

Contents

Statics and dynamics – what does it do for us?	i
Newton’s three laws	ii
1 Math review	1
1.1 Unit systems - SI and U.S.	1
1.2 Geometry: Ancient Euclid and modern vectors	2
1.3 Circles and their properties	2
1.4 Triangles and ratios of their sides (sine, cosine, tangent)	2
1.4.1 Formulas involving sine and cosine	3
1.4.2 Sine and cosine as functions (Euler, circa 1730)	3
1.5 Types of scalars: Variable, specified, constant	3
1.6 Differentiation	4
1.6.1 Definition of an ordinary derivative of a scalar function	4
1.6.2 Definition of a partial derivative of a scalar function	4
1.6.3 Definition of the total derivative of a scalar function	4
1.6.4 Short table of derivatives frequently encountered in engineering	5
1.6.5 Example: Partial and ordinary differentiation	5
1.6.6 Good product rule for differentiation (for scalars, \vec{v} ectors, matrices,)	5
1.6.7 Quotient rule for derivatives: Use exponents and the product rule	5
1.6.8 Chain rule for derivatives	5
1.6.9 Implicit differentiation: A useful tool for calculating derivatives	6
1.7 Integration and a short table of integrals	6
1.8 Solutions of <i>polynomial</i> equations (roots)	7
1.9 Computer solutions of algebraic and differential equations	7
1.10 Optional: Continuous solutions of <i>nonlinear</i> algebraic equations	7
2 Vectors	9
2.1 Examples of scalars, vectors, and dyadics	9
2.2 Definition of a vector	10
2.3 Zero vector $\vec{0}$ – a vector whose magnitude is zero	10
2.4 Unit \hat{v} ectors – a vector whose magnitude is 1 (typeset with a special \hat{h} at)	10
2.5 Equal vectors (=) vectors with the same magnitude and direction	11
2.6 Vector addition (+)	11
2.7 Vector multiplied or divided by a scalar (* or /)	11
2.8 Vector negation and subtraction (-)	12
2.9 Vector dot product (\cdot)	12
2.9.1 Properties of the dot-product (\cdot)	12
2.9.2 Uses for the dot-product (\cdot)	13
2.9.3 Special case: Dot-products with orthogonal unit vectors	13
2.9.4 Examples: Vector dot-products (\cdot)	13
2.9.5 Dot-products to change vector equations to scalar equations (see Hw 1.31)	13

2.10	Vector cross product (\times)	14
2.10.1	Uses for the cross-product (\times) in geometry, statics, motion analysis,	14
2.10.2	Determinants and cross-products (with right-handed unit vectors)	15
2.11	Optional: Scalar triple product ($\cdot \times$ or $\times \cdot$)	16
2.11.1	Scalar triple product and the volume of a tetrahedron	16
2.11.2	($\times \cdot$) to change vector equations to scalar equations (see Hw 1.31)	16
2.12	Optional: Vectors vs. column matrices in the context of $\vec{F} = m \vec{a}$	16
3	Position vectors and vector geometry	17
3.1	Position of a point (or particle) (see examples in Hw 3)	17
3.2	Distance	18
3.3	Example: Cable lengths, angles, surface area (orthogonal walls)	19
4	Vector basis	21
4.1	What is a vector basis?	21
4.2	Expressing a vector in terms of the basis $\vec{a}_1, \vec{a}_2, \vec{a}_3$	21
4.3	Concept: What is the vector vs. how is it expressed	22
5	Rotation matrices	23
5.1	Uses for the rotation matrix ${}^aR^b$ (for geometry, statics, motion analysis, stress)	23
5.2	Simple rotation matrix examples	24
5.2.1	Example: Calculating angles between unit vectors $\hat{a}_x, \hat{a}_y, \hat{a}_z$ and $\hat{b}_x, \hat{b}_y, \hat{b}_z$	24
5.2.2	Example: Calculation of rotation matrix inverse (use transpose)	24
5.2.3	Example: Rotation matrix for express, dot, and cross	24
5.3	Forming rotation matrices and matrix multiplication	25
5.3.1	Example: Forming the simple rotation matrix ${}^aR^n$	25
