\,\widehat{\mathbf{b}}_{\text{y}}\\
\vec{\mathbf{r}}^{\,B_6/B_0} &= -1.0\,\widehat{\mathbf{b}}_{\text{x}} + 5.0\,\widehat{\mathbf{b}}_{\text{y}}\\
\vec{\mathbf{r}}^{\,B_7/B_0} &= -2.0\,\widehat{\mathbf{b}}_{\text{x}} + 7.0\,\widehat{\mathbf{b}}_{\text{y}}\\
\vec{\mathbf{r}}^{\,B_8/B_0} &= -4.0\,\widehat{\mathbf{b}}_{\text{x}} + 5.0\,\widehat{\mathbf{b}}_{\text{y}}\\
\vec{\mathbf{r}}^{\,B_9/B_0} &= -2.0\,\widehat{\mathbf{b}}_{\text{x}} + 0.0\,\widehat{\mathbf{b}}_{\text{y}}
\end{aligned}
$$

A commercial algorithm for calculating the area of the polygon B shown above is to:

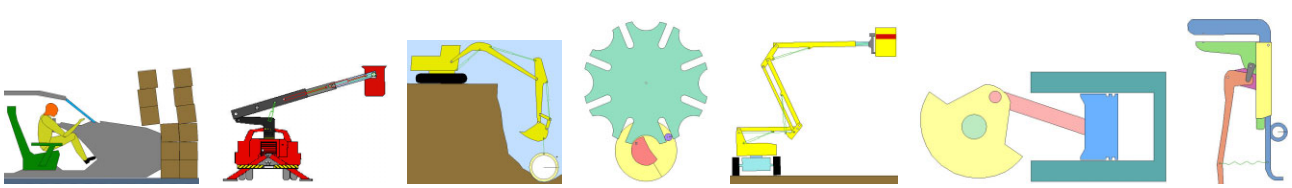
- Label a vertex B_0 and number the remaining vertices sequentially in a counter-clockwise fashion.
- Form $\vec{\mathbf{r}}^{\,B_i/B_0}$, B_i's position from B_0 $(i = 1, 2, \ldots)$
- Calculate $\vec{\mathbf{A}}_2$ and $\vec{\mathbf{A}}_4$, the "vector-areas" of the triangles defined by vertices $B_0\,B_2\,B_3$, and $B_0\,B_4\,B_5$, respectively. Formulas for the vector-areas of each triangle are given below along with the vector sum of these areas $\vec{\mathbf{A}}$ and the polygon's area (the magnitude of $\vec{\mathbf{A}}$).

Result: (Just fill in the blanks. You only need to calculate $\vec{\mathbf{A}}_2$, $\vec{\mathbf{A}}_4$, and $\mathbf{A}$)

$\vec{\mathbf{A}}_1$	=	$\frac{1}{2} * \vec{\mathbf{r}}^{\,B_1/B_0} \times \vec{\mathbf{r}}^{\,B_2/B_0}$	=	$2\,\widehat{\mathbf{b}}_{\text{z}}$
$\vec{\mathbf{A}}_2$	=	$\frac{1}{2} * \vec{\mathbf{r}}^{\,B_2/B_0} \times \vec{\mathbf{r}}^{\,B_3/B_0}$	=	
$\vec{\mathbf{A}}_3$	=	...	=	$8.6\,\widehat{\mathbf{b}}_{\text{z}}$
$\vec{\mathbf{A}}_4$	=	...	=	
$\vec{\mathbf{A}}_5$	=	...	=	$2.25\,\widehat{\mathbf{b}}_{\text{z}}$
$\vec{\mathbf{A}}_6$	=	...	=	$1.5\,\widehat{\mathbf{b}}_{\text{z}}$
$\vec{\mathbf{A}}_7$	=	...	=	$9\,\widehat{\mathbf{b}}_{\text{z}}$
$\vec{\mathbf{A}}_8$	=	$\frac{1}{2} * \vec{\mathbf{r}}^{\,B_8/B_0} \times \vec{\mathbf{r}}^{\,B_9/B_0}$	=	$5\,\widehat{\mathbf{b}}_{\text{z}}$
$\mathbf{A}$	=	$\sum_{i=1}^{8} \vec{\mathbf{A}}_{\text{i}}$	=	
Area	=	$\lvert\vec{\mathbf{A}}\rvert$	=	27.8

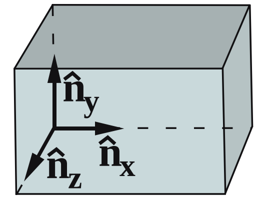
Accounting for overlapped areas is done with **positive** and **negative** signs on vectors.

Note: Compute cross products with the distributive property $(\vec{\mathbf{a}}+\vec{\mathbf{b}}) \times (\vec{\mathbf{c}}+\vec{\mathbf{d}}) = \vec{\mathbf{a}} \times \vec{\mathbf{c}} + \vec{\mathbf{a}} \times \vec{\mathbf{d}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{b}} \times \vec{\mathbf{d}}$ and the **cross-product-definition with the right-hand rule** (not determinants or special formulas in a book). Also, use the fact that $\widehat{\mathbf{b}}_{\text{x}}$, $\widehat{\mathbf{b}}_{\text{y}}$, $\widehat{\mathbf{b}}_{\text{z}}$ are orthogonal unit vectors.



Planar objects, courtesy of Working Model and Design-Simulation Technologies

2.15 ♣ Scalar triple product with bases (Section 2.11).
The figure shows right-handed orthogonal unit vectors $\widehat{\mathbf{n}}_\text{x}$, $\widehat{\mathbf{n}}_\text{y}$, $\widehat{\mathbf{n}}_\text{z}$.

