Courtesy NASA

Chapter 1

Math review

Math is a foundation for science, medicine, engineering, construction, and business. Math provides ***concepts*** (pictures, words, ideas), ***calculations*** (mathematical operations, symbols, equations, definitions), and ***context*** (situations in which the concepts and calculations are relevant and useful). More generally, math is a language and set of rules that helps us count, quantify, calculate, manipulate, relate, define, extrapolate, and abstract "stuff".[1] Advances in math depend on pictures[2] words, symbols, equations, and precise ***definitions***. For example, consider the following ***definition*** of π.

Object	Example	Approximate age of human comprehension
Picture		Toddlers
Spoken word	"circle"	Pre-school
Written word	"circle", "diameter", "circumference"	Elementary school
Symbol	d for diameter, c for circumference	Middle school
Equation	$c = \pi \, d$	Middle/high school
Definition	$\pi \triangleq \frac{c}{d}$	($\triangleq$ means "***defined as***") University

1.1 Unit systems - SI and U.S.

Units quantify the measurement of "stuff". The ***SI*** system was first adopted by France on December 10, 1799 and is now used in all countries other than Liberia, Myanmar, and the United States.

The ***SI*** (metric) system uses a base-10 number system and decimals (not fractions) and has measures for length, mass, force, temperature, time, etc.

Countries using SI units (green) vs. U.S. units (grey).

NIST (National Institute of Standards & Technology) defines physical constants and conversion factors (e.g., conversion from U.S. to SI units).

Length	1 inch $\triangleq$ 2.54 cm		
Mass	1 lbm $\approx$ 0.45359237 kg	1 slug $\approx$ g_{US} lbm	$g_{\text{US}} \approx 32.17404855643044$
Force	1 Newton $\triangleq 1 \frac{\text{kg m}}{\text{s}^2}$	1 lbf $\triangleq 1 \frac{\text{slug ft}}{\text{s}^2}$	1 lbf $\triangleq g_{\text{US}} \frac{\text{lbm ft}}{\text{s}^2}$

Inaccurate unit conversions have caused ***many*** failures. In 1999, NASA lost a $125,000,000 Mars orbiter because one engineering team used SI units while another used U.S. units. In 1983, an Air Canada Boeing 767 ran out of fuel mid-flight because of a kg to lbm unit conversion.[3]

[1]For example, the "idea" of ***value*** (answering **"how much something is worth"**) is quantified through money.

[2]***Art*** is ***not*** reserved for the sophisticated and educated with knowledge and historical context for art. Appreciation for shapes, colors, and emotional expression in art is available to humans on a basic (primitive/subconscious) level.

[3]Ironically, Thomas Jefferson helped United States become the first country (in 1792) to use a monetary system with decimals and a base-10 number system. The historical origin of U.S. units trace to 2575 B.C. and through ancient Egypt,

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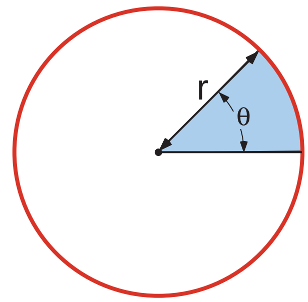
1.2 Geometry: Ancient Euclid and modern vectors

Geometry is the study of figures (e.g., lines, curves, surfaces, solids) and their properties (e.g., distance, area, and volume). Geometry plays a central role in construction, farming, engineering, medicine, science, etc.

Many students spend 2^+ years learning ancient ($\approx$300 BC) 2D Euclidean geometry and trigonometry (trigonometry translates to "triangle measurement"). The invention of ***vectors*** (Gibbs $\approx$ 1900 AD) and its easy-to-use vector addition, dot-products, and cross-products have **greatly simplified** 2D and 3D geometry. Unfortunately, few instructors teach geometry or trigonometry with vectors.

1.3 Circles and their properties



The ratio of ***any*** circle's ***circumference*** to its ***diameter*** is the number[a]

$$\pi \;=\; 3.1415926535897932384626433832795028841971693993751058 2\ldots$$

π is called an "***irrational number***" because it is not a whole number or fraction, nor does it terminate or repeat. It is chaotic, disorderly, and has no discernible pattern (π has been memorized to $67,890^+$ digits).

The ***arc-length*** of a portion of the circle's periphery and the ***area*** of a wedge of the circle can be calculated in terms of the circle's ***radius*** r and the ***angle*** θ as shown right.[6]

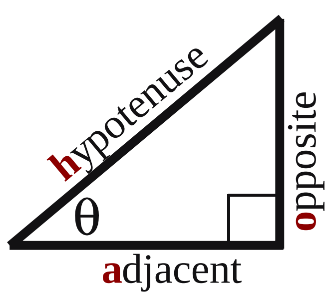
Arc-length	$= \theta\, r$	Area of wedge	$= \frac{\theta}{2}\, r^2$
Circumference	$= 2\, \pi\, r$	Area of circle	$= \pi\, r^2$

[a] The symbol ***π*** was popularized by Euler circa 1750, but the value $\pi \approx 3.14$ was known in Egypt circa 3000 BC.[4]

1.4 Triangles and ratios of their sides (sine, cosine, tangent)

A triangle ("three angles") is a 3-sided planar geometric shape widely used in construction, engineering, and science.