5.3.2	Hug rule (a pattern for quickly forming simple rotation matrices)	25
5.3.3	Example: Forming the simple rotation matrix ${}^bR^a$	26
5.3.4	Example: Rotation matrix multiplication to form ${}^bR^n = {}^bR^a * {}^aR^n$	26
5.4	What is an angle?	27
6	Vector differentiation	29
6.1	Differentiation concepts: Changes in magnitude and direction	29
6.2	Definition: Derivative of a vector in a rigid basis or reference frame	30
6.3	What is a constant vector (i.e., vector fixed in a reference frame)?	30
6.4	Properties of ordinary <u>or</u> partial derivatives of vectors	30
6.5	Example: Derivatives of a vector	31
7	Angular velocity & angular acceleration	33
7.1	Angular velocity concepts: Moon and Earth celestial systems	33
7.2	What is a reference frame?	34
7.2.1	What is a Newtonian (inertial/fixed) reference frame? $\vec{F} = m \vec{a}$	34
7.2.2	Orientation of a rigid body (or reference frame)	34
7.2.3	Optional: Differences between a vector basis, rigid frame, and reference frame)	34
7.3	Angular velocity and the golden rule for vector differentiation	35
7.3.1	Example: Angular velocity and vector differentiation	35
7.3.2	Example: Vector differentiation and angular momentum – spinning body	35
7.3.3	Simple angular velocity	36
7.3.4	Angular velocity negative property	36
7.3.5	Angular velocity addition theorem	36
7.3.6	Angular velocity example: Chaotic plate pendulum	37
7.4	Angular acceleration	37

7.5	Optional: Angular velocity proofs	38
7.5.1	Proof of angular velocity and orthogonal basis vectors	38
7.5.2	Proof of simple angular velocity	38
7.5.3	Proof of angular velocity addition theorem	39
7.5.4	Proof of angular velocity negative property	39
8	Points: Velocity and acceleration	41
8.1	Definition of a point's velocity and acceleration	41
8.2	Velocity and acceleration of two points <u>fixed</u> on a rigid body	43
8.3	Relationship between distance, position, velocity, and acceleration	44
8.3.1	Constant acceleration along a curve	44
8.3.2	Circular motion	44
8.3.3	Analogy between translational motion and planar (2D) rotational motion	44
8.3.4	Example: Free-fall of a sky-diver with constant downward acceleration	45
8.4	Speed and distance-traveled in a reference frame (see Hw 9.11)	45
8.5	Optional: Acceleration vocabulary	46
8.6	Optional: How g 's affect human health (biomechanics)	46
8.7	Optional: Velocity and acceleration proofs	47
9	Constraints: Rods, rolling, gears,	49
9.1	Rods, ropes, and separators	49
9.2	Linear actuator	49
9.3	Ball and socket joint	50
9.4	Revolute joint (also called a hinge or pin joint)	50
9.5	Revolute motor (revolute joint with additional angular constraint)	50
9.6	Prismatic joint (also called a slider joint or square-slot joint)	50
9.7	Contact (an intermittent inequality constraint)	51
9.8	Sliding and rolling (a special type of contact)	51
9.9	Gears and constraint equations	52
10	Particles (points with mass)	55
10.1	System of particles	56
10.2	Linear (translational) momentum for particles	56
10.3	Angular momentum and shift theorem for particles	57
10.4	Kinetic energy for particles	58
10.5	Example: Momentum and energy for a child on a swing	59
10.6	Forming and solving $\vec{\mathbf{F}} = m \vec{\mathbf{a}}$ for translational motion	60
10.7	Example: Velocity, acceleration, and $\vec{\mathbf{F}} = m \vec{\mathbf{a}}$	61
10.8	Optional: Proofs for linear/angular momentum	62
10.8.1	Proof of linear momentum of a system of particles	62
10.8.2	Proof of shift theorem for angular momentum	62
11	Mass, center of mass, centroid	63
11.1	Mass	63
11.2	Center of mass and centroid	63
11.3	Optional: Figures and their length, area, and volume	65