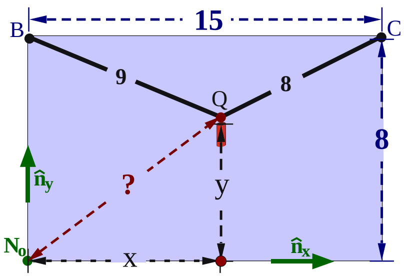


Given	Calculate
$\vec{\mathbf{u}} = 2\,\widehat{\mathbf{n}}_\text{x} + 3\,\widehat{\mathbf{n}}_\text{y} + 4\,\widehat{\mathbf{n}}_\text{z}$	$\vec{\mathbf{u}} \times \vec{\mathbf{v}} \cdot \vec{\mathbf{u}} = \square$
$\vec{\mathbf{v}} = x\,\widehat{\mathbf{n}}_\text{x} + y\,\widehat{\mathbf{n}}_\text{y} + z\,\widehat{\mathbf{n}}_\text{z}$	$\vec{\mathbf{u}} \times \vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = \square\, z - \square\, x - 6\,y$
$\vec{\mathbf{w}} = 5\,\widehat{\mathbf{n}}_\text{x} - 6\,\widehat{\mathbf{n}}_\text{y} + 7\,\widehat{\mathbf{n}}_\text{z}$	$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}} = \square\, z - 45\,x - \square\, y$

Note: Although the order of operations in $\vec{\mathbf{u}} \times \vec{\mathbf{v}} \cdot \vec{\mathbf{u}}$ is unambiguous, parentheses may clarify your work.

$\vec{\mathbf{u}} \times \vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}}$ and it is OK to switch $\cdot$ and $\times$ in scalar triple products. **True/False**.

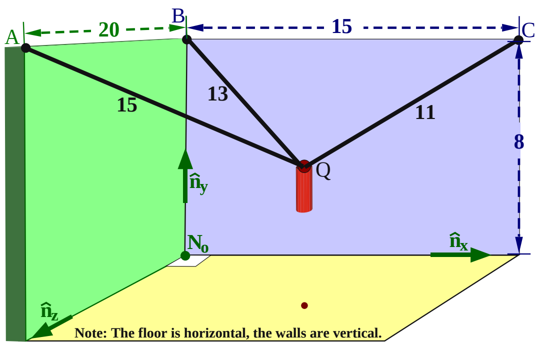
2.16 Locating a microphone (2D). Show work. (Section 1.4)
A microphone Q is attached to two pegs B and C by two cables. Knowing the peg locations, cable lengths, and points B, C, Q, N_o all lie in the same plane, determine the distance between Q and N_o.
Do the problem with Euclidean geometry (law of cosines $2x$), then try vectors (see Hw 1.35).



Quantity	Value
Distance between B to C	15 m
Distance between N_o to B	8 m
Length of cable joining B and Q	9 m
Length of cable joining C and Q	8 m
Distance between N_o and Q If Q is above ceiling, distance ≈ 12 m	9.01 m

Note: Although there are two mathematical answers to this problem, one is above the ceiling and requires the cables to be in compression.

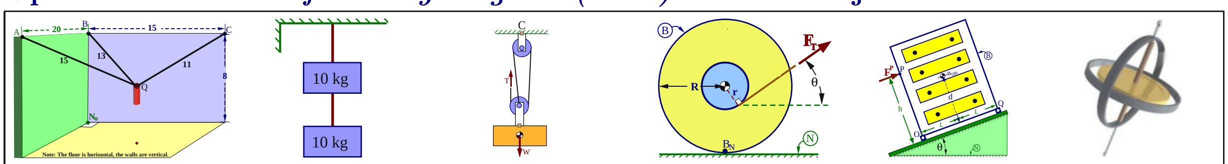
2.17 † Locating a microphone (3D).
A microphone Q is attached to three pegs A, B, and C by three cables. Knowing the peg locations and cable lengths, determine the distance between Q and point N_o. Show work. (If needed, hint below).[2]



Quantity	Value
Distance between A to B	20 m
Distance between B to C	15 m
Distance between N_o to B	8 m
Length of cable joining A and Q	15 m
Length of cable joining B and Q	13 m
Length of cable joining C and Q	11 m
Distance between N_o and Q If Q is above ceiling, distance ≈ 17 m	13.3 m

Note: This is part of the process of a camera targeting a football/baseball in a stadium or laser targeting cancer or …

2.18 Optional: Draw the *free-body diagram (FBD)* for each object below.



Particle Q	Top block	Bottom pulley	Rolling spool B	Bureau B	Entire system
Hw 2.17	Hw 12.12	Hw 12.14	Hw 13.12	Hw 13.8	Hw 11.15

[2]Hint: See Homework 1.35 or Section 3.3. Introduce unknowns x, y, z so Q's position from N_o is $x\,\widehat{\mathbf{n}}_\text{x} + y\,\widehat{\mathbf{n}}_\text{y} + z\,\widehat{\mathbf{n}}_\text{z}$.
Although nonlinear algebraic equations are usually solved with a computer, these can also be solved "by-hand".
Solution at www.MotionGenesis.com. Alternatively, www.WolframAlpha.com solves sets of nonlinear equations, e.g., type
Solve `x^2 + (-20+z)^2 + (-8+y)^2 = 225,   x^2 + z^2 + (-8+y)^2 = 169,   z^2 + (-15+x)^2 + (-8+y)^2 = 121`