<ins>S</ins>oh<ins>C</ins>ah<ins>T</ins>oa is a ***mnemonic*** for memorizing the definitions of ***<ins>S</ins>ine***, ***<ins>C</ins>osine***, and ***<ins>T</ins>angent*** (ratios of various sides of a right triangle).



$$\sin(\theta) \;\triangleq\; \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos(\theta) \;\triangleq\; \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan(\theta) \;\triangleq\; \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin(\theta)}{\cos(\theta)} \tag{1}$$

The ***Pythagorean theorem*** in equation (2) relates lengths of sides of a right triangle. Combining the definitions of $\sin(\theta)$ and $\cos(\theta)$ with the Pythagorean theorem gives the second relationship to the right.

$$\text{hypotenuse}^2 = \text{adjacent}^2 \;+\; \text{opposite}^2 \qquad \sin^2(\theta) \;+\; \cos^2(\theta) \underset{(1)}{=} 1 \tag{2}$$

Note: Numbers under = refer to equation numbers, e.g., (1) under = means "refers to equation (1)".

Greece, and Rome. The ***inch*** approximates the width of a man's thumb. The ***foot*** approximates a foot <ins>with</ins> shoe and was somewhat standardized in England to King Henry I. The ***mile*** "mille passus" is 1000 paces (2 steps) of a Roman soldier. An Australian study found that switching from British units to metric units freed $\frac{1}{2}$-year in science education. U.S. lawmakers have consistently failed to legislate changes in federal systems, e.g., road signs, NASA, DOD, and NSF.

[4] An ***angle*** involves two lines (or vectors) and is measured in radians or degrees. A radian is the ratio of the arc-length of a circle to its radius. A degree is an archaic unit of angle measurement based on the ancient Babylonian year which had 360 days (12 months $*$ 30 days). Each degree represents one day of Earth's travel about the sun and the degree symbol's circular appearance $^\circ$ is a reminder that 360° measures the Earth's quasi-circular travel around the sun.

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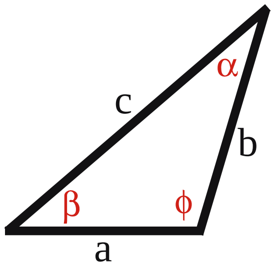
1.4.1 Properties of sine and cosine and useful trigonometric formulas



Law of cosines	Euclid of Alexandria Egypt, 300 BC
Law of sines	Ptolemy of Alexandria Egypt, 100 AD
Addition formula for sine	Ptolemy of Alexandria Egypt, 100 AD

$$c^2 = a^2 + b^2 - 2\,a\,b\,\cos(\phi) \qquad \textit{Law of cosines} \tag{3}$$

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\phi)}{c} \qquad \textit{Law of sines} \tag{4}$$

$$\sin(-\alpha) = -\sin(\alpha) \tag{5}$$

$$\cos(-\alpha) = \cos(\alpha) \tag{6}$$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha) \qquad \textit{Addition formula for sine} \tag{7}$$

$$\cos(\alpha+\beta) \underset{(7)}{=} \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \qquad \textit{Addition formula for cosine} \tag{8}$$

1.4.2 Optional: Sine and cosine as functions (Euler, circa 1730)

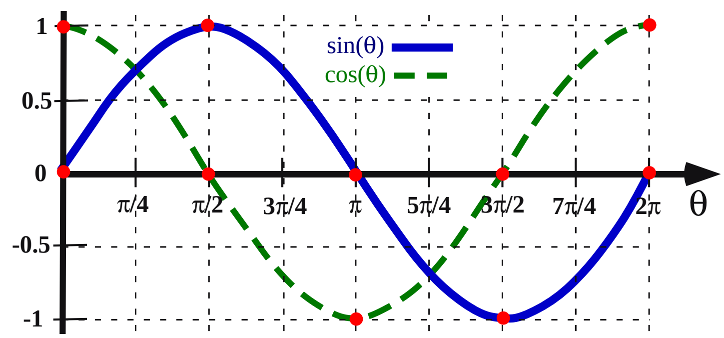
Euler's interpretation of ***cosine*** and ***sine*** as ***functions*** (not just ratios of sides of a triangle) was a major advance for trigonometry and functions.[5]

$$\cos(\theta) \triangleq \frac{\textbf{\textit{a}}\text{djacent}}{\textbf{\textit{h}}\text{ypotenuse}}$$

Cosine function

$$\sin(\theta) \triangleq \frac{\textbf{\textit{o}}\text{pposite}}{\textbf{\textit{h}}\text{ypotenuse}}$$

Sine function



Sine and cosine are two of the most important functions in mechanics

[5] The Babylonians used properties of right triangles for thousands of years before their proofs by Pythagoras of Samos [≈500 BC]. The definitions of ***sine***, ***cosine***, and ***tangent*** as ratios of sides of a right triangle predate 140 BC when the Greek Hipparchus made sine, cosine, and tangent tables. Euler's interpretation of sine, cosine, and tangent as ***functions*** was a breakthrough for math. Gibb's invention of vectors (≈1900 AD) significantly simplified 3D geometry and trigonometry and proofs of ***law of cosines***, ***law of sines***, and ***sine addition formula***, from which other trigonometric formulas are derived [***cosine addition formula***, ***half-angle formulas***, ***double-angle formulas***, etc.].

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