11.4	Optional: Integrals for mass, center of mass, and centroid	65
11.5	Experiments, tables, CAD/CAE, and medical scanning	66
11.6	Optional: Measuring mass	66
11.7	Optional: The language and etymology of "mass"	66

12 Concepts: Moments/products of inertia	67
12.1 Moment of inertia (also see Section 14.2)	67
12.1.1 Example: Moments of inertia of a particle (repeated in Hw 10.2)	68
12.1.2 Example: Moment of inertia concepts (repeated in Hw 10.3)	68
12.1.3 Demonstration: How moment of inertia affects a rolling cylinder	68
12.1.4 Demo: How moment of inertia affects a spinning book (repeated in Hw 10.4)	68
12.1.5 Shift theorem for moment of inertia (parallel axis theorem)	69
12.2 Products of inertia (also see Section 14.3)	69
12.2.1 Example: Products of inertia of a particle (repeated in Hw 10.2)	69
12.2.2 Conceptual example of products of inertia (repeated in Hw 10.9)	70
12.2.3 Demo: How product of inertia affects a rotating rattleback (repeated in Hw 10.18)	70
12.2.4 Optional: Two sign conventions (\pm) for products of inertia	70
13 Dyadics	71
13.1 Dyadics and matrices	72
13.2 Dyadic symmetry	72
14 Inertia dyadics	73
14.1 Inertia dyadic: A “suitcase” for moments and products of inertia	73
14.1.1 Inertia dyadic for a particle or system	73
14.1.2 Shift theorem for an inertia dyadic (for an arbitrary system S)	73
14.1.3 Example: Inertia dyadic of a particle (also see example in Section 14.5)	74
14.2 Moments of inertia and inertia dyadics (also see Section 12.1)	74
14.3 Products of inertia and inertia dyadics (also see Section 12.2)	75
14.4 Inertia matrix (a symmetric matrix)	75
14.5 Example: Inertia properties for a child on a swing	76
14.6 Example: Mass properties of three particles on a parallelepiped	77
14.7 Optional: Motivating concepts for inertia dyadics	78
15 Rigid bodies	79
15.1 What is a rigid body? (see examples in Hw 11, laws of motion in Chapter 20).	79
15.2 Angular momentum of a rigid body (also see Sections 20.4, 20.9)	80
15.3 Kinetic energy of a rigid body	81
15.3.1 Example: Kinetic energy of inverted pendulum on cart	81
15.4 Example: Momentum, kinetic energy (simple aircraft)	82
15.5 Optional: Proof of angular momentum for a rigid body B	83
15.6 Optional: Proof of kinetic energy for a rigid body B	84
16 Force and resultant	85
16.1 Force and the law of action/reaction	85
16.2 Statics, dynamics, and resultant forces	86
16.3 Resultant of a set of vectors (e.g., forces on a point, body, or system)	86
16.4 Resultant force on a system (internal force cancellation)	86
16.5 Contact and distance forces	87
16.6 Applied and constraint forces	87
16.7 Optional: The definition and philosophy of force	87
17 Moment and torque	89
17.1 Moment of a vector	89
17.1.1 Moment of a set of vectors	89
17.1.2 Shift theorem for the moment of a set of vectors	89
17.1.3 $\hat{\mathbf{u}} \cdot \vec{\mathbf{M}}^{S/P} = \hat{\mathbf{u}} \cdot \vec{\mathbf{M}}^{S/O}$ if $\hat{\mathbf{u}}$ is parallel to line \overline{OP} (useful for MG road-maps in Section 21.1)	90

17.1.4	Moment arm of a vector about a point (example at end of Section 17.6)	90
17.2	Moment of internal forces (a separate law of mechanics?)	90
17.3	Statics, dynamics, and moments of forces	90
17.4	Definition of static equilibrium	90
17.5	Torque of a set of vectors (moment of a couple)	91
17.6	Neuromuscular biomechanics example: Muscle tension for curling	92
17.7	Optional: Proof of shift theorem for moment of a set of vectors	94
17.8	Optional: Minimum moment magnitude and central axis (wrench)	94
18	Replacement of forces and bound vectors	95
18.1	Replacement of distance forces (not a unique process)	95
18.2	Replacement of contact forces	96
18.3	Applicability of equivalence/replacement?	96
18.4	Replacement of forces associated with constraints	97
19	Encyclopedia of applied force and torque	99
19.1	Weight, mass, and gravity	99
19.2	Uniform gravity on a particle or body (see appendix for g for planets)	99
19.3	Universal gravitational attraction between two particles	100
19.4	Electrostatic forces and Coulomb's law for charged particles	101
19.5	DC (direct current) permanent magnet motors	101
19.6	Kinetic friction and the Continuous Friction Law	102
19.7	Translational spring between two points	103
19.8	Rotational (torsional) spring	103
19.9	Translational damper between two points	104
19.10	Rotational (torsional) damper	104
19.11	Light linear springs and dampers in <i>series</i>	105
19.12	Light linear springs and dampers in <i>parallel</i>	105
19.13	Viscous damper between two surfaces	106
19.14	Fluid forces (lift and drag)	106
19.14.1	Demo: Experimental determination of drag forces on a coffee filter	107
19.14.2	Closed-form solutions with aerodynamic drag	107
19.15	Baseball aerodynamic lift/drag/torque - a case study in philosophy	108
19.16	Optional: Proofs for spring and gravity forces	111
19.16.1	Optional: Proof of light linear springs and dampers in <i>series</i>	111
19.16.2	Optional: Proof of light linear springs and dampers in <i>parallel</i>	111
19.16.3	Optional: Proof for uniform gravity on a body or set of particles	111
20	Newton/Euler dynamics	113
20.1	Newton's law of motion for a particle	113
20.2	Newton's equation for a system	113
20.3	Linear momentum principle	114
20.4	Angular momentum principle	114
20.5	Euler's equation for a rigid body with simple angular velocity	114
20.6	Euler's equations for a rigid body (3D)	115
20.7	Optional: Lagrange's equations of the second kind	115
20.8	Impulse/momentum	116
20.9	Conservation of linear/angular momentum	116
20.10	Optional: Proof: Linear momentum principle ($\vec{F}^S = m^S * {}^N\vec{a}^{S_{cm}}$)	117
20.11	Optional: Proof: Angular momentum principle	117

21 Dynamics with MG road-maps	121
21.1 MG road-maps for efficient statics and dynamics	122
21.1.1 MG road-map: Projectile motion (2D)	123
21.1.2 MG road-map: Rigid body pendulum (2D)	123
21.1.3 MG road-map: Inverted pendulum on cart (x and θ) (2D)	123
21.1.4 MG road-map: Rotating rigid body (3D)	124
21.1.5 MG road-map: Bridge crane equations of motion (2D)	124
21.1.6 MG road-map: Particle on spinning slot (2D)	124
21.1.7 MG road-map: Motion of a chaotic double pendulum (3D)	125
21.1.8 MG road-map: Particle pendulum (2D) – angle and tension	125
22 Power and work	127
22.1 Power/energy-rate principle (for any system S in a Newtonian frame N).	127
22.2 Power of a force, set of forces, or torque on a rigid body	129
22.3 Forces that do not contribute to power (workless forces)	129
22.4 Power/energy-rate to size a torque-motor	129
22.5 Power/energy-rate for a commercial spring scale	130
22.6 Definition of work with an integral or differential equation	131
22.7 Work/energy principle	132
22.8 Optional: Proofs with power	132
22.8.1 Optional: Proof of the power/energy-rate principle	132
22.8.2 Optional: Proof for power of a set of forces (or <u>torque</u>) on a rigid body B	133
23 Potential energy	135
23.1 Work/Energy principle	135
23.2 Conservation of mechanical energy (special case of work/energy)	135
23.3 Potential energy - a special type of work for “conservative” forces	136
23.3.1 Potential energy for a constant force or a uniform gravitational field	136
23.3.2 Potential energy for central forces (e.g., springs and inverse-square-law gravity)	137
23.3.3 Potential energy for a simple rotational spring	137
23.4 Optional: Proofs of potential energy	138
23.4.1 Optional: Proof of potential energy for a constant force in reference frame N	138
23.4.2 Optional: Proof of potential energy for a central force	138
24 MIPS: Classic particle pendulum	139
24.1 Modeling the classic particle pendulum	139
24.2 Identifiers for the classic particle pendulum	140
24.3 Physics: Equations of motion of the classic particle pendulum	141
24.3.1 $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ for the classic particle pendulum	141
24.3.2 Angular momentum principle for the classic particle pendulum	142
24.3.3 Euler’s rigid body equation for the classic particle pendulum	142
24.3.4 Kinetic energy for the classic particle pendulum	143
24.3.5 Power/energy-rate principle for the classic particle pendulum	143
24.3.6 Conservation of mechanical energy for the classic particle pendulum	143
24.3.7 Lagrange’s method for the classic particle pendulum	144
24.4 Solution of the classic particle pendulum ODE	144
24.4.1 Numerical solution of pendulum ODE via MotionGenesis and/or MATLAB [®]	144
24.4.2 Optional: Exact (closed-form) solution of the classic particle pendulum ODE	144
24.4.3 Simplification and analytical solution of the classic particle pendulum ODE	145
24.5 Interpretation of results for the classic particle pendulum	145

25 Example: Inverted pendulum on cart	147
25.1 Kinematics (space and time)	147
25.2 Rotation matrix, angular velocity, angular acceleration	148
25.3 Position vectors, velocity, acceleration	148
25.4 Forces, moments, and free-body diagrams (2D)	149
25.5 Mass, center of mass, inertia (required by dynamics)	150
25.6 Newton/Euler laws of motion for A and B separately (inefficient)	150
25.7 Dynamics of a rigid body with simple angular velocity (special 2D case)	150
25.8 Optional: Angular momentum principle (2D alternative to Section 25.7)	151
25.9 Equations of motion via MG road-maps/D'Alembert (efficient)	151
25.10 Matrix form of equations of motion (for solution, controls, ...)	151
Homework	152
Homework 1: Vectors (basis independent)	155
Homework 2: Vector addition and dot/cross-products (with basis)	161
Homework 3: Optional/Advanced: Position vectors and geometry	169
Homework 4: Vector bases and rotation matrices	173
Homework 5: Vector differentiation	185
Homework 6: Angular velocity and angular acceleration	193
Homework 7: Velocity and acceleration	201
Homework 8: Constraints	217
Homework 9: Particle mass, linear/angular momentum, kinetic energy, etc.	223
Homework 10: Moments and products of inertia, dynamic Celt (rattleback)	237
Homework 11: Rigid body: Momentum, energy, and equations of motion.	249
Homework 12: Optional: Forces, force models, and statics	263
Homework 13: Optional: Moments, torques, and static equilibrium	271
Homework 14: $\vec{F} = m \vec{a}$ and FBDs for translating mechanical systems	283
Homework 15: $\vec{M} = \frac{d\vec{H}}{dt}$ and FBDs for rotating rigid bodies	289
Homework 16: Optional: Power, work, potential energy, conservation of energy	303
Homework 17: Optional: MIPS simulation project	313
Appendices and index	315
Appendix: Mass and geometry properties of common objects	315
Appendix of constants	320
Summary of equations	321
Bibliography	325
Table of customizable notation at: www.MotionGenesis.com \Rightarrow Textbooks \Rightarrow Resources	i